A MATHEMATICAL MODEL FOR PREDICTING THE LIGHT IRRADIANCE INSIDE THE PLANT TISSUE CULTURE VESSELS

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In micro-propagation industry, light irradiance inside the plant culture vessel is one of the most important environmental factors affecting the growth of plantlets inside the culture vessels. Practically it is difficult to measure the light irradiance inside such culture vessel. A mathematical model was developed to predict the light irradiance, the internal transmittance and the transmittance distribution at any plant level along the vessel height. In order to validate the model, three types of culture vessels; GA-7 box vessel, square box vessel, and round vessel were selected to apply the model and the error of the model was analyzed. The mathematical model predicts a Mean Relative Error (MRE) of 0.35 % for square vessel type, 0.26 % For GA-7 box vessel, and 0.68 % for round vessel type. The characteristics of square based vessels and round based vessels were investigated. Also a comparison between these two main categories was investigated. From the comparison it was found that square vessels give irradiance distribution uniformity better than round vessels for zero side wall inclination, but for inclination angle greater than one degree, the uniformity of distribution of irradiance for round vessels is better than that of square vessels.

1. INTRODUCTION

Micro-propagation, the cloning of plants in small vessels, is quickly emerging as an important component of many production industries worldwide. In micro-propagation in vitro industry, the planlets are cultured in a small closed system of culture vessels. Many sets of successive horizontal shelves are arranged in the culture room. Fluorescent lamps are usually used as a light source to provide the light irradiance required for the growth of planlets inside the culture vessels.

Light irradiance inside the culture vessel is considered as one of the most important environmental factors affecting the growth of plantlets (Aitken-Chrisie et al., 1994). There are many factors affecting the light irradiance inside the culture vessel. As mentioned by Fujiwara and Kozai (1995), these factors are the type and number of the lighting sources, the position of the lighting sources, the position of the vessel on the culture shelf, the optical characteristics of the shelf, and the materials and shape of the culture vessels. Kitaya et al. (1994) investigated experimentally the effects of light intensity and direction on the photoautotrophic growth and morphology of potato plantlets in vitro. They found that the shoot length was shorter in the sideward lighting treatment than in the downward lighting treatment at different values of light intensity.

Many trials have been done to measure and predict the light irradiance inside the culture vessels because of its important role in the growth of the palnlets inside the

culture vessels. It is also important to develop a lighting system which can provide suitable light irradiance inside the culture vessels at a reduced electric cost. According to the survey of Standaert de Metsenaere (1991), electricity accounts for 5 – 6 % of the total costs. The main constituent of the total cost is due to the lighting systems (65 %).

NOMENCLATURE

A	element area inside culture vessel, m^2	θw	angle between the normal line passing through a point on a horizontal level inside vessel and the point on the upper edge of the side wall
DI	direct irradiance at a point on a horizontal level inside culture vessel, W/m^2	$\theta\backslash c$	angle between the normal line passing through a point P on the vessel side wall and the point on the edge of the cover
$DI_{co\ ver}$	direct irradiance due to rays transmitted out the vessel cover, W/m^2	$\theta\backslash w$	angle between the normal line passing through a point P on the vessel side wall and the point on the upper edge of the side wall
$DI_{ve\ ssel}$	direct irradiance for entire vessel, W/m^2	α	angle between line (cn) and the projection of the incident ray for round vessel (Fig. 4)
$DI_{w\ all}$	direct irradiance due to rays transmitted out the side wall of round vessel, W/m^2	γ	angle between line (cn) and the center line(on) for round vessel (Fig. 3)
$DI_{w\ alls}$	direct irradiance due to rays transmitted out the side walls of square vessel, W/m^2	δ	inclination angle of the vessel side wall with the vertical
$DI_{wc\ over}$	direct irradiance through the solid angle of the vessel cover, W/m^2	$\rho'_{//}$	directional-hemispherical spectral specular reflectance polarized parallel to the incidence plane
$DI_{wv\ essel}$	direct irradiance through a hemisphere with no existence of the vessel, W/m^2	$\rho'_{\perp}$	directional-hemispherical spectral specular reflectance polarized perpendicular to the incidence plane
$DI_{w\ wall}$	direct irradiance through the solid angle of the side wall of round vessel, W/m^2	τ	transmittance due to absorption
$DI_{w\ walls}$	direct irradiance through the solid angle of the side wall of square vessel, W/m^2	φ	azimuth angle for a ray transmitted inside vessel
F	radiation shape factor	**Subscripts:**	
I_P	irradiance at a point on the side wall of vessel, W/m^2	b	arbitrary area located on a horizontal level inside vessel
IR	irradiance due to diffuse reflection, W/m^2	I	number of areas located on side wall number 1
$i\backslash$	light intensity, W/m^2.Sr.	i	index of area located on side wall number 1
R	unpolarized effective reflectance	J	number of areas located on side wall number 2
$R_{//}$	effective reflectance polarized parallel to the incidence plane	j	index of area located on side wall number 2
$R_{\perp}$	effective reflectance polarized perpendicular to the incidence plane	K	number of areas located on side wall number 3
R_d	diffuse reflectance of vessel material.	k	index of area located on side wall number 3
		L	number of areas located on side wall number 4
T	unpolarized effective transmittance	l	index of area located on side wall number 4
$T_{//}$	effective transmittance polarized parallel to the incidence plane	n	part number of square vessel divisions
		$part1$	first part or round vessel
$T_{\perp}$	effective transmittance polarized perpendicular to the incidence plane	$part2$	second part of round vessel
		r	division number for the side wall of round vessel
$TI_{ves\ sel}$	total vessel irradiance, W/m^2	$total$	The four side walls for square vessel or side wall for round vessel
$Tr_{co\ ver}$	transmittance of the vessel cover	wn	part number of the side walls for square vessel
		$wall1$	side wall number 1
		$wall2$	side wall number 2
		$wall3$	side wall number 3
		$wall4$	side wall number 4
Tr_{ref}	transmittance due to reflection	wwn	part number of the side walls for square vessel if

Tr_{vessel}	total vessel transmittance		*that part was not existing*
Tr_{wall}	transmittance of the side wall for round vessel		
Tr_{walls}	transmittance of the side walls for square vessel		
Greek Symbols:			
Θ	angle of incidence over the side wall of vessel		
θ	angle of incidence over a horizontal level inside vessel		
θc	angle between the normal line passing through a point on a horizontal level inside vessel and the point on the edge of the cover		

Fugiwara et al. (1989) studied experimentally the effect of various closures for various empty vessels on light irradiance inside these vessels. They used four different types of closure and three different types of vessel. Ibaraki et al. (2005) used a sensor film, for measuring integrated solar radiation to estimate the light irradiance distribution pattern inside a small test tube as a culture vessel under downward lighting conditions. They constructed a planlet model inside the culture vessel whose leaves were made of the sensor film to simulate a potato plantlet. Huang and Chen (2005) investigated experimentally the uniformity of internal transmittance for the tissue culture vessels that are usually applied in the plant tissue culture industry. They found that uniformity of internal transmittance for the round based vessels was better than that of a square or rectangular box.

Ciolkosz et al. (1996) presented a study for evaluating four analytical models of fluorescent lighting systems for calculating the light irradiance, utilizing four calculation techniques: point source calculations; the Lumen method; the Integrated Area Fourier Series (IAFS); and the application distance photometry (ADP), in addition to a regression empirical model. They constructed a full scale mock up of a micro-propagation shelf and they recorded the light irradiance at various locations under various light source mounting heights and configurations for evaluating the results from the models. They reported that the IAFS technique was the best one to calculate the light irradiance at the vessel cover level. It gave 5.1 % Mean Relative Absolute Error (MRAE) when calculating average light irradiance on a shelf, and 4.5 % when calculating mean relative deviation, which is a measure of uniformity. They used the regression empirical model based on IAFS values to predict light irradiance at the plant level. This model gave MRAE of 9.2 %, with a corresponding value for Mean Relative Deviation (MRD) of 12.1 %. The result of this study was adopted to serve as a design issue for micro-propagation lighting systems (Ciolkosz et al., 1997). After comparing the measured light irradiance at different vessel conditions, Ciolkosz et al. (1997) found that the agar and plant material in the vessels had only little effect on the light irradiance at both the vessel cover level and plant level. Chen (2005) developed three models for estimating lighting distribution for plant micro-propagation at vessel cover level: line; ribbon; and empirical regression models. He found that the ribbon model had the best fitting ability. He also evaluated the effect of height and spacing of fluorescent tubes on the distribution uniformity of light irradiance.

It is obvious that most of the previous theoretical studies focused on estimating analytically the light irradiance at the vessel cover level and used empirical regression to find it at the plant level. They did not take into account the effect of the vessel's material and hence its radiative properties and the shape of the vessel to estimate the light irradiance at the plant level and the effect of the adjacent vessels.

The objective of this study is to develop an accurate mathematical model to predict the light irradiance at any point inside the culture vessel: over the vessel base or along the vessel height and the transmittance distribution at any plant level along the vessel height. In the present study, the effect of the vessel's material and the shape of the vessel have been taken into account.

2. MATHEMATICAL MODEL

2.1 Assumptions

For the mathematical model, the following assumptions are made:
1- Radiation intensity incident in a hemisphere covering the used vessel is uniform and equal to unity.
2- The temperature variation through the thin vessel walls is neglected.
3- The interference of incident and reflected waves through the vessel w is neglected, as the thickness of the vessel wall is greater than the light wavelength values of light.
4- Deviation of the transmitted ray due to refraction in the vessel wall is neglected as long as the fluid is the same for both sides of the wall.
5- The absorption coefficient and the refractive index of the vessel material are independent of the light wavelength, (0. 02 mm^{-1}, 1.59 for polycarbonate and 0. 056 mm^{-1}, 1.47 for polypropylene, respectively).
6- Reflection inside the culture vessel is uniform.
7- Any radiation reaching the bottom of the vessel is completely absorbed.
8- The second reflections inside the vessel are neglected.
9- Radiation is unpolarized. So, the effective reflectance and transmittance can be calculated respectively as follows:

$$R = \frac{(R_{//} + R_{\perp})}{2} \qquad , \qquad (1)$$

where for multiple reflection inside the vessel walls,

$$R_{//} = \rho_{//}\left[1 + \frac{(1-\rho_{//})^2\tau^2}{1-\rho_{//}^2\tau^2}\right] \qquad \text{and} \qquad R_{\perp} = \rho_{\perp}\left[1 + \frac{(1-\rho_{\perp})^2\tau^2}{1-\rho_{\perp}^2\tau^2}\right]$$

Also, $\qquad T = \frac{(T_{//} + T_{\perp})}{2} \qquad , \qquad (2)$

Where $\qquad T_{//} = \frac{\tau(1-\rho_{//})^2}{1-\rho_{//}^2\tau^2} \qquad$ and $\qquad T_{\perp} = \frac{\tau(1-\rho_{\perp})^2}{1-\rho_{\perp}^2\tau^2}$

In equations (1) and (2) $\qquad \rho'_{//}(\Theta) = \left[\frac{\tan(\Theta-\chi)}{\tan(\Theta+\chi)}\right]^2, \quad \rho'_{\perp}(\Theta) = \left[\frac{\sin(\Theta-\chi)}{\sin(\Theta+\chi)}\right]^2,$

$$\tau(\Theta) = \exp\left(\frac{-aL}{\cos(\chi)}\right)$$

where χ is the angle of refraction which is related to the incident angle Θ by Snell's law as:

$$\chi(\Theta) = \sin^{-1}\left(\frac{\sin\Theta}{n}\right)$$

The types of the culture vessels were classified according to their base shape into two categories: vessels with square base, and vessels with round base.

2.2 Relations for vessels with square base

The culture vessels with square base consist of two parts: the vessel box and the vessel cover. For square vessels, the vessel box has four side walls, which may be vertical or tilted outward with an angle δ, as shown in Fig.1. An incoming ray of intensity $i\grave{}(\theta,\varphi)$ penetrates the vessel side wall into the vessel to point (o) on a certain horizontal level of position defined by Z_1 and Z_2 and at a vertical distance of H away from the vessel cover. To calculate the direct irradiance due to all incoming rays through the four side walls, every side wall is divided into two parts. The first part is that within the azimuth angle φ_1 and to the right of line (dc), and the second part is that within the azimuth angle φ_2 and to the left of line (dc). The direct irradiance at point (o) due to incoming rays with unity value and uniform intensity through each of the first and the second parts of the wall is the integration of incoming rays transmitted out of the respective part of the wall over the solid angle of the part at (o):

$$DI_{w_1} = \int_0^{\varphi_1} \int_{\theta_{w1}}^{\pi/2} T(\Theta)\cos\theta\sin\theta d\theta d\varphi \tag{3}$$

and

$$DI_{w_2} = \int_0^{\varphi_2} \int_{\theta_{w2}}^{\pi/2} T(\Theta)\cos\theta\sin\theta d\theta d\varphi \tag{4}$$

Where the polar angles θ_{w1} and θ_{w2} are functions of the azimuth angle φ

The angle Θ is the incidence angle of the incoming ray upon the surface of the side wall and is related to angles θ and angle φ by the following relation:

$$\Theta = \cos^{-1}(\sin\theta\cos\varphi\cos\delta - \cos\theta\sin\delta) \tag{5}$$

The total irradiance at point (o) of ordinates Z_1 and Z_2 due to all radiation incoming into the vessel through the four side walls within eight azimuth angles can be obtained from:

$$DI_{walls} = \sum_{n=1}^{n=8} DI_{w_n} = \sum_{n=1}^{n=8} \int_0^{\varphi_n} \int_{\theta_{wn}}^{\pi/2} T(\Theta)\cos\theta\sin\theta d\theta d\varphi \tag{6}$$

The actual transmittance of vessel side walls is defined in this work as the ratio between the total irradiance at a certain point on a horizontal level due to radiation incoming into the vessel through the side walls and the total irradiance at the same point due to radiation incoming into the vessel through the same solid angle of the side walls if the side walls were not existing.
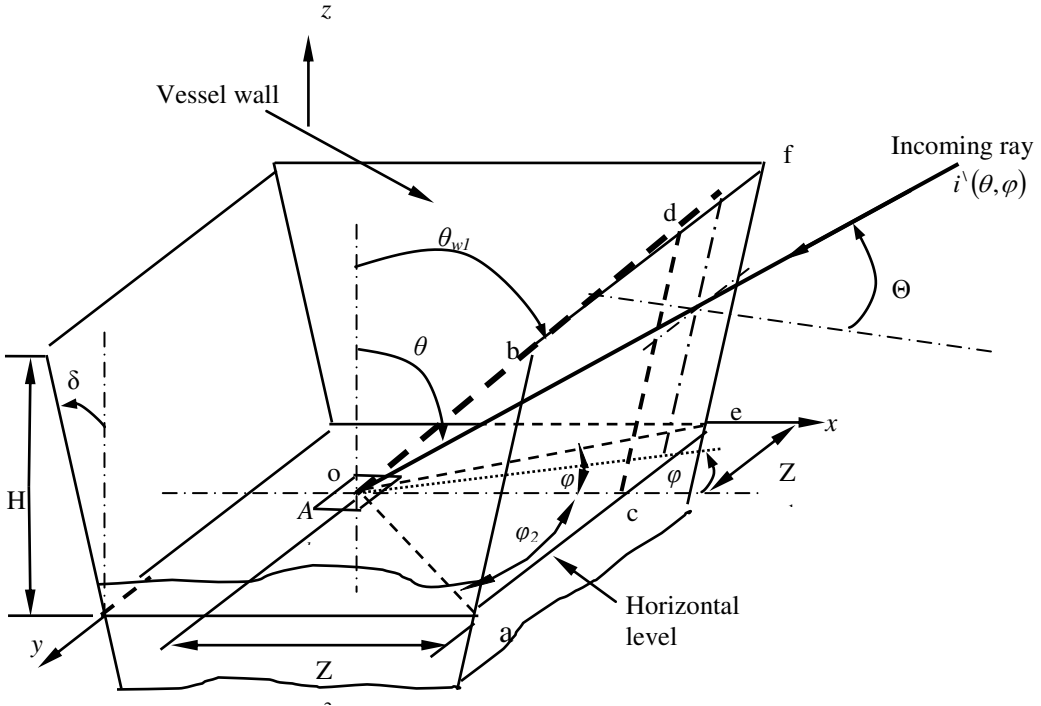
Fig. 1 Schematic diagram showing a ray incoming into a square vessel through its inclined side wall and incident upon a horizontal level.

The vessel walls transmittance can be calculated as follow:

$$Tr_{walls} = \left(\frac{DI_{walls}}{DI_{wwalls}} \right) \tag{7}$$

where:

$$DI_{wwalls} = \sum_{n=1}^{n=8} DI_{ww_n} = \sum_{n=1}^{n=8} \int_0^{\varphi_n} \int_{\theta_{wn}}^{\pi/2} \cos\theta \sin\theta d\theta d\varphi \tag{8}$$

The total irradiance at point (o) due to radiation incoming into the vessel through the vessel cover within the same eight azimuth angles as for the side walls can be obtained from Eq.(6) noting that $\Theta = \theta$ for the vessel cover. Then:

$$DI_{cover} = \sum_{n=1}^{n=8} \int_0^{\varphi_n} \int_0^{\theta_{cn}} T(\theta) \cos\theta \sin\theta d\theta d\varphi \tag{9}$$

Using a similar definition for the actual transmittance of the vessel cover as that for the side walls then:

$$Tr_{cover} = \left(\frac{DI_{cover}}{DI_{wcover}} \right) \tag{10}$$

Where:

$$DI_{wcover} = \sum_{n=1}^{n=8} \int_{0}^{\varphi_n} \int_{0}^{\theta_{cn}} \cos\theta \sin\theta d\theta d\varphi \tag{11}$$

The irradiance at an arbitrary rectangle of area A_b at a certain horizontal level due to the diffuse reflection from the four side walls for square vessel with inclined side walls can be calculated as follows:

$$IR_{b-total} = IR_{b-wall_1} + IR_{b-wall_2} + IR_{b-wall_3} + IR_{b-wall_4} \tag{12}$$

Where:

$$IR_{b-wall_1} = \sum_{i=1}^{I} \left(\frac{A_i}{A_b} F_{i-b} I_{Pi} R_d \right) \tag{13}$$

$$IR_{b-wall_2} = \sum_{j=1}^{J} \left(\frac{A_j}{A_b} F_{j-b} I_{Pj} R_d \right) \tag{14}$$

$$IR_{b-wall_3} = \sum_{k=1}^{K} \left(\frac{A_k}{A_b} F_{k-b} I_{Pk} R_d \right) \tag{15}$$

$$IR_{b-wall_4} = \sum_{l=1}^{L} \left(\frac{A_l}{A_b} F_{l-b} I_{Pl} R_d \right) \tag{16}$$

where I_P is the irradiance at a certain point (P) on the side wall 1, 2, 3, or 4 due to radiation penetrating the vessel through the other three side walls and the cover. It can be calculated as follows:

$$I_P = \sum_{n=1}^{n=6} \left(\int_{0}^{\varphi_n} \int_{\theta_{wn}^{\backslash}}^{\pi/2} T(\Theta) \cos\theta \sin\theta d\theta d\varphi \right)_{wall} + \sum_{n=1}^{n=8} \left(\int_{0}^{\varphi_n} \int_{0}^{\theta_{cn}^{\backslash}} T(\theta) \cos\theta \sin\theta d\theta d\varphi \right)_{cover} \tag{17}$$

The transmittance due to reflection is defined in this work as the ratio between the diffuse reflection reaching a certain point on a horizontal level and the direct irradiance at the same point due to radiation coming through a hemisphere with no vessel.

The transmittance due to reflection can thus be calculated as follows:

$$Tr_{ref} = \left(\frac{IR_{b-total}}{DI_{wvessel}} \right) \tag{18}$$

Where:

$$DI_{wvessel} = \int_{0}^{2\pi} \int_{0}^{\pi/2} \cos\theta \sin\theta d\theta d\varphi = \pi \tag{19}$$

The total vessel irradiance at a certain point on the horizontal level can be obtained from:

$$TI_{vessel} = DI_{vessel} + IR_{b-total} \tag{20}$$

where:

$$DI_{vessel} = DI_{walls} + DI_{cover} \tag{21}$$

The total vessel transmittance at a certain point on a horizontal level is defined in this work as the ratio between the total vessel irradiance coming to a certain point on a horizontal level and the direct irradiance at the same point due to radiation coming through a hemisphere without the existence of the vessel. Thus:

The total vessel transmittance can be calculated as follows:

$$Tr_{vessel} = \frac{TI_{vessel}}{DI_{wvessel}} = \frac{TI_{vessel}}{\pi} \tag{22}$$

2.3 Vessels with round base

Figure 2 shows an incoming ray of intensity $i^{\backslash}(\theta, \varphi)$ penetrating a round vessel side wall tilted outwards with the vertical by angle δ at point (a). This ray is incident upon the horizontal level at point (o).

To calculate the total irradiance at point (o) due to the radiation incoming into the vessel through its side wall, the vessel is divided into two identical parts as shown in Fig. 3. The two parts are separated by the (no) center line. The side wall portion in each part is divided relative to point (o) into 180 strips for each part. Each strip has the length of the side wall and its width forms 1 degree relative to point (o).

The direct irradiance at point (o) due to all of the incoming rays of intensity equal to unity through the side wall of part one or part two is the integration of all incoming rays transmitted out of the inclined side wall of that part over its solid angle. It can be obtained as follows:

$$DI_{wall} = 2\int_0^\pi \int_{\theta_w}^{\pi/2} T(\Theta)\cos\theta\sin\theta\, d\theta\, d\varphi \tag{23}$$

where:
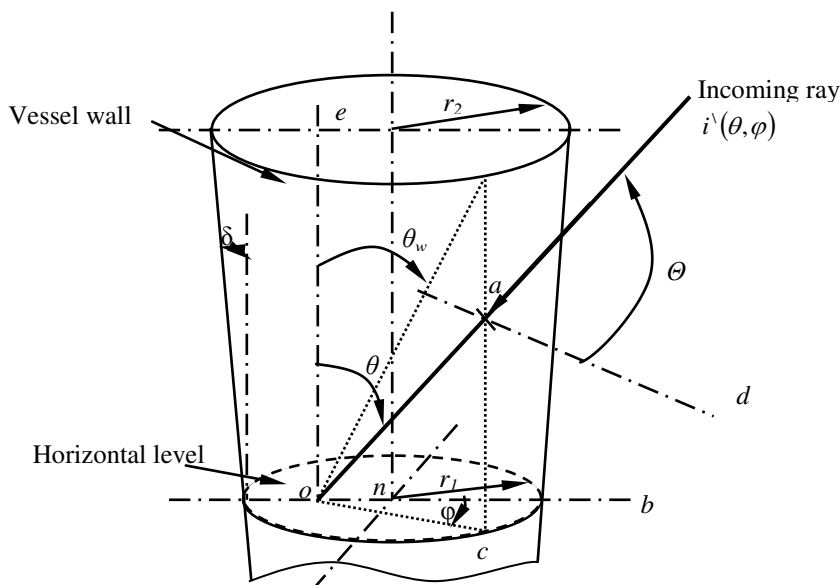
Fig. 2 Schematic diagram showing an incoming ray through the
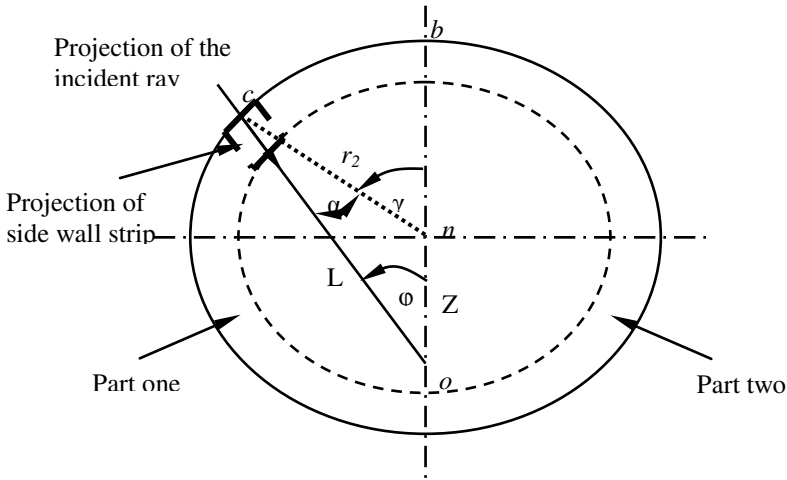inclined vessel side wall and incident upon a horizontal level.

Fig. 3 Plan view of the round
vessel with inclined side wall.

$$\Theta = \cos^{-1}\left[\sin\theta\cos(\gamma - \varphi)\cos\delta - \cos\theta\sin\delta\right] \tag{24}$$

The actual transmittance for the side wall of the round vessel according to the definition of wall transmittance mentioned before can be calculated as follows:

$$Tr_{wall} = \left(\frac{DI_{wall}}{DI_{wwall}}\right) \tag{25}$$

where:

$$DI_{wwall} = 2\int_{0}^{\pi}\int_{\theta_w}^{\pi/2}\cos\theta\sin\theta d\theta d\varphi \tag{26}$$

The total irradiance at point (o) due to all radiation incoming into the vessel through the vessel cover wall can be obtained similar to square vessel from:

$$DI_{cover} = 2\int_{0}^{\pi}\int_{0}^{\theta_c}T(\theta)\cos\theta\sin\theta d\theta d\varphi \tag{27}$$

The actual transmittance for the side wall of the round vessel can be calculated as follows:

$$Tr_{cover} = \left(\frac{DI_{cover}}{DI_{wcover}}\right) \quad , \tag{28}$$

where:

$$DI_{wcover} = 2\int_{0}^{\pi}\int_{0}^{\theta_c}\cos\theta\sin\theta d\theta d\varphi \tag{29}$$

The irradiance from a strip of area A_r as a part of the side wall to an area A_b located on the horizontal level at a distance Z_b from (n) as shown in Fig. 4, due to diffuse reflection, can be calculated as follows:

$$IR_{b-r} = \frac{A_r}{A_b} F_{r-b} I_{Pmean} R_d \tag{30}$$

The average irradiance I_{Pmean} from each strip on the side wall, Fig. 4, can be calculated as follows:

$$I_{Pmean} = \left(\frac{\sum_1^m I_P}{m} \right) \quad , \tag{31}$$

where m is the number of the measuring points on each strip of area A_r.

To calculate the irradiance at point (P), located anywhere on the side wall of round vessel, the vessel is assumed to be divided into two identical parts as shown in Fig. 5. Thus:
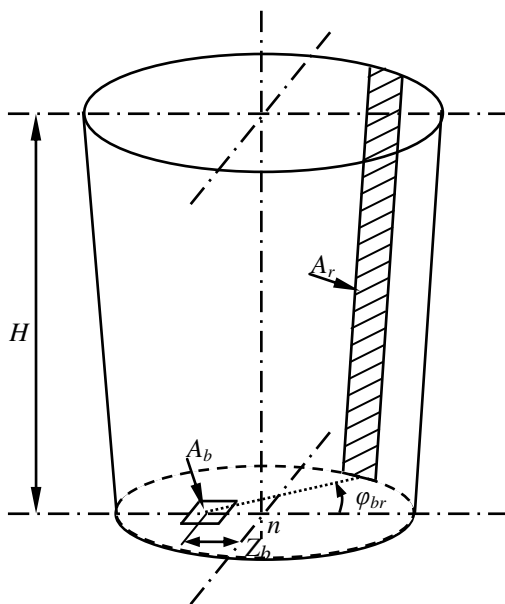
Fig. 4 Schematic diagram to show the position of A_b and A_r for round vessel with inclined side wall.

$$I_P = 2\left[\left(\int_0^{\pi/2} \int_0^{\theta'_c} T(\theta)\cos\theta\sin\theta\, d\theta\, d\varphi \right)_{cover} + \left(\int_0^{\pi/2} \int_{\theta'_w}^{\pi/2} T(\Theta)\cos\theta\sin\theta\, d\theta\, d\varphi \right)_{wall} \right] \tag{32}$$
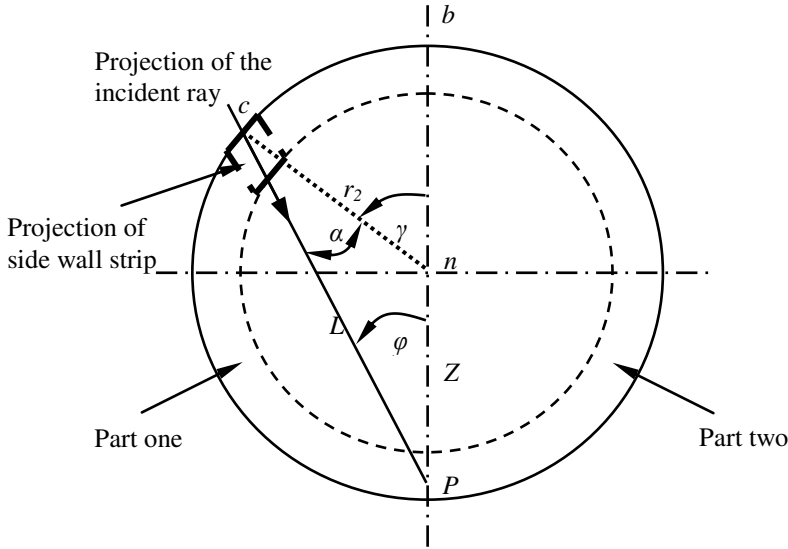
Fig. 5 Plan view of round vessel with inclined side
wall showing projection position of point P.

The irradiance from the whole side wall to a certain area A_b on a certain horizontal level can be calculated as follows:

$$IR_{b-total} = 2\sum_{r=1}^{90} IR_{b-r} \tag{33}$$

The transmittance due to reflection for round vessel, using a similar definition as before, can be calculated as follows:

$$Tr_{ref} = \left(\frac{IR_{b-total}}{DI_{wvessel}}\right) = \frac{2\sum_{r=1}^{90} IR_{b-r}}{\pi} \tag{34}$$

The total vessel irradiance and transmittance at a point on a certain horizontal level can be obtained from:

$$TI_{vessel} = DI_{vessel} + IR_{b-total} \tag{35}$$

The total vessel transmittance can be calculated as follows:

$$Tr_{vessel} = \frac{TI_{vessel}}{DI_{wvessel}} = \frac{TI_{vessel}}{\pi} \tag{36}$$

3 NUMERICAL CALCULATIONS

The integrations to calculate irradiance and transmittance for culture vessels used in the present work are numerically evaluated by using Simpson quadrature. First, these values are calculated at the vessel bottom level, then at different horizontal levels along the vessel height.

3.1 Specifications of culture vessels

Three types of the widely used culture vessels in the plant tissue culture industry were selected. Their specifications are given below.

3.1.1 GA-7 box

Figure 6 shows a schematic diagram for the shape and dimensions of the GA-7 box. The vessel type GA-7 box is manufactured by Magenta Co., USA. The cover of the vessel is made of Polypropylene, and the wall material is Polycarbonate. The wall thickness is 1.7 mm.

3.1.2 Square box

Figure 7 shows a schematic diagram for the shape and dimensions of the square box (Vito-Vent, Vitro Co., France). Both the vessel cover and walls are made of Polypropylene. There are two cavities at the cover position as a special design for permitting fingers to grid and remove the cover. The wall thickness is 1.7 mm.

3.1.3 Round vessel

Figure 8 shows a schematic diagram for the shape and dimensions of the round vessel (LA-RH, LAB Associate, The Netherlands). Both the vessel cover and walls are made of Polypropylene. This type is of inclined side wall. Also the same type is used in this research by assuming a normal side wall. The wall thickness is 1.7 mm.
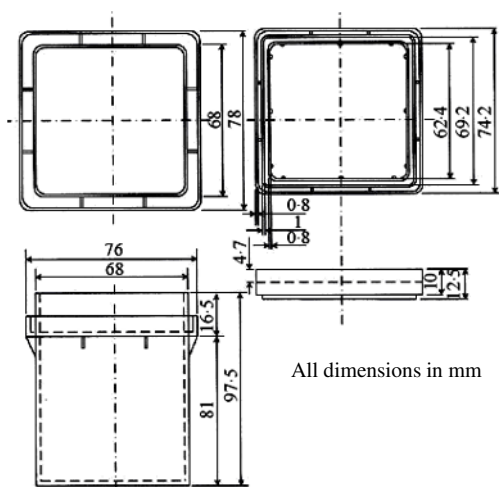
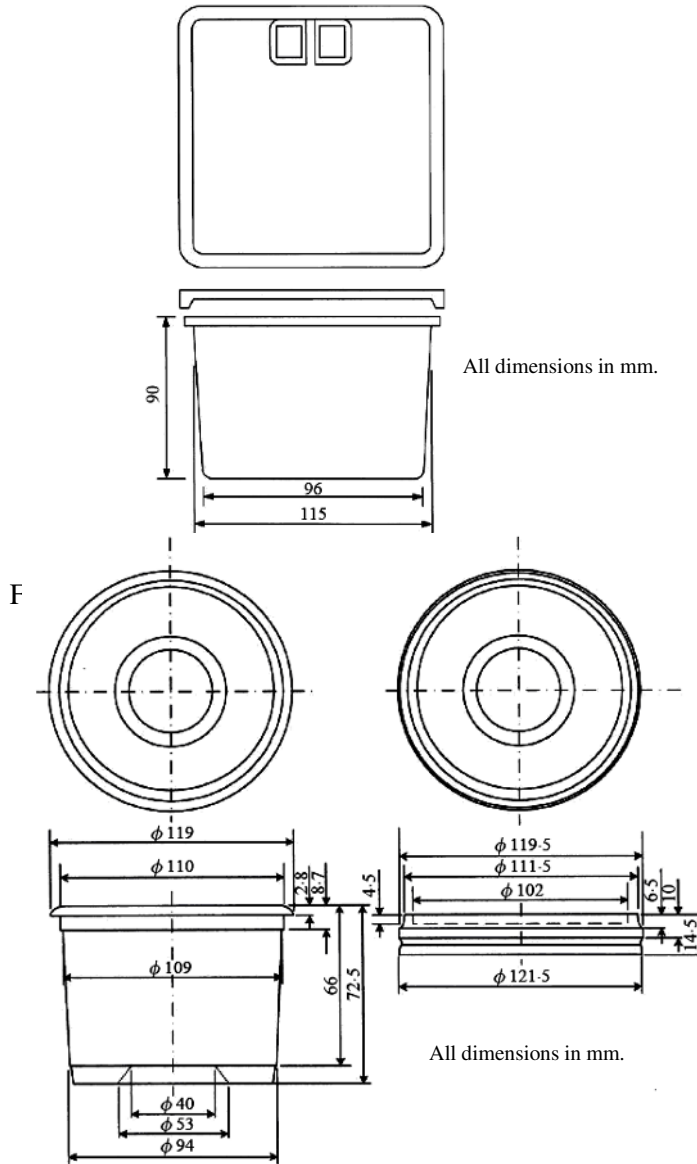Fig. 6 Shape and dimensions of GA-7 box .

Fig. 8 Shape and dimensions of round vessel.

3.2 Method of calculations

The model was applied to both the vessel bottom level and different horizontal levels along the vessel height. The main dimensional aspects of the selected vessel shape were used in the calculations. The vessel bottom or any horizontal level along the vessel height for both GA-7 box vessel type and square box vessel type was divided into 36 elements, as shown in Fig. 9. Each element has a width of X/6, where X is the horizontal level width. For round vessel type, 37 square elements were selected on the

vessel bottom or any horizontal level along the vessel height; as shown in Fig. 10, to calculate the irradiance and transmittance.
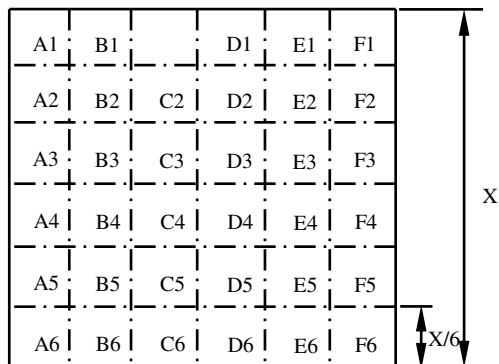
Fig. 9 Elements on any horizontal level at which the
irradiance and transmittance are calculated for square
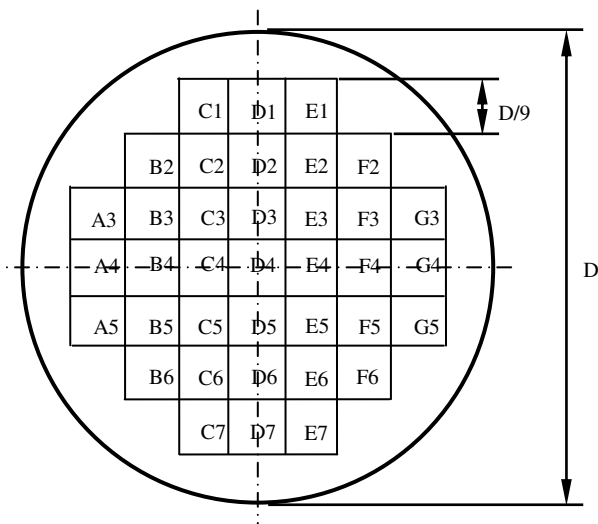vessel and GA-7 box vessel type.

Fig. 10 Elements on any horizontal level at which the
irradiance and transmittance are calculated for round
vessel type.

3.3. Transmittance Uniformity

The criterion of mean relative deviation is used in this work to evaluate the uniformity performance for the irradiance transmittance inside the culture vessel.
It can be calculated as follows:

$$MRD = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_{avg} - Y_i}{Y_{avg}} \right| \qquad , \tag{37}$$

where:

MRD is the mean relative deviation.

n is the number of the calculated segments.

Y_i is the calculated irradiance transmittance of each element of one level in the culture vessel.

Y_{avg} is the arithmetic average of Y_i; $Y_{avg} = \frac{1}{n} \sum_{i=1}^{n} Y_i$

3.4. The Comparison Criteria

The results obtained numerically were compared with measured values, wherever available in the literature. Two bases are used here to compare between the computed and measured results. The first one is the percentage relative error. This base is used to evaluate the ability of the model to predict the irradiance transmittance for each element of the vessel bottom. The second base is the mean relative absolute error. This base is used to evaluate the ability of the model to predict the average irradiance transmittance for the vessel bottom.

The percentage relative error can be obtained from:

$$\% RE = \frac{T_{mi} - T_{ci}}{T_{mi}} \times 100 \qquad , \tag{38}$$

where:

T_{mi} is the measured value at a certain element

T_{ci} is the calculated value at the same element

The mean relative error can be calculated as follows:

$$MRE = \frac{1}{n} \sum_{i=1}^{n} \frac{\left| T_{mi} - T_{ci} \right|}{T_{mi}} \tag{39}$$

4. RESULTS AND DISCUSSION

4.1 Comparison for the plant tissue culture vessels

In this section a comparison based on the relative percentage error is carried out between the measured transmittance by Huang and Chen, (2005) and the computed transmittance using the present model, over the vessel bottom for the square vessel, GA-7 box vessel and round vessel type

Huang and Chen measured the internal irradiance inside the culture vessel by removing its bottom and placing the LI-190 SA quantum sensor at the bottom level for the same type of culture vessel. The measurements were executed on a full-scale micro-propagation shelf which contains two fluorescent luminaries mounted on the upper side of an identical shelf. The length of fluorescent tubes, Philips TLD 36 W/39, was 1.2 m. The height of tubes was 0.35 m. They calculated the transmittance of vessel as the ratio between the internal irradiance and external irradiance.
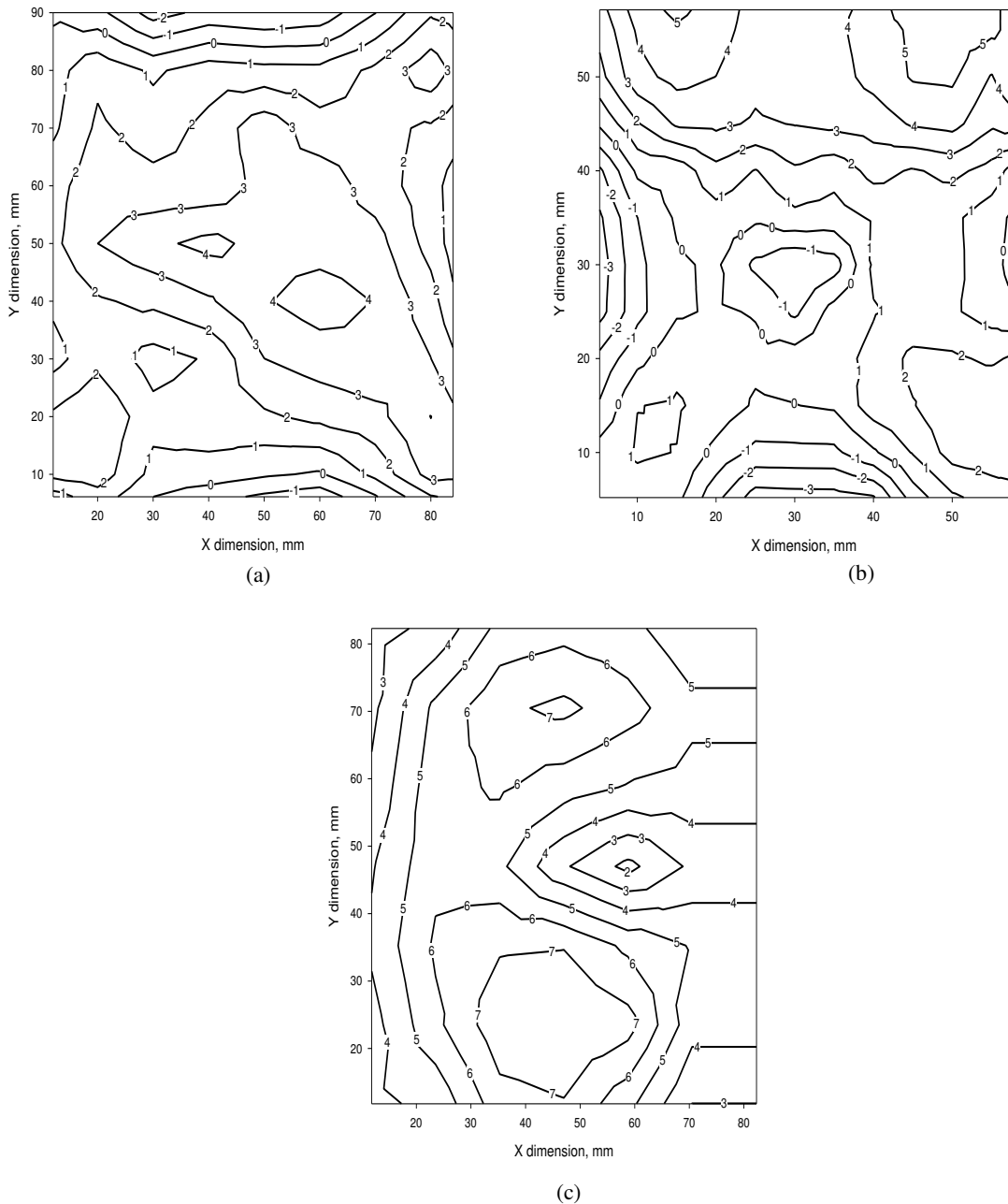
(a)

(b)

(c)

Fig. 11 Contour lines of % relative error between the model results and Huang and Chen, (2005) experimental results for: (a) the square vessel type, (b) GA-7 box vessel, and (c) the round vessel type.

In this work, the percentage relative error of vessel transmittance for the square vessel type over the vessel bottom is ranged from 0 % to 4 % as shown in Fig.11 (a) and the mean relative error is 0.0035. Figure 11(b) shows the contour lines of the

percentage relative error over the vessel bottom for the GA-7 box vessel type. The percentage relative error is ranged from -3 % to 5 %. The mean relative error is 0.0026. Figure 11(c) shows the contour lines of the relative error over the vessel bottom for the round vessel type. The relative error of irradiance transmittance over the vessel bottom was ranged from 2 % to 7 %. The mean relative error is 0.0068. These deviations of the calculated irradiance transmittance from the measured values are attributed to two main reasons: in this model the light intensity was assumed uniform around the culture vessel, but actually the light intensity around the culture vessel is not uniform. The second reason is that the absorption coefficient and the refractive index of the vessel material were assumed in this model independent of wavelength, but actually these radiative properties are slightly dependent on the wavelength. Generally the error is not large, and the computed values can present the transmittance quite well. This gives confidence in the estimations based on the present model.

4.2 performances of the tissue culture vessels

Figure 12 shows the percentage contribution of the cover, walls, and reflection average irradiance to the total average vessel irradiance for the four vessel types used in this work. The cover contribution decreases with the height of the horizontal level towards the vessel bottom. This is due to the inverse dependence of the solid angle of the cover on the square of the distance from the cover. However, the side wall contribution increases towards the vessel bottom, because of increasing of the side wall area with the distance from the vessel cover.

The reflection represents the smaller contribution of the total vessel irradiance along the vessel height for the four types. For the two types of square base: square vessel and GA-7 box vessel, the reflection percentage contribution increases slightly with increasing the depth of the horizontal level from the vessel cover. On the other hand, the reflection percentage contribution is nearly constant for the two types of round base.

## 4.3 Effect of the aspect ratio on the characteristics of square vessels

The aspect ratio for vertical square vessels is defined as the ratio of the vessel height (H) to the width of the vessel bottom (X). A range of $0.25 - 3$ for the aspect ratio was considered. As shown in Fig. 13(a), increasing the aspect ratio leads to a decrease in the cover irradiance and an increase in the walls irradiance and a linear increase in the reflection irradiance. This leads to almost constant total vessel irradiance for different values of aspect ratio.
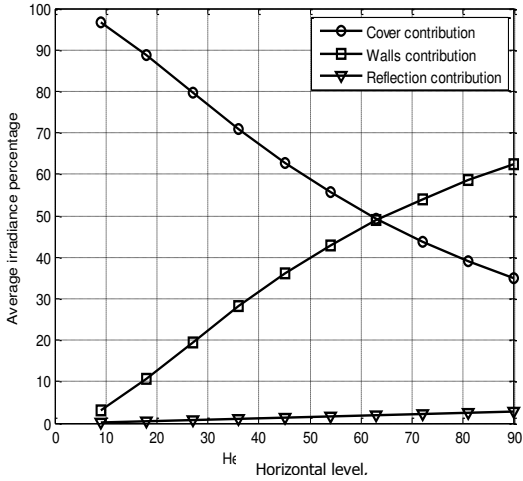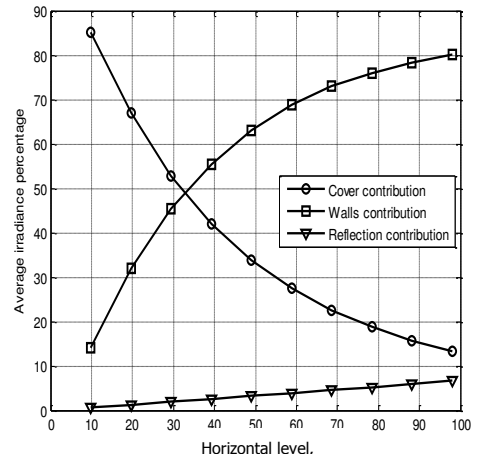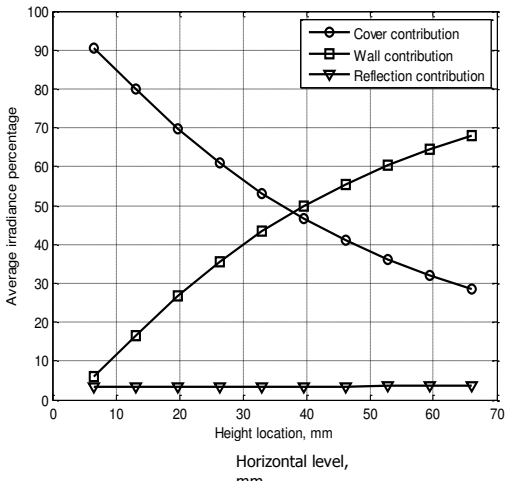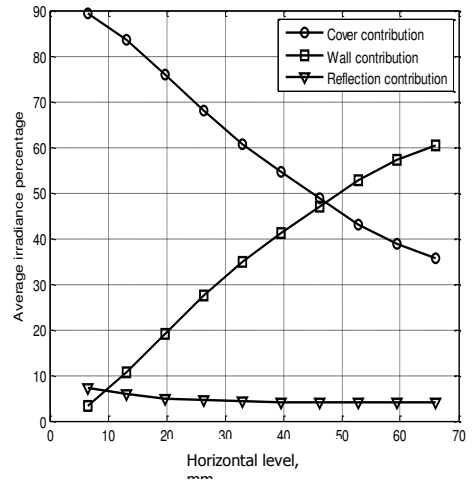
Fig. 12 Cover, walls, and reflection percentage contributions for: (a) square vessel type, (b) GA-7 box type, (c) round vessel type of inclined side wall, and (d) round vessel type of normal side wall

The aspect ratio has no significant effect on the cover transmittance, as shown in Fig 13(b). However, increasing the aspect ratio leads to a slight decrease in the walls transmittance and a linear increase in the reflection transmittance. However no significant change in the total vessel transmittance is shown with the changes in the aspect ratio. Figure 13(c) shows the effect of the aspect ratio on the mean relative deviation of the irradiance transmittance for the bottom level for square vessels. This figure shows that the small mean relative deviation where the best irradiance uniformity occurs at the small ratio of the chosen range of 0.25 and it reaches a maximum value at a ratio of 1 corresponding to the worst irradiance uniformity. For an

aspect ratio higher than 1, the irradiance distribution uniformity starts to be enhanced gradually, but the uniformity is still not good.
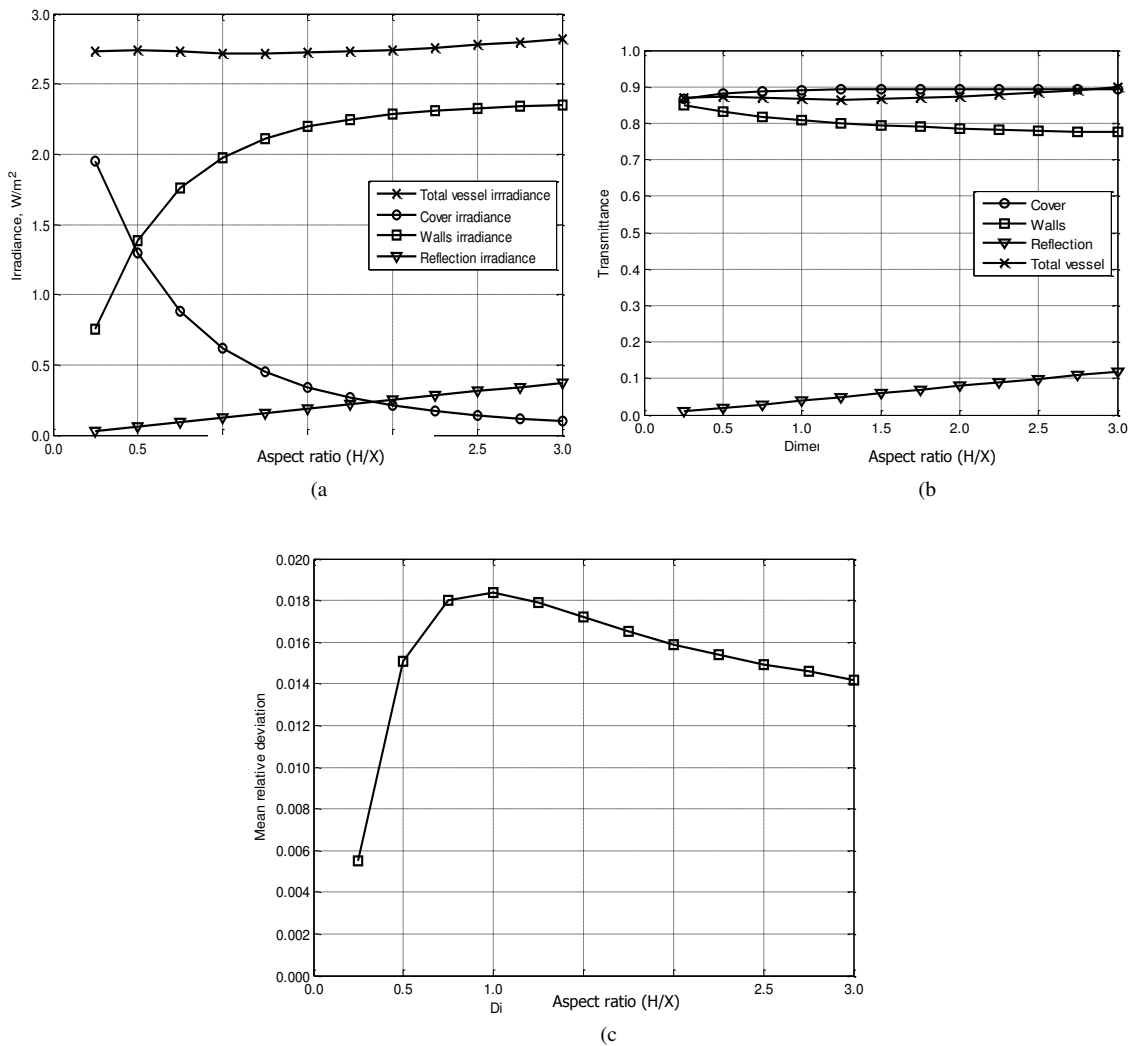

(a

(b

(c

Fig. 13 Effect of the aspect ratio on: (a) irradiance, (b) transmittance, and (c) mean relative deviation, at the bottom for square vessels.

All these changes with aspect ratio are attributed to the changes in the vessel height, the side walls area, and the vessel cover area with increasing the aspect ratio.

It may be concluded from the current study that the aspect ratio of square vessels has a small effect on the total average vessel irradiance and vessel transmittance, with preference of low aspect ratio for the downward lighting systems, and high aspect ratio for the sideward lighting systems. It is recommended to avoid using square vessels with aspect ratio value of one because of the worst uniformity of irradiance distribution of these vessels.

4.4 Effect of the aspect ratio on the characteristics of vertical round vessels

The aspect ratio for round vessels is defined as the ratio of the vessel height (H) to the diameter of the vessel bottom (D). The values of (H/D) considered for this study were between 0.25 - 3.

The irradiance reaching the vessel bottom from the cover is reduced as H/D increases, while that reaching the bottom through the vessel side wall is increased. Also, increasing the aspect ratio causes a very slight decrease in the reflection irradiance, as shown in Fig. 14(a). Figure 14(a) also shows that the total vessel irradiance slightly decreases with the increase in the aspect ratio. Figure 14(b) illustrates the effect of H/D on the irradiance transmitted to the bottom of vessel with round base. The aspect ratio has no significant effect on the cover transmittance. However, an increase in aspect ratio leads to a slight decrease in the side wall transmittance and a slight linear decrease in the reflection transmittance leading to a small decrease in the total vessel transmittance. Figure 14(c) shows the effect of the aspect ratio on the mean relative deviation of the irradiance transmittance at the bottom level for round vessels. This figure shows that the small mean relative deviation, hence best irradiance distribution uniformity, is achieved at a ratio of 0.25 and it reaches a maximum value at a ratio of 1, then it starts to be enhanced gradually, but the uniformity is still not good. These changes in the characteristics are attributed to the changes in the vessel height, the side wall area, and the vessel cover area with increasing the aspect ratio. Thus, it is preferable to use round vessels with values of aspect ratio lower than 0.5 with downward lighting systems, for enhancing the irradiance distribution uniformity.

4.5 Effect of the side wall inclination on the characteristics of square vessels

The increase in the side walls inclination, between 0 – 10 degrees at $H/D = 1$ for square vessel, causes an increase in the cover area and consequently in the irradiance reaching the vessel bottom from the cover (see Fig. 15(a)). However, the increase in the inclination of the side walls increases the distance the light crosses through the side wall, causing a decrease in the side walls irradiance. And thus a slight decrease in the reflection irradiance also. The net result is a decrease of the total vessel irradiance with the increase of the side walls inclination, as shown in Fig. 15(a). Similar effect on the transmittance is shown in Fig. 15(b). Also, as shown in Fig. 15(c), an increase in the side walls inclination for square vessels enhances the uniformity of irradiance distribution at the bottom level. For vessels with square base, to obtain higher total vessel irradiance at the vessel bottom it is recommended to use a square vessel with side walls of zero inclination. However increasing the side walls inclination slightly enhances the uniformity of irradiance distribution at the vessel bottom.

4.6 Effect of the side wall inclination on the characteristics of round vessels

For the same conditions as that used for square vessels, the behavior of round vessels at different inclination angles is shown in Fig. 16(a-c). Generally, the behavior of round vessel is close to that of square vessel except for the total irradiance and the cover, side
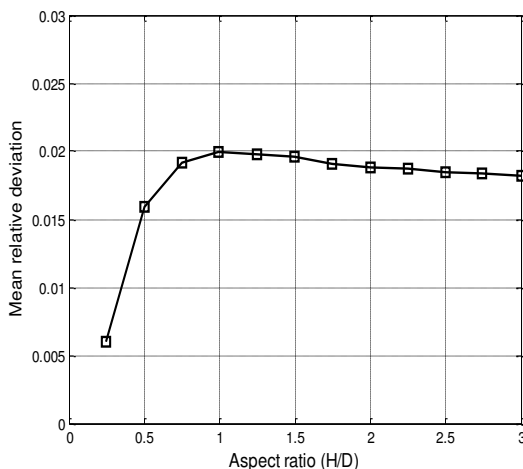
wall, and total transmittance for round vessel are almost unchanged with the increase of inclination angle. Thus, the uniformity of irradiance distribution at the vessel bottom for round vessels can be enhanced by increasing the side wall inclination without affecting the total vessel irradiance at the vessel bottom.
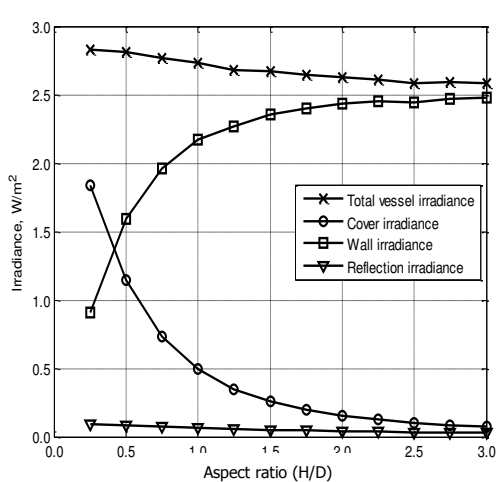
(a)
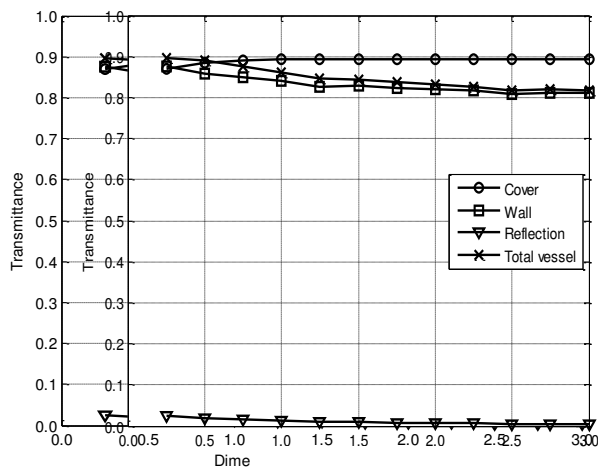
(b)

(c)

Fig. 14 Effect of the aspect ratio on: (a) irradiance, (b) transmittance, and (c) mean relative deviation, at the bottom for round vessels.

5 CONCLUSIONS

The present work proposes a mathematical model able to predict the internal light irradiance and the transmittance of culture vessels, taking into account the vessel material and shape for growth environment with uniform light intensity. Three types of

culture vessels from those used in the plant tissue culture industries were used: square vessel, GA-7 box vessel and round vessel. The following conclusions are obtained:

- There is a confidence in the calculated results of this model as its results for the internal transmittance can predicts the results measured by Huang and Chen (2005) on the vessel bottom with small relative error.
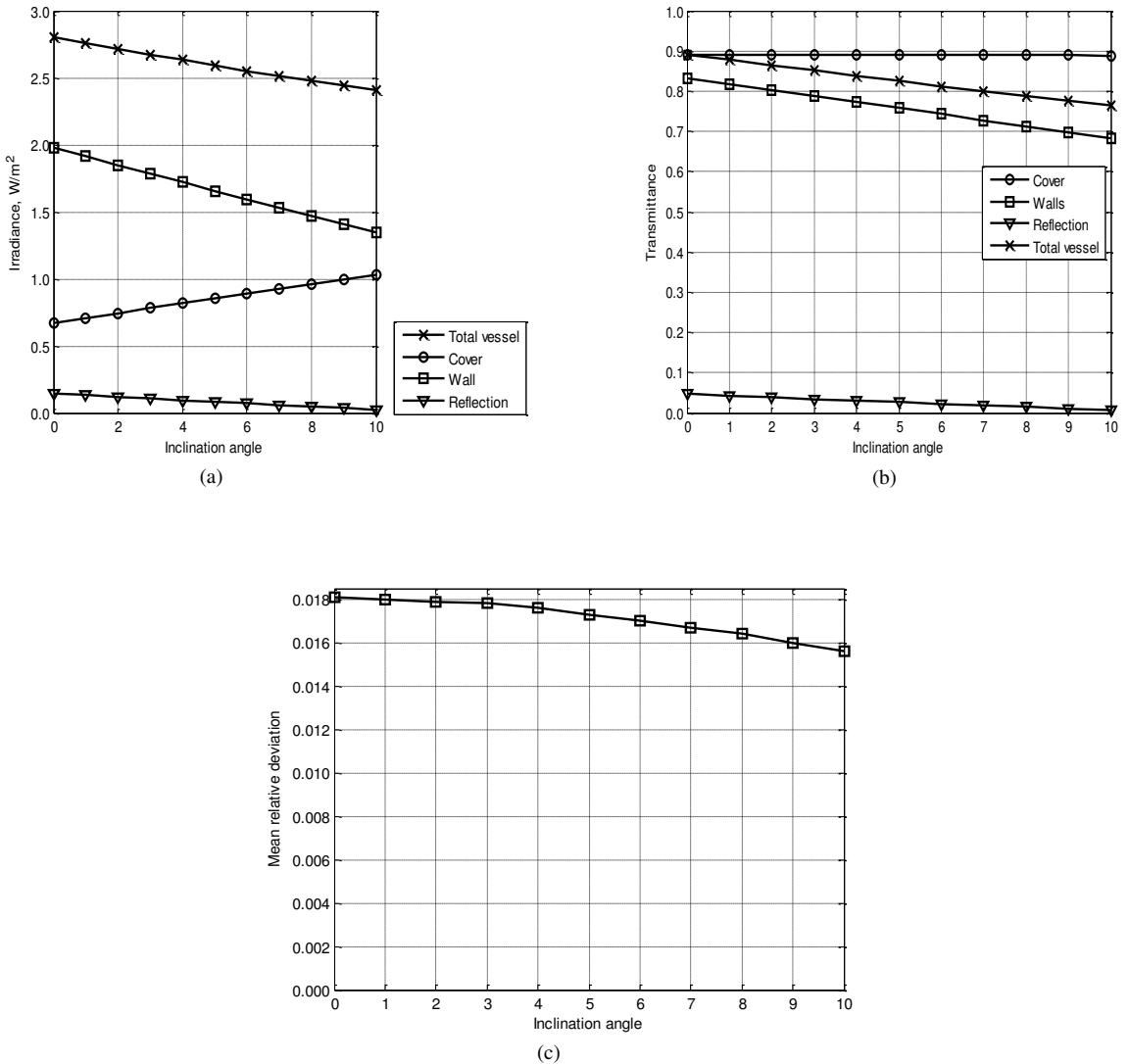

(a)

(b)

(c)

Fig. 15 Effect of the side wall inclination on: (a) irradiance, (b) transmittance, and (c) mean relative deviation, at the bottom for square vessels.

- The aspect ratio of vessels with square base has a small effect on the average total vessel irradiance and vessel transmittance, with preference of low aspect ratio for the downward lighting systems, and high aspect ratio for the sideward lighting systems. it is recommended to avoid using square vessels with aspect ratio value of one to get better uniformity of irradiance distribution.

- For square vessels, higher total vessel irradiance at the vessel bottom is achieved with side wall of zero inclination and increase in the side walls inclination enhances the uniformity of irradiance distribution at the vessel bottom.
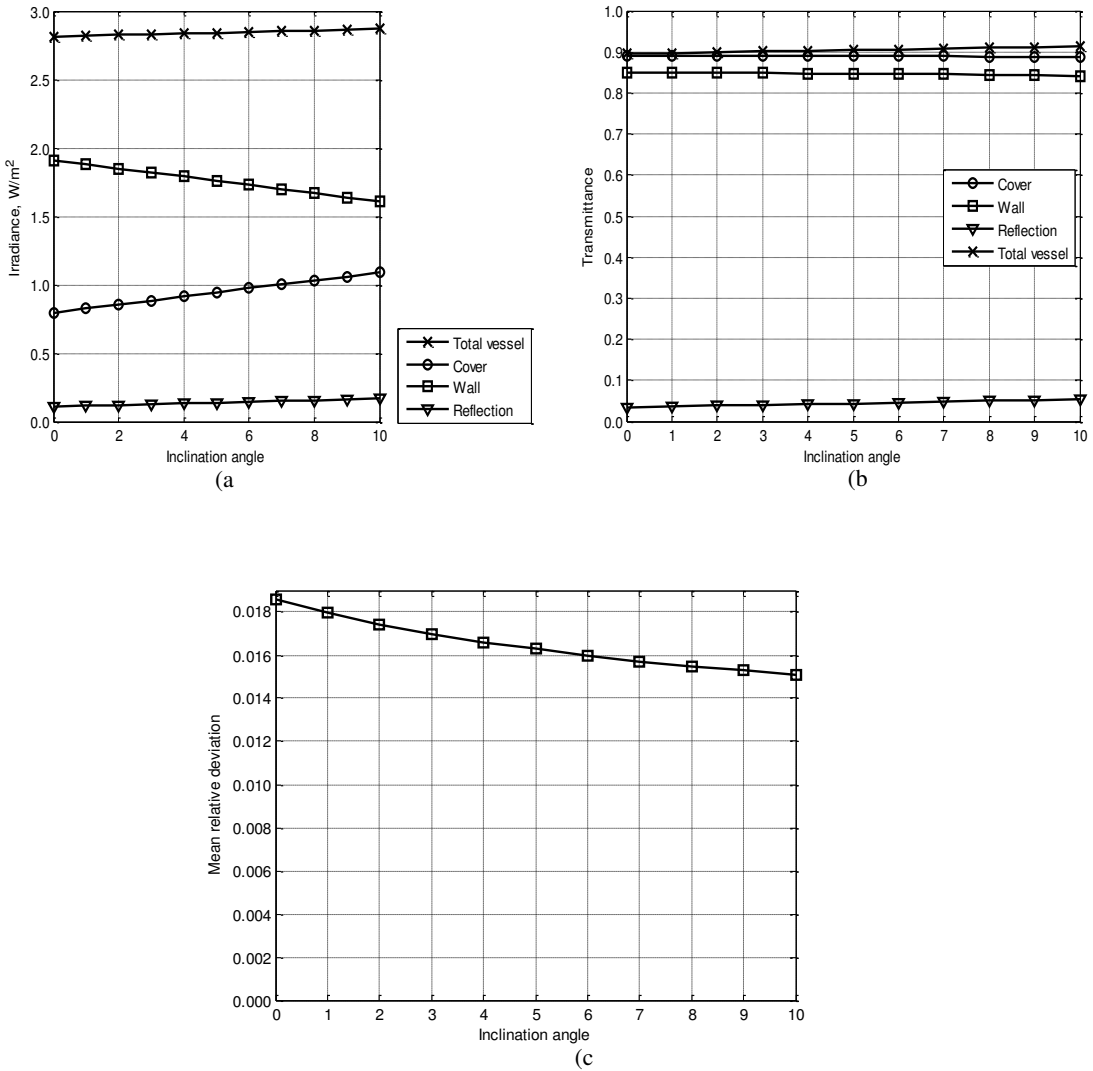

Fig. 16 Effect of the side wall inclination on: (a) irradiance, (b) transmittance, and (c) mean relative deviation, at the bottom for round vessels.

- Vessels with round base of low aspect ratio are recommended with downward lighting systems. The irradiance distribution uniformity for round based vessels can be enhanced by increasing the side wall inclination without affecting the total vessel irradiance at the vessel bottom.
- Vessels with square base are more preferable than that with round base because their total vessel transmittance is better for aspect ratio higher than 1.
- Square vessels always give irradiance distribution uniformity at the vessel bottom better than that of round vessels for zero side wall inclination. But, for higher inclination angle, the uniformity of distribution of irradiance for round vessels is always better than that of square vessels.

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نموذج رياضي للتنبؤ بالإشعاعية الضوئية
داخل أوعية إستزراع الأنسجة النباتية

في صناعة إستزراع الأنسجة النباتية، تعتبر الإشعاعية الضوئية واحدة من أهم العوامل البيئية التي تؤثر في نمو الشتلات النباتية داخل أوعية الإستزراع. و لما كان من الصعب قياس الإشعاعية الضوئية من الناحية العملية داخل أوعية إستزراع الأنسجة النباتية فقد أدت الحاجة إلى ضرورة التنبؤ بقيمة الإشعاعية الضوئية داخل أوعية الإستزراع. في هذه الدراسة نقدم نموذج رياضي قادر على التنبؤ بقيمة الإشعاعية الضوئية و درجة تمريرها عند أي نقطة داخل الأوعية وكذلك إنتظام توزيع الإشعاعية النافذة عند أي مستوي رأسي للنبات داخل الأوعية مع شدة ضوء منتظمة خارج الأوعية أخذاً في الإعتبار شكل و نوع المادة المصنعة منها الأوعية. و قد تم مقارنة النتائج التي تم الحصول عليها من هذا النموذج من خلال تطبيقه على بعض أنواع أوعية الإستزراع المعروفة مع قياسات عملية لقيمة الإشعاعية الضوئية لنفس هذه الأنواع أخذت على قاعدة هذه الأوعية. و من خلال هذه المقارنة وجد أن النموذج قادر على التنبؤ بالقيمة المتوسطه للإشعاعية الضوئية على قاعدة هذه الأوعية بنسبة خطأ تصل إلى 0.35 % في الأوعية المربعة, 0.26 % في الأوعية الخاصة نوع (GA-7), و 0.68 % فى الأوعية المستديرة. ومن خلال هذا النموذج تم دراسة خواص أوعية الإستزراع التي لها قاعدة مربعة و التي لها قاعدة دائرية. و تم أيضاً المقارنه بين خواص هذين النوعين. و قد وجد من هذه المقارنة أن الأوعية ذات القاعدة المربعة تعطي توزيع أفضل للإشعاعية الضوئية عن الأوعية ذات القاعدة الدائرية في حالة تعامد جدران الأوعية, بينما الأوعية ذات القاعدة الدائرية تعطي قيمة أفضل عن تلك ذات القاعدة المربعة لو زادت زاوية ميل الجدران عن درجة واحدة لنفس السمات الشكلية.