

## EVALUATION OF USING CONVENTIONAL ANALYTICAL METHODS FOR THE DESIGN OF CYLINDRICAL SHELLS WITH VERTICAL PLATE THROUGH A COMPREHENSIVE 3-DIMENSIONAL ANALYSIS

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**Ahmed Abdel-Raheem Fargaly,**

*Civil Engineer in Assiut University.*

**Aly G. Aly, Fayez k. and Hosny M. S.**

*Professor doctor in civil engineering, Faculty of Engineering Assiut University.*

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*Two-dimensional structures like plates and cylindrical shells must be analyzed in the two directions simultaneously to get a true analysis of these structures, but in traditional methods these structures are analyzed by solving each direction alone as one-dimensional structures without taking into consideration the effect of the other direction. This approximation in analysis tends to give big differences in deformations, displacements, and straining actions. Due to theory of plasticity, the plane strain in X-direction tends to contribute of strain in Y-direction.*

*To make a fruitful comparison between spatial and traditional methods one model solved in textbooks was taken into consideration. This model is a cylindrical shell solved by spatial method with the same conditions of the solved traditional method model, but with 3-d model. The straining actions of two methods were compared. Normal force in the spatial method was bigger than traditional method by 4 times, the transverse bending moments in traditional method were much bigger than corresponding values in spatial method by values ranging between 40-140%, and the longitudinal normal stresses in traditional method were bigger than spatial method in compression by 40% and in tension by 100%.*

*This proved that traditional method overestimates the straining action and stresses compared to the spatial method based on 3 dimensional analyses.*

**KEYWORD:** shells - spatial analysis – finite element.

### 1. INTRODUCTION

In the last century it was logic to seek simple approaches for solving complicated structures, one of which is shell structure. This was due to the lack of modern technologies for dealing with sophisticated structures.

Recently the appearance of digital computers with very large memories encouraged the use of advanced methods like finite element technique and the availability of computer programs as a soft ware to deal with such complicated highly indeterminate structures.

Finite element method has been used to analyze shell structure. The structure has been divided into small shell elements. SAP2000 program is used in the analysis. The model provides the actual behavior of the structure as a one-unit spatial structure. This mathematical model is more realistic for explaining the actual behavior of the structure and obtaining more accurate results.

In this study, light is focused on common configuration of cylindrical shells provided with vertical plates. Shell structures are classical and broad topic in reinforced concrete structures. A general structural behavior of cylindrical shell roofs with different span-to-radius ratios is first discussed to distinguish between a “shell” behavior and a “beam” behavior of the structure. In traditional method the following four assumptions are usually made in the computation:

- i. The structure is monolithic.
- ii. The material is elastic, homogeneous and isotropic.
- iii. Plane sections remain plane after deformation.
- iv. Form of cross section is not changed.

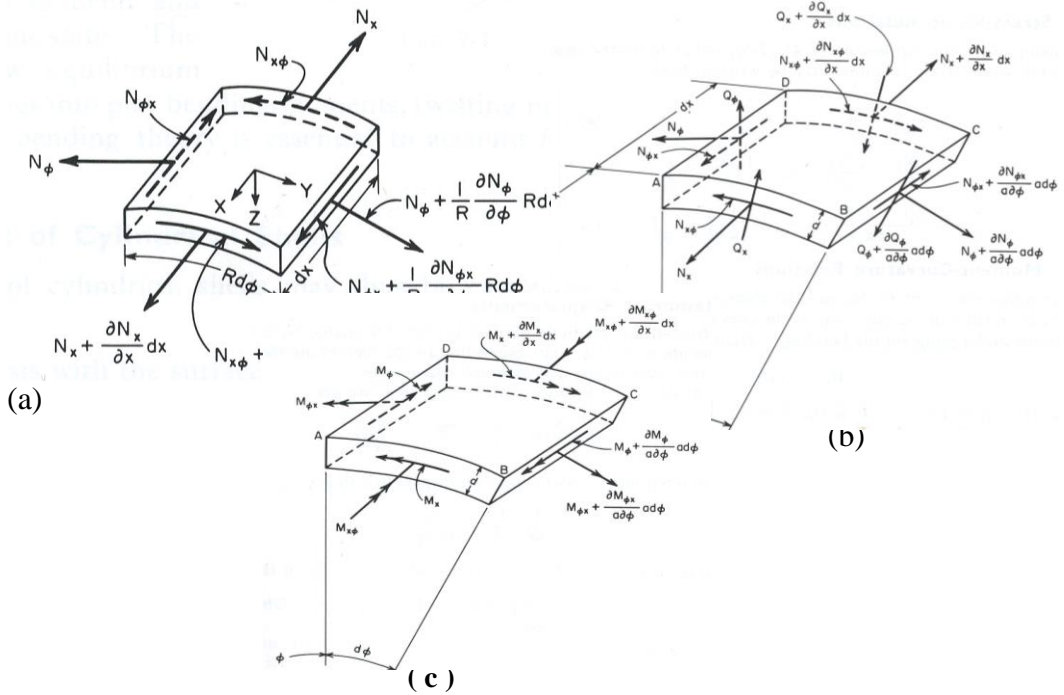
## 2- DESCRIPTION OF TRADITIONAL METHOD PROCEDURE

The shell roofs discussed in the study are within the range of thin elastic shells. Some assumptions are adapted in thin elastic shell theory. The first assumes that the shell must be thin. A shell is considered to be thin if  $d/R \leq 1/20$  [1], where  $d$  and  $R$  are respectively the thickness and the radius of the shell. Mathematically, this assumption implies that the ratio  $d/R$  can be neglected compared to unity in derivation of the thin elastic shell theory [2]. Structurally, this assumption infers a small deflection of shells, a negligible stress normal to the middle surface as reference surface of the shell, and a preservation of normal to the middle surface of the shell. It is noted that the designation of the middle surface as the reference surface assumes a homogeneous material of the shell. The theory deals with uniformly distributed loads only (live load and dead load). The other assumption is an extension of the Bernoulli hypothesis in beam theory [3], which states that plane sections remain plane after deformation. Thus, it assumes that there is no strain in the direction of the normal and that the strain is linearly distributed across the thickness of the shell.

Under these assumptions, the stresses in a shell element (Figure 1) are only normal stresses  $N_x$ ,  $N_\phi$ , shear stress  $N_{x\phi}$  (membrane theory) or might include transverse shear stresses  $Q_x$ ,  $Q_\phi$  and couples  $M_x$ ,  $M_\phi$ ,  $M_{x\phi}$  (bending theory).

Moment in longitudinal direction =  $\frac{wl^2}{8}$ , where  $w = 2 \cdot p \cdot \phi_0 \cdot R + p_b$ ,  $p$  = load/unit area,  $\phi_0$  = half central angle,  $p_b$  = load of beam /m,  $R$  = radius of cylindrical shell

and longitudinal normal stresses =  $\frac{M \cdot y}{I}$ .



**Figure 1:** Stresses in membrane theory (a) and bending theory (b, c)

If the edge beams are supported on a continuous foundation, the shell behaves like an arch, i.e. there are normal stresses only. If the edges are free, both the transverse and long direction will carry the load. This particular kind of shells is often referred to as simply supported shells, and discussed in this study long shell depending on the ratio  $R/l$ . That is why beam method is also valid for solving long shell problem. Shells restrained (with vertical beams) at edges transfer a greater part of the load in the transverse direction than do shells with simply supported cross section especially in shells of considerable lengths. At the restrained cross section shells the transverse moments are given by:

$$M_{\phi} = -\frac{pR^2}{\phi_0^4} (0.0134\phi_0^6 - 0.1519\phi_0^4\phi^2 + 0.2173\phi_0^2\phi^4 - 0.0435\phi^6) \dots\dots(2-a)[4]$$

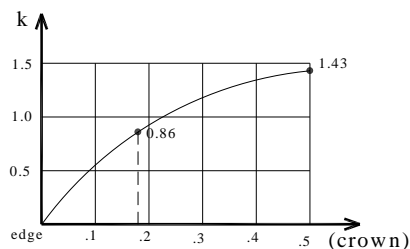
$M_{\phi}$  =Normal force

The normal force in long shells with  $R \leq 10m$  may be estimated from the following relation: -

$$N_{\phi} = k \cdot p \cdot R \dots\dots\dots(2-a),$$

$N_{\phi}$  =Membrane force

Where k is constant as shown in figure 2 [4]



**Figure 2:** k values curve

### 3- DESCRIPTION OF COMPUTER ANALYSIS

The intent was to investigate the differences between an acceptable “exact” model and various gross approximations used in practice. From theory of plates and using FEM in the analysis the stresses is distributed in both directions transverse and longitudinal. A shell section is a set of material and geometric properties that describe the cross section of one or more shell elements. Sections are defined independently of the shell elements, and are referenced during the definition of the elements.

Each shell element has its own element local coordinates system used to define material properties, loads, and output. The axis of local system is denoted 1, 2, and 3 the first two axes lay in the plane of the elements with a specified orientation, the third axis is normal.

The Shell element internal forces (also called stress resultants) are the forces and moments that result from integrating the stresses over the element thickness. These internal forces are:

- Membrane direct forces:

$$F_{11} = \int_{-t/2}^{+t/2} \sigma_{11} dx_3$$

$$F_{22} = \int_{-t/2}^{+t/2} \sigma_{22} dx_3$$

- Plate bending moments:

$$M_{11} = \int_{-t/2}^{+t/2} t \sigma_{11} dx_3$$

$$M_{22} = \int_{-t/2}^{+t/2} t \sigma_{22} dx_3$$

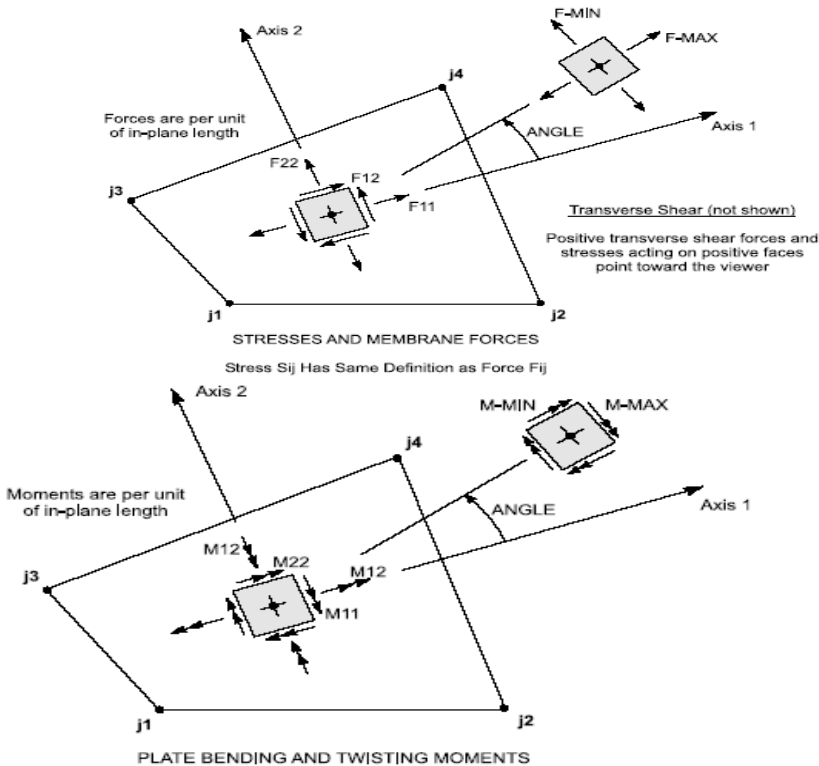
Where  $x_3$  represents the thicknesses coordinate measured from the mid-surface of the element. It is very important to note that these stress resultants are forces and moments per unit of in-plane length. They are present at every point on the mid-surface of the element.

The sign conventions for the stresses and internal forces are illustrated in figure 3. Stresses acting on a positive face are oriented in the positive direction of the element local coordinate axes. Stresses acting on a negative face are oriented in the negative direction of the element local coordinate axes. A positive face is one whose outward normal (pointing away from element) is in the positive local 1 or 2 directions. Positive internal forces correspond to a state of positive stress that is constant through the thickness. Positive internal moments correspond to a state of stress that varies linearly through the thickness and is positive at the bottom. Thus:-

$$\sigma_{11} = \frac{F_{11}}{t} - \frac{12M_{11}}{t b^3} x_3$$

$$\sigma_{22} = \frac{F_{22}}{t} - \frac{12M_{22}}{t b^3} x_3$$

The stresses and internal forces are evaluated at the standard 2-by-2 Gauss integration points of the element and extrapolated to the joints. Although they are reported at the joints, the stresses and internal forces exist throughout the element. (SAP2000 Analysis Reference, 1997) [5].

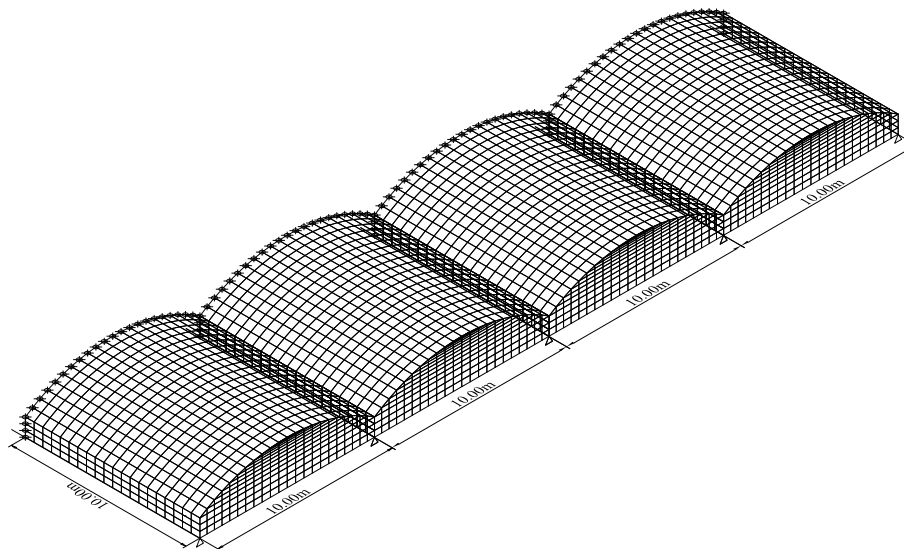


**Figure 3:** Shell element forces and stresses

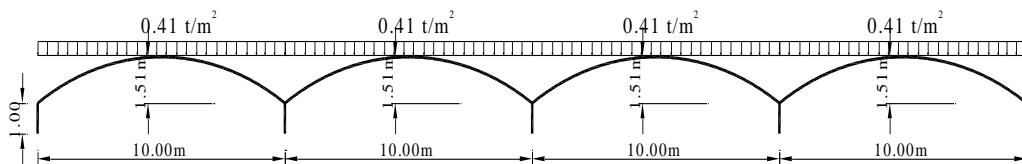
### 4 – COMPARISON BETWEEN TRADITIONAL METHOD AND SPATIAL METHOD THROUGH SOME SOLVED EXAMPLES (CYLINDRICAL SHELL WITH VERTICAL PLATES)

At the same structure geometry the analysis was applied to get the actual differences between the two methods of analysis.

The present study includes structural spatial analysis of 4 bays, cylindrical-shells with vertical plates as shown in figure 4, the geometry and the mesh model of shell, and the same dimension in traditional method example.



(a) 4- bays 3-d shells with vertical plates model

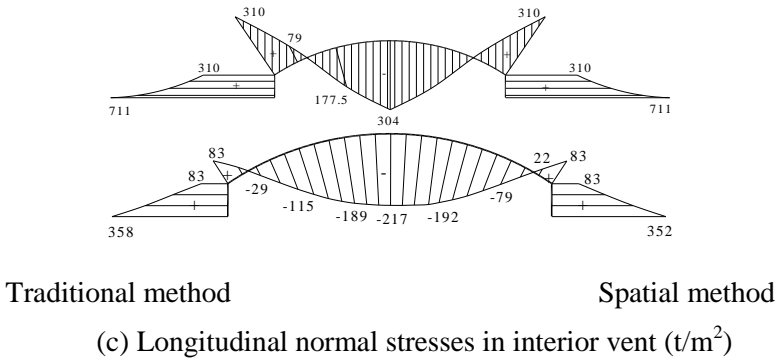
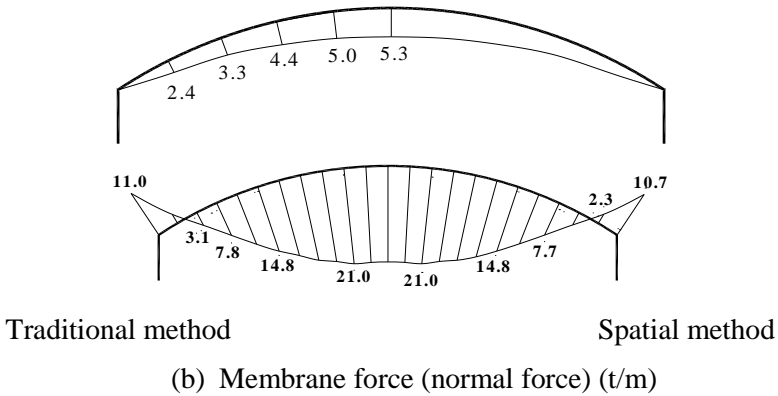
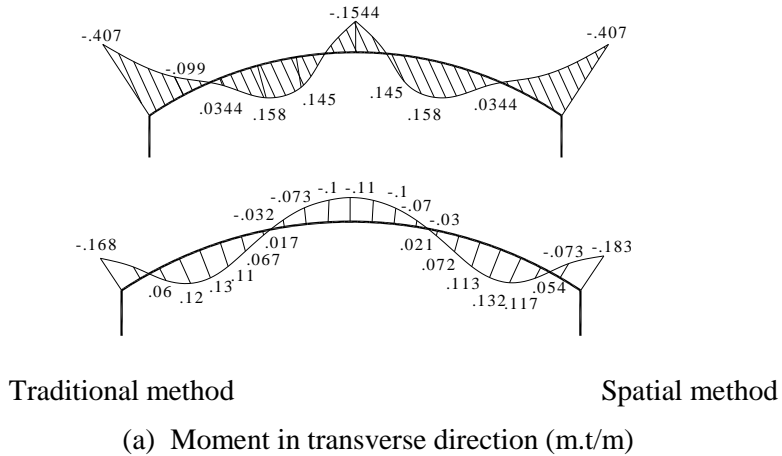


(b) Cross section of shells

**Figure 4:** Geometry, and Loading for 4-bays Shells (spatial method)

As shown in figure 5 (a) the moments were calculated using traditional and spatial methods, the bending moments show a negative values in top of the vertical plates and the top crown of the shell, and positive value between these ranges in both methods. Membrane forces were calculated by traditional and spatial methods as shown in figure 5 (b), showing compression in all the shell cross-section in traditional and spatial methods. Longitudinal normal stresses were calculated in shell and plates in figure 5 (c) by traditional and spatial (the indicated value of stress represents the arrange value of the two stresses at

extreme fibers. The values of shear stresses (S12 and S23) are very small with respect to the values given in S11 and S22 stresses, so it will be neglected). The positive values of stresses appear in the shell crown till 1.40m from the vertical plates, and tension stresses appear in the vertical plates and near it by 1.40m from plates as shown in each methods.



**Figure 5:** comparison of straining action of traditional and spatial methods

## 5- RESULTS ANALYSIS

Figure 5 (a) shows transverse bending moment in both methods, it is clear that values obtained from traditional method are bigger than resulting from spatial three dimensional method by about 140% at the shell beam connection and about 40% at the crown. These values of transverse moments in both methods are dispersed this attribute to the cross section of shell in traditional method is solved as one unit and in spatial method as continuous segments and effect of longitudinal direction.

Figure 5 (b) shows membrane force in both methods, values obtained by 3-dimensional analysis in mid span of the shell are much bigger than corresponding values from traditional method by about 300%. Furthermore, membrane force appeared at the shell beam connection due to 3-diminsional analyses with +ve sign reaching about half the values in mid span of the shell vent. This is attributing to the arch effect of distributing forces in vent and balancing normal stresses in longitudinal direction.

Figure 5 (c) shows normal stress in both methods, concerning the longitudinal normal stresses, the stresses resulting from traditional method are much bigger than those resulting from spatial method by amount ranging between 270% at the top of vertical beam and 100% at the bottom of vertical beam, and in crown of shell the normal stresses in traditional method bigger by 70% than spatial method. This divergence is attribute to the bigger values of normal force appear in spatial method balancing the effect of longitudinal bending moments.

## CONCLUSIONS

Three-dimensional analysis of shells as spatial structures by using the well-known computer program SAP2000 based on finite element technique proved the significant inaccuracy of traditional methods given in textbooks and used by most of structural engineers dealing with every direction separately ignoring the integrity of the structure as a whole.

Therefore it is recommended, in the design of cylindrical shell structures, to analysis them as a spatial structures in three dimensions to avoid uneconomic side effect of using traditional approximate method. However, for engineers who are dealing with conventional methods, reduction factors should be considered for the calculated straining actions. These factors need more case studies before can be adopted.

## REFERENCES

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### تقييم طرق التحليل التقليديه المستخدمه لتصميم الياطات القشرية الاسطوانيه من مقارنتها بالتحليل ثلاثى الابعاد

اعتاد المهندسون الإنشائيون من خلال ما يقدم في الكتب المرجعية تصميم و تحليل المنشآت القشرية الاسطوانيه باستخدام الطريقة التقليدية و التي تتعامل مع كل اتجاه في القشره على حده دون الاخذ في الاعتبار المنشا كوحده واحده في الفراغ مما دفع الباحث الى تحليل بعض نماذج من هذه المنشآت بطريقة العناصر المحددة و باستخدام برنامج sap 2000 في ثلاثة ابعاد و بمقارنة النتائج تبين ان هناك فروقا كبيرة لا يمكن التغاضي عنها و على سبيل المثال فان قيم عزوم الانحناء في الاتجاه العرضى تزيد في الطريقة التقليدية عن نظيراتها في الطريقة الفراغية بمقدار 40% الى 140% طبقا لموضع القطاع.

كما ان قيم الاجهادات العمودية في الطريقة التقليدية تزيد بمقدار 40% في الضغط و 100% في الشد عن نظيراتها في الطريقة الفراغية.

و لذلك فاننا نوصى بضرورة استخدام معامل تصغير للقيم المستنبطة بالطريقة التقليدية لتنتمشى مع القيمه الحقيقه المستنتجه بالطريقه الفراغيه لضمان البعد الاقتصادي في التصميم غير ان هذا المعامل يحتاج لمزيد من دراسة الحالات لامكانية إقراره.