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MOMENT CAPACITY OF STEEL FIBER REINFORCED CONCRETE BEAMS

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Using discrete steel fibers in reinforced concrete improves some mechanical properties of concrete such as the tensile strength, shear strength, toughness and the flexural strength. In this research work, the effect of adding steel fibers on the moment capacity of steel fiber reinforced concrete (SFRC) is studied and new relationships accounting for the variations in the modulus of rupture resulted from the added steel fibers are presented. The relations are expressed in terms of the fiber content, fiber aspect ratio and the concrete compressive strength and then employed in predicting the moment capacity of SFRC beams.

The applicability of the proposed formula in predicting the moment capacity of SFRC beams is checked by analyzing fifty-two beams of different fiber content, concrete strengths and densities ranging from normal to light-weight. The comparisons of the predictions agreed with the corresponding experimental values.

KEYWORDS: *Concrete; Flexural Strength; Steel fibers; Reinforced Concrete; SFRC*

INTRODUCTION

Steel fiber reinforced concrete (SFRC) is primarily made of concrete and discrete steel fibers. The role of the randomly oriented, discontinuous fibers is to bridge across the cracks that develop in concrete. So, existence of steel fibers in concrete members improves the tensile strength, ductility and toughness in addition to the flexural strength. It is also recognized that replacing shear reinforcement by steel fibers enhances the shear strength of such members. Since the fibers are randomly distributed, the improvements in flexural strength resulting from the steel fibers are not large enough to fully substitute the continuous bars in flexural reinforced elements. So, a lot of researches have been carried out, over the past thirty years, on combining the SFRC and steel bars to achieve optimum conditions in flexural elements through using steel fibers together with the longitudinal steel bars [1].

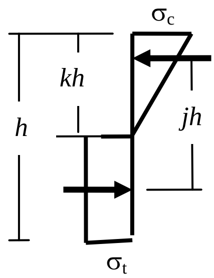
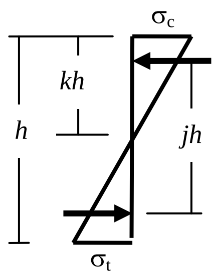
RESEARCH SIGNIFICANCE

This work studies the effect of fiber content on the modulus of rupture of SFRC members, which in role affects the flexural capacity of such composites. The current research is directed to obtain a new formula capable to predict the flexural capacities of SFRC beams reinforced with longitudinal steel bars.

FLEXURAL CAPACITY OF SFRC BEAMS

Several methods that predict the flexural strength of SFRC beams of no longitudinal steel bars are found in the literature [2-5]. Most of them basically use the direct tensile strength, the split strength or the compressive strength in estimating the flexural capacity of SFRC members. Some of such methods are listed in Table 1.

Table1: Summary of methods that predict the flexural strength of SFRC.

Method	***Stress distribution***					
	kh	***jh***	σ_c	σ_t	***Tensile force, T***	
Dwarkanath @ Nagaraj[2]	*0.3h*	*0.55h*		$1.08\ \sigma_{dt}$	$T=b.h\ (1-k)\sigma_t$	
Swamy & Mang[3]	*0.25h*	*0.54h*		$1.0\ \sigma_{dt}$		
Hannant[4]	*0.33h*	*0.55h*		$0.6\ \sigma_{sp}$		
Ghalib[5]	*0.5h*	*0.67h*	$\sigma_c=\sigma_t$	$1.0\ f_r$	$T=0.5b.h(1-k)\sigma_t$	

σ_c : Compressive stress *σ_t : Tensile Strength*
σ_{dt} : Direct tensile strength *σ_{sp} : Split tensile strength*
f_r : Modulus of rupture ***M_p =(C or T) (jh)***
b: width of beam cross-section. *h: total depth of beam cross section*
d: effective depth of the beam section

On the other hand, the design considerations for steel fiber concrete, ACI 544.4R [6] suggests the nominal moment capacity of SFRC section shown in Fig. 2 to be given by the following equations

$$M_n = A_s . f_y (d - \frac{a}{2}) + 0.5(h + e - a).b.(h - e).\sigma_t \qquad (1-a)$$

$$e = [0.003 + \varepsilon_t\,(fibers)]\,\frac{c}{0.003} \qquad (1-b)$$

$$\sigma_t = 0.007772\,v_f\,(l/d).F_b \qquad (1-c)$$

Where:

σ_t is the tensile strength of the fibrous concrete.
F_b is the bond efficiency of the fiber which varies from 1 to 1.2 depending on the fiber characteristics
$A_s.f_y$ is tensile force in the steel reinforcement.
v_f and *l/d* are the fiber volumetric and aspect ratios, respectively

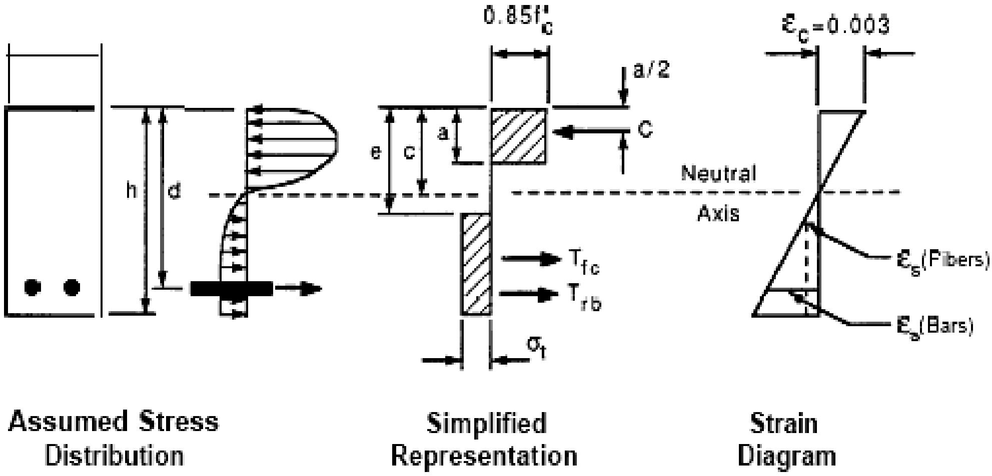


Fig. 1: Design assumptions of reinforced concrete beams containing steel fibers [6].

The current research work, assumes the moment capacity of SFRC beams with steel bars to be composed of two parts: the first is the moment capacity of the conventional reinforced concrete (M_p) and the second is the moment caused by the increase in the modulus of rupture of the SFRC beams resulted from the added steel fibers (M_f). Referring to Fig. 2b and 2c, the two parts can be expressed as follows:

$$M_f = \Delta f_r .b.h(0.67h)/4 = 0.167\Delta f_r .b.h^2 \qquad (2-a)$$

$$M_c = A_s . f_y (d - \frac{a}{2}) \qquad (2-b)$$

$$M_p = M_c + M_f \qquad (2-c)$$

Where

$$a = A_s . f_y /(0.85 f'_c .b)$$

A_s and f_y are the area and the yield strength of longitudinal steel bars, respectively.
Δf_r is the increase in the modulus of rupture caused by the presence of steel fibers.

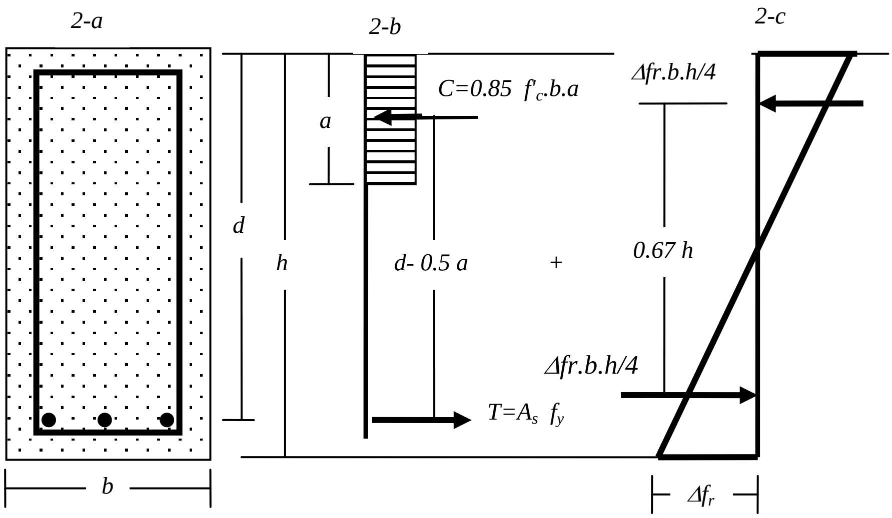


Fig. 2: Flexural analysis of SFRC beams with longitudinal steel.
(a) Beam cross-section (b) Reinforced concrete contribution (c) Fiber contribution.

In order to investigate the effect of the fiber reinforcement index on the modulus of rupture, 40 tests of the modulus of rupture, are gathered from literature [7-11] and analyzed using the regression analysis. The analyzed specimens, listed in Table 2, have different concrete densities, concrete grades, fiber volumetric ratios and different fiber aspect ratios. The increase in the modulus of rupture, Δf_r that represents the difference between the modulus of rupture of SFRC, f_r and that of plain (no fiber) concrete, f_p is reported in Table 2. The ratio $\Delta f_r / f_p$ is plotted against $v_f (l/d)$ as shown in Fig. 3 and the following relationships are obtained using the linear and nonlinear regression

$$\Delta f_r = [0.61(v_f \cdot l/d)] f_p \quad \text{(Linear regression)} \qquad (3\text{-a})$$

$$\Delta f_r = [0.21(v_f \cdot l/d)^2 + 0.36(v_f l/d)] f_p \quad \text{(Nonlinear regression)} \qquad (3\text{-b})$$

Where the modulus of rupture, f_p of the plain concrete is estimated by the ACI 318M-05 Building Code [17] such that $f_p equals\, 0.7\lambda\sqrt{f_c^{'}}$ where the parameter λ equals 1 for normal-weight concrete (NWC), 0.85 for sand light-weight concrete (LWC) and 0.75 for all LWC.

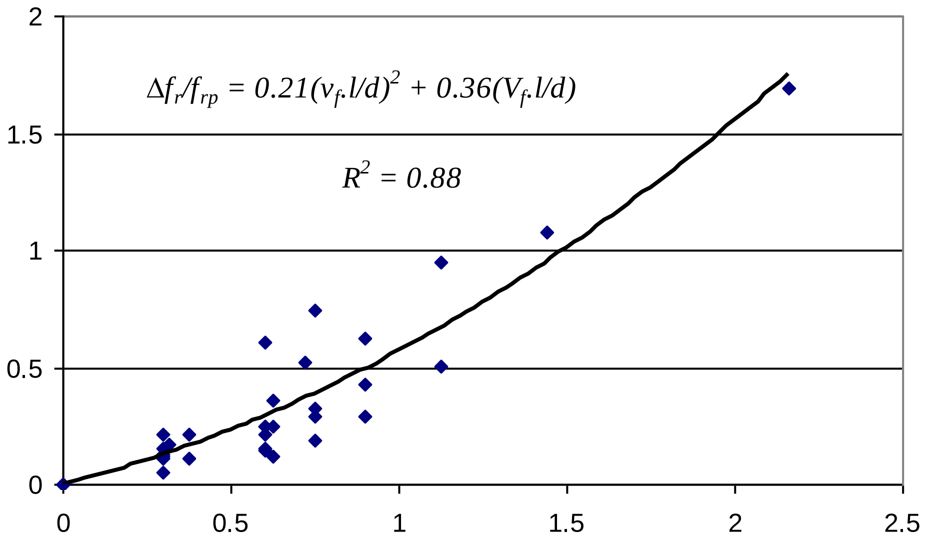
The applicability of the new method in predicting the flexural capacity of SFRC sections is checked by analyzing 52 beams tested experimentally by different researchers [12-15]. The analysis is conducted using Eqn 2 in connection with Eqn.3. Table 3 lists the basic information of the analyzed beams. The beams have different fiber content, different densities (normal weight and light-weight concrete) and their compressive strengths range from 16 to 90.5 MPa. The predicted moment capacities, M_p of the analyzed beams are compared with the corresponding experimental results,

M_{exp} and plotted on Fig. 4. The plot of the predicted versus the experimental moment capacity shows acceptable predictions for the moment capacities using the proposed approach for both the NWC as well as the LWC beams.

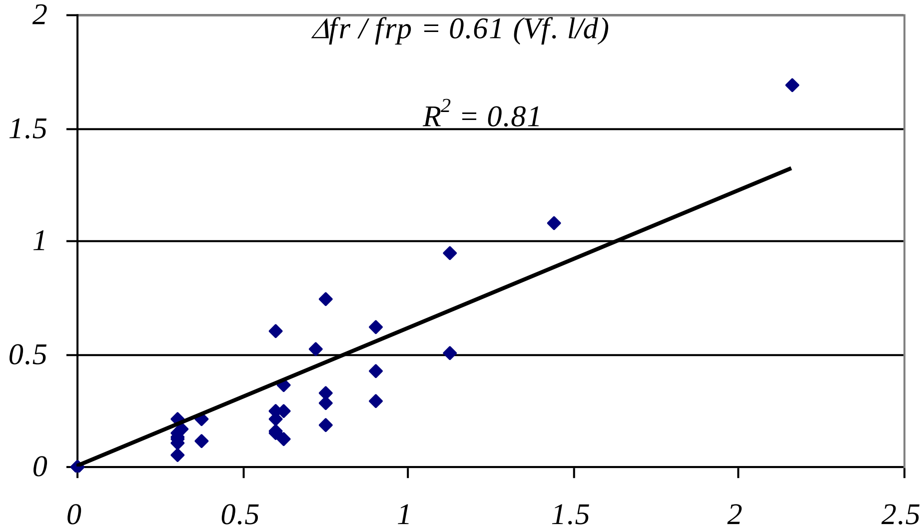
Table 2: Modulus of rupture of SFRC composites.

1 Author	2 v_f%	3 l/d	4 F'_c [MPa]	5 f_p [MPa]	6 f_r [MPa]	7 $\Delta f_r/f_p$
Dwarkanath[2] (1991)	0		-	4.36	-	0
	1	72	-		6.64	0.522
	2	72	-		9.06	1.08
	3	72	-		11.74	1.69
Abdel-Wahab[7] *(1997) (NSC)*	0		19.57	2.9	-	0
	0.5	62.5	22.63		3.4	0.17
	1	62.5	22		3.95	0.36
	1	62.5	19.84		3.63	0.25
	1	62.5	28.48		3.25	0.12
Ashour and Wafa[13] (1993) *(HSC)*	0		86.14	8.93		0
	0.5	75	87.11		9.94	0.11
	1	75	88.11		10.6	0.19
	1.5	75	90.5		13.64	0.53
Ashour and Wafa[14] (1997)	0	75	80.19	8.97		0.0
	0.75	75	80.87		9.84	0.10
	1.5	75	82.32		14.05	0.57
Campione and Papia[8] *(2001) (LWC)*	0		20.6	4.1		0
	0.5	60	20.6		4.3	0.05
	1	60	20.6		4.7	0.15
	2	60	20.6		5.85	0.43
Rjoub & Hunaiti[9] *(2002) (NWC)*	0	-	25.1	3.73	-	0
	0.5	75	25.0		4.53	0.21
	1	75	27.1		4.8	0.29
	1.5	75	29.6	4.1	8	0.95
Bani Yasin[10] (2004) (cubes 100mm)	0		23.74	6.96		0
	0.5	60	25.13		7.88	0.13
	1	60	26.6		8.45	0.21
	1	75	26.17		9.25	0.33
	0		51.72	9.26		0
	0.5	60	54.83		11.2	0.21
	1	60	56.12		14.8	0.60
	1	75	55.58		16.06	0.73
Aqayleh[11] *(2004) (cubes 100mm)*	0	60	21.99	6.33		0.00
	0.5	60	22.43		7.03	0.11
	1	60	23.48		7.32	0.16
	1.5	60	24.94		8.17	0.29
	0	60	52.85	9.13		0.00
	0.5	60	54.16		10.51	0.15
	1	60	55.22		11.43	0.25
	1.5	60	56.46		14.8	0.62

v_f, f_r, f_p, l/d and Δf_r are same as in Table 1
@ f'_c is obtained from cubes (100x100x100mm) [$f'_c = 0.78 f_{cu}$)



a. Effect of fiber reinforcement index on the modulus of rupture of SFRC [Nonlinear regression].



b. Effect of fiber reinforcement index on the modulus of rupture of SFRC [linear regression].

Fig. 3: Effect of fiber reinforcement index on the splitting strength of SFRC:
a. Nonlinear regression. b. Linear regression

Table 3: Flexural strength of SFRC beams (with longitudinal bars)

Author (Concrete Type)	Beam #	Cross section dimensions			Material Properties					M_{exp} [kN.m]	M_p [kN.m]
		b [mm]	d [mm]	h [mm]	f'c [MPa]	A_s [mm^2]	f_y [MPa]	V_f	l/d		
Abu Hijab[12] (NWC) 1999	N1	200	208	250	30.46	308	415	0	75	27.34	25.01
	N2	200	208	250	30.46	308	415	0	75	26.75	25.01
	N3	200	208	250	30.38	308	415	0	75	28.65	25.00
	N4	200	208	250	30.38	308	415	0	75	24.26	25.00
	N5	200	258	300	30.63	308	415	0	75	36.00	31.41
	N6	200	258	300	30.63	308	415	0	75	35.82	31.41
	N7	200	208	250	30.46	308	415	0.5	75	28.99	27.14
	N8	200	208	250	30.46	308	415	0.5	75	29.43	27.14
	N9	200	208	250	30.38	308	415	0.5	75	29.44	27.13
	N10	200	208	250	30.38	308	415	0.5	75	26.23	27.13
	N11	200	258	300	30.63	308	415	0.5	75	37.20	34.48
	N12	200	258	300	30.63	308	415	0.5	75	36.72	34.48
	N13	200	208	250	30.46	308	415	1	75	30.44	29.27
	N14	200	208	250	30.46	308	415	1	75	30.28	29.27
	N15	200	208	250	30.38	308	415	1	75	30.77	29.26
	N16	200	208	250	30.38	308	415	1	75	28.85	29.26
	N17	200	258	300	30.63	308	415	1	75	37.68	37.56
	N18	200	258	300	30.63	308	415	1	75	37.80	37.56
Abu Hijab[12] (LWC λ=0.85) 1999	L19	200	208	250	10.04	308	415	0	75	22.50	21.80
	L20	200	208	250	10.04	308	415	0	75	22.70	21.80
	L21	200	208	250	10.04	308	415	0	75	25.20	21.80
	L22	200	208	250	10.04	308	415	0	75	25.00	21.80
	L23	200	158	200	10.04	308	415	0	75	16.10	15.41
	L24	200	158	200	10.04	308	415	0	75	16.00	15.41
	L25	200	208	250	13.70	308	415	0.5	75	27.00	24.29
	L26	200	208	250	13.70	308	415	0.5	75	27.50	24.29
	L27	200	208	250	13.70	308	415	0.5	75	28.80	24.29
	L28	200	208	250	13.70	308	415	0.5	75	29.00	24.29
	L29	200	158	200	13.70	308	415	0.5	75	18.20	17.46
	L30	200	158	200	13.70	308	415	0.5	75	18.80	17.46
	L31	200	208	250	15.44	308	415	1	75	28.80	26.05
	L32	200	208	250	15.44	308	415	1	75	29.00	26.05
	L33	200	208	250	15.44	308	415	1	75	30.00	26.05
	L34	200	208	250	15.44	308	415	1	75	31.10	26.05
	L35	200	158	200	15.44	308	415	1	75	20.00	18.73
	L36	200	158	200	15.44	308	415	1	75	19.60	18.73
Ashour & Wafa[14] 1993	B1	170	265	300	86.14	628	437	0	75	105.65	69.70
	B2	170	265	300	87.11	628	437	0.5	75	116.46	74.14
	B3	170	265	300	88.11	628	437	1	75	122.81	78.63
	B4	170	265	300	90.53	628	437	1.5	75	130.38	83.33
	B5	170	265	300	86.14	628	437	0	75	104.61	69.70
	B6	170	265	300	87.11	628	437	0.5	75	115.77	74.14
	B7	170	265	300	88.11	628	437	1	75	118.52	78.63
	B8	170	265	300	90.50	628	437	1.5	75	120.82	83.33

Ashour	B1-0	100	125	150	80.19	157	412	0	75	8.73	7.78
et al[15]	B1-0.75	100	125	150	80.87	157	412	0.75	75	9.60	8.72
(1997)	B1-1.5	100	125	150	82.32	157	412	1.5	75	10.95	9.68
	BN1	177.8	324	381	32.75	1019	448.2	1.75	100	169.17	148.15
Craig[16]	BN2	177.8	324	381	40.34	1019	317.2	1.75	100	146.12	119.74
1987	BN3	177.8	324	381	34.48	1019	317.2	1.75	100	130.07	116.50
	BHS4	203.2	254	304.8	54.47	1019	448.2	0	100	105.44	104.95
	BHS5	203.2	254	304.8	68.95	1019	448.2	1.75	100	120.58	129.86

NWC: Normal weight concrete *LWC: Light weight concrete.*
HSC: High strength concrete *NSC: Normal strength concrete.*

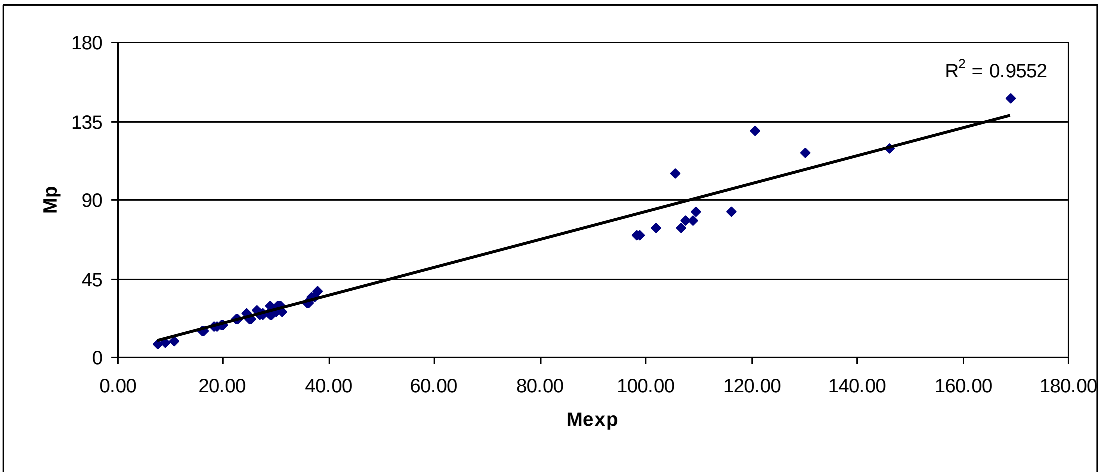


Fig. 4: Predicted versus experimental moment capacities of SFRC beams.

CONCLUSIONS

A method capable for predicting the moment capacity of SFRC beams with longitudinal tension steel bars is presented. The method adds the moment caused by the increase in the modulus of rupture of SFRC beams, in terms of the fiber reinforcement index to that adopted in the conventional RC beams. The comparisons of the experimental flexural moments with the predicted values showed acceptable predictions for both the normal and high strength concrete members whether they are constructed from NWC or LWC.

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مقاومة الأنعطاف للجوائز الخرسانية المسلحة بالألياف

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يُعد استخدام الألياف المعدنية من الوسائل المؤدية لتحسين بعض الخواص الميكانيكية لهذه لخرسانة كمقاومة الشد ومقاومة القص والمتانة ومقاومة الانحناء، لذا جاءت هذه الدراسة لتقييم تأثير استخدام الألياف على مقاومة الانعطاف في الجسور الخرسانية المسلحة بالألياف.

وقد قدم هذا البحث علاقات جديدة أخذت بالحسبان التغير في معامل تمزق الخرسانة نتيجة احتوائها للألياف المعدنية، وقد عُبر عن ذلك بدلالة كل من: محتوى الألياف، ونسبة طولها إلى قطرها، ومقاومة الخرسانة على الأنضغاط كما وُظفت هذه العلاقات في تقدير مقاومة الانعطاف للجوائز الخرسانية المسلحة بالألياف المعدنية.

تم التحقق من قدرة العلاقات المقدمة في هذا البحث على توقع مقاومة الانعطاف بتحليل اثنين وخمسين جائزا وفق العلاقات الجديدة، حيث اختلفت هذه الجوائز فيما بينها في محتوى الألياف، ومقاومة الخرسانة، والكثافة (خرسانة عادية وخرسانة خفيفة) إضافة إلى نسبة الحديد الطولي، وقد أوضحت الدراسة توافقاً جيدا في المقاومة المتوقعة باستخدام العلاقات المقترحة مع القيم التجريبية المناظرة لها.