Journal of Engineering Sciences, Assiut University, Vol. 35, No. 3 pp. 665-680, May 2007

ADAPTIVE FUZZY IDENTIFICATION FOR NONLINEAR SISO SYSTEMS

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(Received December 27, 2006 Accepted June 14, 2007)

This paper investigates how SISO nonlinear systems can be adaptively identified using fuzzy systems which are independent of human knowledge. The proposed methodology uses the on-line data to build up the fuzzy system which approximate the nonlinear dynamics. After filtering the input, the nonlinear system is approximated by a set of fuzzy rules that describes the local linear systems. The Lyapunov direct method is utilized to derive the adaptive law of the proposed identification procedure. Theoretical results are simulated on a one-link robot. Results show that the proposed on-line identifier can consistently track mechanical friction and pay-load variations.

KEYWORDS: *Adaptive fuzzy models, Identification, Filter design, Lyapunov direct method, One-link robot.*

1. INTRODUCTION

Difficulties encountered in conventional modeling can arise from poor understanding of the underlying phenomena, inaccurate values of various process parameters, or from the complexity of the resulting model. A complete understanding of the underlying mechanisms is virtually impossible for a majority of real systems. For instance, in robotic systems variations of the pay-load are usually ignored in the modeling stage to simplify the equation of motion [1]. If all sources of uncertainty have been included, the resulting differential equations become very complex and hardly to deal with. Even if the structure of the model is determined [2], a major problem of obtaining accurate values for the parameters remains. It is the task of the system identification to estimate the parameters from measured data.

System identification, whether *on-line* or *off-line*, is an essential part of system design. Typical applications are the simulation, prediction and the control system design. In recent years, rapid development of intelligent control methodologies such as neural network [3,4], fuzzy logic theory[5-7], and rule-based expert systems [8] have provided alternative tools to tackle the problem of system identification. Fuzzy, neuro-fuzzy [3,6] and genetic-fuzzy systems [9,10] have been widely considered in literature. The aim is to establish optimal fuzzy systems that locally approximate the nonlinear system. Optimization is carried out using different criterion like stochastic and gradient methods [3,6,7,11]. However, most of these algorithms are advocated for off-line identification i.e. discrete-domain. Although, discrete fuzzy approximation models of

continuous-time systems are useful in many engineering applications, continuous-time models are often desired for the subsequent control system design [12,13].
Fuzzy identification of nonlinear systems is generally based on a fuzzy model that is constituted by a set of fuzzy *if-then* rules that maps inputs to outputs. A fuzzy model has excellent capability in complex and uncertain system description and is particularly suitable for modeling the nonlinear system by a set of fuzzy local models that are combined using a fuzzy inference mechanism corresponding to various operating points [14,15].

The aim of this paper is to develop a fuzzy input-output model expressed in the continuous-time domain which is an on-line identification for nonlinear SISO systems. The dynamic system is described by a group of fuzzy rule sets. Each fuzzy rule set is formed by a local linear dynamic system. The method utilizes the fact which states that any dynamic system (linear or nonlinear) can be approximated by a finite number of such rule sets, [7].

The paper is organized as follows. Section 2 introduces the underlying identification problem statement. In Section 3, the fuzzy system followed in this paper is introduced. The Section also includes Sub-Section for filter design. In Section 4, the adaptive law is derived. Section 5 demonstrates the implementation methodology. Simulation tests for one-link robots are given in Section 6. Section 7 offers our concluding remarks.

2. PROBLEM FORMULATION

Suppose an *n*-order SISO nonlinear system is bounded input bounded output (BIBO) stable system. It is expressed as follows:

$$\begin{aligned} y^{n}(t) &= f(u^{n-1}(t), \cdots, u(t), y^{n-1}(t), \cdots, y(t)) \\ &= f(U(t), Y(t)) \end{aligned} \tag{1}$$

where $y(t)$ is the output of the plant, $Y(t)$ is the vector of higher derivatives of the output, $Y(t) = [y^{n-1}(t), \cdots, y(t)]^T$, $Y(t) \in \mathbf{R}^{\mathbf{n}}$, $u(t)$ is the input of the plant, and $U(t)$ is the higher derivatives of the input, $U(t) = [u^{n-1}(t), \cdots, u(t)]^T$, $U(t) \in \mathbf{R}^{\mathbf{n}}$, $f(.) \in \mathbf{R}^{\mathbf{n}} \times \mathbf{R}^{\mathbf{n}} \to \mathbf{R}$ is an unknown smooth mapping defined on a compact set $\mathbf{Q} \subset \mathbf{R}^{\mathbf{n}} \times \mathbf{R}^{\mathbf{n}}$.

The nonlinear system can be approximated by a piecewise local linear system. The local linear system may be obtained by taking the Taylor series expression of nonlinear function around each equilibrium point.

Although, the exact mathematical expression of the real system is difficult to derive; the dynamics of local linear models at various operating points can be identified on-line from the measured input $u(t)$ and output $y(t)$ data pairs. Then, by associating the local linear system with the fuzzy membership function, the fuzzy dynamic model is formed. From the point of view of the fuzzy logic system, the fuzzy input-output model can be seen as a generic fuzzy system (see Fig. 1) which includes stable filter, fuzzy rule base, fuzzy inference engine and defuzzification. The coming Section gives details of the fuzzy logic system and the stable filter used in this work.

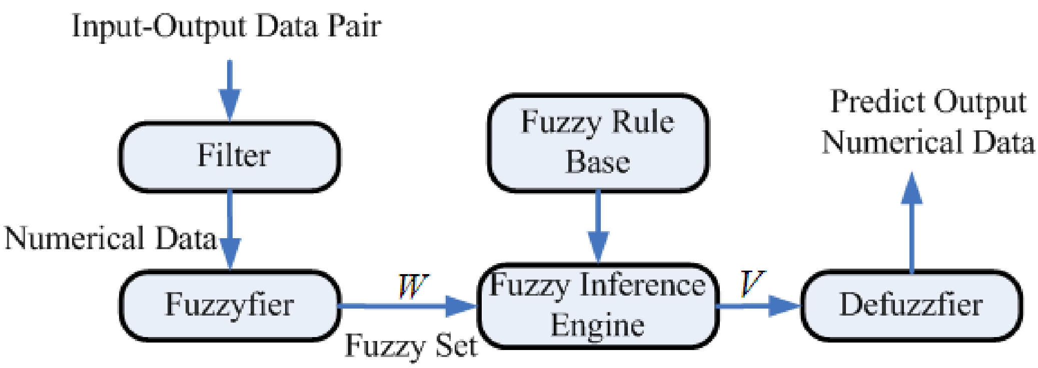

Fig. 1. Fuzzy input-output model.

3. FUZZY LOGIC AND FILTER DESIGN

3.1. Fuzzy Rule Base

The characteristics of the nonlinear system are described by a group of fuzzy rule sets shown as follows:

$$R^l: \textbf{if } \textit{input } u(t) \textbf{ is } A^l \textbf{ and } \textit{output } y(t) \textbf{ is } B^l \textbf{ then } \textit{the system output}$$
$$y^l(t) = \theta_U^{*l} U(t) + \theta_Y^{*l} Y(t), \qquad l = 1,2,\cdots nk \tag{2}$$

The antecedent in each rule is the input and output fuzzy sets and the consequent part is the crisp function representing the local linear characteristic. The parameter vectors θ_U^{*l} and θ_Y^{*l} represents the nominal parameters of the fuzzy model. $U(t)$ and $Y(t)$ are filtered higher derivative vectors of input and output, respectively. Note that it is neither desirable nor practical to obtain the actual derivatives of signals, which are inherently noisy.

The antecedents of the rule set describe fuzzy regions of the system states and input, and the consequent part of the rule is crisp function expressed in the state space equation. This fuzzy state-space model requires that the states in the antecedents are either measurable or estimated accurately.

3.2. Filter Design

The general practice in continuous-time identification is to filter the signal first and then obtain the filtered higher derivative [16]. Therefore, the higher derivative vectors $U(t)$ and $Y(t)$ in each rule set can be replaced by the output $w_p^{(u)}(t)$, $w_p^{(y)}(t)$ of two filters acting on the input $u(t)$ and output $y(t)$, Fig.1. The rule set can then be expressed as:

$$R^{l}: \textbf{if } \textit{input } u(t) \textbf{ is } A^{l} \textbf{ and } \textit{output } y(t) \textbf{ is } B^{l} \textbf{ then } \textit{the system output}$$
$$y^{l}(t) = \hat{\theta}_U^{l} w_p^{(u)}(t) + \hat{\theta}_Y^{l} w_p^{(y)}(t) = \hat{\theta}_l w_p(t) \tag{3}$$

where $l = 1, \cdots, nk$, $\hat{\theta}_l = [\hat{\theta}_U^{l}, \hat{\theta}_Y^{l}] = [b_1^l \; b_2^l \cdots b_n^l \; a_1^l \; a_2^l \cdots a_n^l]$ is the parameter vector to be identified, and $w_p(t) = [w_p^{(u)}(t), w_p^{(y)}(t)]^T \in \mathbf{R}^{2\mathbf{n}}$.

The consequent part of the rule sets is derived from the linear transfer function of the local system dynamics. Suppose the transfer function of the local dynamic system in the l^{th} rule set is expressed as:

$$P^{l}(s) = \frac{y_p^{l}(s)}{u^{l}(s)} = \frac{b_n^l s^{n-1} + b_{n-1}^l s^{n-2} + \cdots + b_1^l}{s^n + a_n^l s^{n-1} + a_{n-1}^l s^{n-2} + \cdots + a_1^l} \tag{4}$$

where $y_p^l(s)$, $u^l(s)$ are the Laplace transform of the output and input of the local linear system.

To identify the parameters of the transfer function, the higher derivatives of the output signals are required. A monic Hurwitz polynomial of degree n given in Equation (5) is introduced.

$$\hat{\lambda}(s) = (s + \lambda)^n = s^n + \lambda_n s^{n-1} + \cdots + \lambda_1 \tag{5}$$

The main requirement is to make sure that this filter covers the bandwidth of interest in order to ensure all modes of the local linear model are accounted for. From (4), we obtain (6) as follows:

$$y_p^l(s) = \frac{b^{*l}\psi(s)}{\hat{\lambda}(s)} u(s) = \frac{a^{*l}\psi(s)}{\hat{\lambda}(s)} y_p(s) \tag{6}$$

where $\psi(s) = [1 \; s \cdots s^{n-1}]^T$, $a^{*l} = [\lambda_1 - a_1^l \; \lambda_2 - a_2^l \cdots \lambda_n - a_n^l]$ and $b^{*l} = [b_1^l \; b_2^l \cdots b_n^l]$. Suppose the vectors $w_p^{(u)}(t)$ and $w_p^{(y)}(t) \in \mathbf{R}^{\mathbf{n}}$ satisfy the following linear relation:

$$\begin{aligned} \dot{w}_p^{(u)} &= \Lambda w_p^{(u)} + b_\lambda u(t) \\ \dot{w}_p^{(y)} &= \Lambda w_p^{(y)} + b_\lambda y_p(t) \end{aligned} \tag{7}$$

where

$$\Lambda = \begin{vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ -\lambda_1 & \cdots & \cdots & & -\lambda_n \end{vmatrix}, \quad b_\lambda = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{vmatrix} \tag{8}$$

Therefore, (6) can be rewritten as

$$\hat{y}_p^l(s) = b^{*l} w_p^{(y)}(s) + a^{*l} w_p^{(u)}(s) \tag{9}$$

The time response of the output of local dynamic system is expressed as

$$\hat{y}_p^l(t) = b^{*l} w_p^{(y)}(t) + a^{*l} w_p^{(u)}(t) = \theta^{*l} w_p(t) \tag{10}$$

where $\theta^{*l} = [b^{*l}, a^{*l}] \in \mathbf{R}^{2\mathbf{n}}$ and $w_p^T(t) = [w_p^{(u)}(t), w_p^{(y)}(t)]^T \in \mathbf{R}^{2\mathbf{n}}$.

So that, the output of each local linear system is a linear combination of the filter output.

3.3. Fuzzy Inference Engine

Fuzzy logic principles are used to combine the IF-THEN rules into a mapping from fuzzy sets in $W_i = A^l \times B^l$ to fuzzy sets in identifies output $W_o = C^l$. For each rule set, the fuzzy implication R^l is equal to the following equation:

$$\mu_{A^l \times B^l \to C^l}(\bar{x}, \bar{y}) \tag{11}$$

where $\bar{x} \in [u(t), y(t)],\ \bar{y} \in y^l(t)$.

The truth value of the proposition $y(t) = y^l(t)$ is calculated by Eq. (12) in which the product operation rule of fuzzy implication is adapted.

$$\begin{aligned} \mu_{W_o}(y^l) &= \left| u(t)\ is\ A^l and\ \ y(t)\ is\ B^l \right| \cap \left| R^l \right| \\ &= \left| \mu_{A^l}(u(t)) \mu_{B^l}(y(t)) \right| \cap \left| R^l \right| \end{aligned} \tag{12}$$

For simplicity, we assume $\left| R^l \right| = 1$. Therefore, the truth value of the consequence obtained is as follows:

$$\mu_{W_o}(y^l) = \mu_{A^l}(u(t)) \mu_{B^l}(y(t)) \tag{13}$$

3.4. Fuzzification

The fuzzifier performs a mapping from crisp point $(u(t), y(t))^T \in W_i$ into a fuzzy sets (A^l, B^l). The fuzzy sets (membership functions) represent the meaning of the linguistic variables of the input and output measurements of the SISO nonlinear system. Throughout this work, Gaussian membership functions are used to compute the truth of each rule. This is a requirement for the fuzzy system to be regarded as universal approximator; [7].

The universe of discourse of $u(t)$ and $y(t)$ are equally divided into fuzzy m subspaces, Fig. 2. The number of fuzzy partition m represents the granularity of the fuzzy model. It is directly related to the matching accuracy of the model of the nonlinear system. The larger the fuzzy partition, the more accurate the fuzzy model is. The number of rules is m^2. It should be noted that as the number of rules increases, the dimension of the parameter matrix increases as well. Thus, the choice of the number of rules is based on a compromise between matching performance and the computation load.

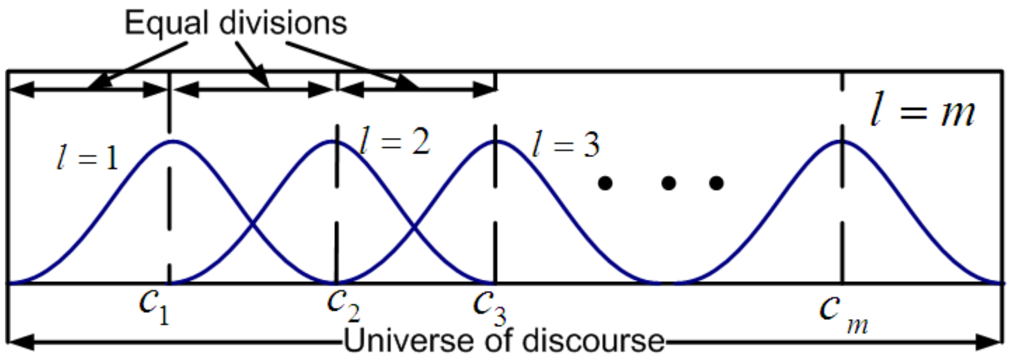

Fig. 2. Fuzzy partition of the universe of discourse.

3.5. Defuzzification

The defuzzfier performs a mapping from fuzzy sets in W_o to a crisp point $y \in W_o$. There are several methods for defuzzification [3,6] such as maximum defuzzification, center average defuzzification, and height defuzzification. In this work, center average defuzzifier is adopted. The final value output $y(t)$ of the nonlinear system inferred from nk implication is given an average of all $y^l(t)$ with the weight $\mu_{W_o}(y(t))$:

$$\hat{y}(t) = \frac{\sum_{l=1}^{nk} \mu_{W_o}(y^l) \times y^l(t)}{\sum_{l=1}^{nk} \mu_{W_o}(y^l)} \tag{14}$$

Let $\zeta^l = \mu_{W_o}(y^l) / \sum_{l=1}^{nk} \mu_{W_o}(y^l)$, $Z = [\zeta^1 \ \zeta^2 \cdots \zeta^{nk}] \in \mathbf{R}^{\mathbf{nk}}$. Then Eq. (14) becomes

$$\begin{aligned} \hat{y}(t) &= \sum_{l=1}^{nk} \zeta^l y^l(t) = \sum_{l=1}^{nk} \xi^l \hat{\theta}^l w_p(t) \\ &= Z\hat{\Theta} w_p(t) \end{aligned} \tag{15}$$

where

$$\hat{\Theta} = \begin{vmatrix} b_1^1 & b_2^1 & \cdots & b_n^1 & a_1^1 & a_2^1 & \cdots & a_n^1 \\ b_1^2 & b_2^2 & \cdots & b_n^2 & a_1^2 & a_2^2 & \cdots & a_n^2 \\ & & & & \vdots & & & \\ b_1^{nk} & b_2^{nk} & \cdots & b_n^{nk} & a_1^{nk} & a_2^{nk} & \cdots & a_n^{nk} \end{vmatrix} \in \mathbf{R}^{\mathbf{nk}} \times \mathbf{R}^{2\mathbf{n}} \tag{16}$$

is the parameter matrix to be identified.

Z can be viewed as the degree vector denoting the contribution of each local linear system on the overall system. It includes the influence of the membership function for the antecedent and consequent reference fuzzy sets, and operators for the logic connectives, inference and defuzzification.

At current stage of the research, we have selected the fuzzy system and the coming Section clarify the adjustment of the parameter matrix $\hat{\Theta}_{nk\times 2n}$. The parameter matrix $\hat{\Theta}_{nk\times 2n}$ is a set of the free parameters that can be adjusted during the identification in order to make the output of fuzzy model $\hat{y}(t)$ track the actual output of the plant.

4. FUZZY ON-LINE IDENTIFICATION

From the above discussion, we deduce that for a given nonlinear system, there exists an approximate fuzzy input-output model. Therefore, the nonlinear on-line identification is transformed into the problem of on-line identification of each local linear system. The task of fuzzy on-line identification is to tune the parameter matrix $\hat{\Theta}_{nk\times 2n}$ in the fuzzy model to a nominal parameter matrix $\hat{\Theta}^{*}_{nk\times 2n}$. In this work, the performance index indicating the goodness of the approximation is defined as the square of difference between the identification data $y(t)$ and the fuzzy model estimation $\hat{y}(t)$.

$$J(\hat{\Theta}) = e^2(\hat{\Theta}) = (\hat{y}(t) - y(t))^2 \tag{17}$$

The nominal parameter matrix $\Theta^{*}_{nk\times 2n}$ is determined as the minimum of $J(\Theta)$ as follows:

$$\Theta^{*}_{nk\times 2n} = \mathbf{arg\ \ min}\ \ J(\hat{\Theta})$$

Consequently, the identification problem has been transformed to an optimization problem. The on-line identification should arrive at the following objective:

$$\lim_{t\to\infty} \hat{\Theta} = \Theta^{*} \ \text{ or } \lim_{t\to\infty} \left| e(t) \right| = 0 \tag{18}$$

4.1. Error Analysis

Fuzzy on-line identification aims at identifying the parameters of each local linear system. At the beginning of identification, the parameter vector series $\hat{\theta}^{l}(l = 1,2,\cdots,nk)$ is set arbitrarily. As time goes by, the parameters in each local system should be tuned to the “optimal” values. Now, the on-line identification problem is focused on deriving the update law for the parameter matrix in the fuzzy model.

Consider the error between the identification data $y(t)$ and the fuzzy model estimation $\hat{y}(t)$:

$$\begin{aligned} e(\Theta) &= \hat{y}(t) - y(t) = \sum_{l=1}^{nk} \zeta^{l}(\hat{\theta}^{l} - \theta^{*l}) w_p(t) \\ &= \sum_{l=1}^{nk} \zeta^{l} \tilde{\theta}^{l} w_p(t) = \mathrm{Z}\tilde{\Theta} w_p(t) \end{aligned} \tag{19}$$

where

θ^{*l} is the optimal value of the parameter vector in the l^{th} rule,

$\hat{\theta}^{l}$ is the identified value of the parameter vector in the l^{th} rule,

$\tilde{\theta}^{l} = \hat{\theta}^{l} - \theta^{*l}$ is the error parameter vector in the l^{th} rule,

$\tilde{\Theta} = \Theta^{*} - \hat{\Theta} \in \mathbf{R}^{\mathbf{nk}} \times \mathbf{R}^{2\mathbf{n}}$ is the error in parameter matrix,

Θ^{*} is a constant nominal parameter matrix.

The objective of identification can be stated as the determination of an algorithm for adjusting $\hat{\theta}^{l}$ or $\hat{\Theta}$ so that the error parameter vector $\tilde{\theta}^{l}$ or the error parameter matrix $\tilde{\Theta}$ tend to zero as $t \to \infty$. It is clear that if input $u(t)$ and output $y(t)$ are uniformly bounded and $\tilde{\theta}^{l}$ or $\tilde{\Theta}$ tend to zero the error $e(t)$ will also tend zero asymptotically. Hence, if exact parameter estimation is carried out, the convergence of the output error to zero will necessarily follow.

4.2. Adaptive Law Synthesis

The following theorem summarizes the convergence analysis of fuzzy on-line identification algorithm. The direct Lyapunov method is used in the proof.

Theorem. *For a single input/single output nonlinear system, subject to any bounded continuous input* $u(t)$ *with the bounded output* $y(t)$, *there exists a fuzzy model* $\hat{y}(t)$ *in the following form:*

$$R^{l} \textbf{ : if } u(t) \textbf{ is } A^{l} \textbf{ and } y(t) \textbf{ is } B^{l} \textbf{ then } y^{l}(t) = \hat{\theta}_{l} w_{p}(t) \tag{20}$$

where $l = 1, 2 \cdots, nk$ *with the update law as:*

$$\dot{\hat{\Theta}} = -g e(t) Z^{T} w_{p}(t), \qquad g > 0 \tag{21}$$

such that

$$\lim_{t \to \infty} \left| f(Y(t), U(t)) - \hat{y}(t) \right| = 0 \tag{22}$$

Proof. The justification for the choice of the update law (21) is based on the following Lyapunov function candidate:

$$V(\tilde{\Theta}) = \sum_{l=1}^{nk} \frac{\tilde{\theta}^{l^{T}} \tilde{\theta}^{l}}{2} \geq 0 \tag{23}$$

The time derivative is obtained as:

$$\dot{V}(\tilde{\Theta}) = \sum_{l=1}^{nk} \tilde{\theta}^{l^{T}} \dot{\tilde{\theta}}^{l} \tag{24}$$

if $\dot{\tilde{\theta}}^{l}$ is selected as (21), then:

$$\dot{V}(\tilde{\Theta}) = \sum_{l=1}^{nk} \tilde{\theta}^{l^T} \dot{\tilde{\theta}}^{l} = \sum_{l=1}^{nk} \tilde{\theta}^{l^T} e(t) g \ \zeta^{l} w_p(t)$$
$$= \sum_{l=1}^{nk} - g e(t) \tilde{\theta}^{l^T} \zeta^{l} w_p(t) \tag{25}$$

Because $e(t) = f(U(t), Y(t)) - \hat{y}(t) = \sum_{l=1}^{nk} \zeta^{l} \tilde{\theta}^{l^T} w_p(t)$, then (25) becomes:

$$\dot{V}(\tilde{\Theta}) = -g e^{2}(t) \le 0 \tag{26}$$

Therefore, V, $\tilde{\Theta}$ and $\hat{\Theta}$ are uniformly bounded. Also, because $V(t)$ is monotonically decreased and bounded, $\lim_{t \to \infty} V(\tilde{\Theta}) = V_\infty$ exists.

Since both $u(t)$ and $y(t)$ are bounded signals, $u(t)$ and $y(t)$ belong to L_∞ space. According to (7) it is inferred that $w_p^{(u)}(t)$ and $w_p^{(y)}(t)$ belong to L_∞ space. Thus $w_p(t)$ also belongs to L_∞ space. Let

$$c_1 = \sup_{t>0} \left\| w_p(t) \right\| \tag{27}$$

Then we get the inequality:

$$\left\| 1 + \gamma w_p^{T}(t) w_p(t) \right\| \le 1 + \gamma^{*} c_1^{2} \tag{28}$$

Integrating both sides of Eq. (26), we get from $t = 0$ to $t = \infty$:

$$\int_0^\infty e^{2}(\tau) d\tau \le \frac{1 + \gamma^{*} c_1^{2}}{g} (V_o - V_\infty) \quad \text{where } V_o = V(\tilde{\Theta}(0)) \tag{29}$$

According to the definition of L_2 space [17], we know that $e(t) \in L_2$. Since the filtered signals w and $\dot{w}$ belong to L_∞ space, both are bounded. Therefore, $e(t)$ and $\dot{e}(t)$ are bounded as well. By Barbarat's lemma [2], $e(t)$, $\dot{e}(t) \in L_\infty$ and $e(t) \in L_2$, it implies $e(t) \to 0$ as $t \to \infty$. This implies that for a SISO nonlinear system with bounded input and output, there exists a fuzzy model (Eqs. 14 - 15), with the output error $e(t)$, such that:

$$\lim_{t \to \infty} \left| e(t) \right| = 0 \tag{30}$$

Remark 1. The adaptive law (21) is proportional to the gradient of the output squared. This can be easily verified as follows:

$$e(\Theta) = \hat{y}(t) - y(t) = \sum_{l=1}^{nk} \zeta^{l} (\hat{\theta}^{l} - \theta^{*l}) w_p(t)$$
$$= \sum_{l=1}^{nk} \zeta^{l} \tilde{\theta} w_p(t) = Z \tilde{\Theta} w_p(t) \tag{31}$$

and

$$\frac{\partial}{\partial\Theta}(e^2(\hat{\Theta})) = 2e(\Theta)\frac{\partial e(\Theta)}{\partial\hat{\Theta}} = 2e(\Theta)\mathrm{Z}w_p(t) \tag{32}$$

This suggests that the update law (21) can be regarded as the steepest decent solution to the underlying identification question.

Remark 2. For linear dynamic system, perfect identification depends on the nature of the input [18]. The input signal should have the property of persistent excitation in order to guarantee that the error converges to zero. The same condition applies to the nonlinear system and the local linear models.

5. IMPLEMENTATION

The proposed algorithm for the fuzzy on-line identification is concluded from the above analysis. A step by step procedure for the implementation of the algorithm is now stated.

1. Select a stable filter for the input and output of the plant (5). The order of the stable filter is equal to that of the plant n.
2. Initialize the parameter matrix $\hat{\Theta}_{nk\times 2n}$.
3. Apply the filter to the input and output of the plant, then their higher derivatives can be obtained from the regressive vector $w_p(t) = [w_p^{(u)}(t), w_p^{(y)}(t)]$.
4. Fuzzify the input $u(t)$ and output $y(t)$. The membership functions are selected as Gaussian and m linguistic variables for both $u(t)$ and $y(t)$. The number of rules describing the nonlinear system is $nk = m^2$. Therefore, the truth value of the proposition $y(t) = y^l(t)$ is calculated as Eq. (13) and the output weighting vector Z is calculated according to (15).
5. In each rule, the characteristic of the plant is described as the following local linear system:

   R^l **: if** *input* $u(t)$ **is** A^l **and** *output* $y(t)$ **is** B^l **then** $y^l(t) = \hat{\theta}_l w_p(t)$

   The output of the plant $\hat{y}(t)$ is calculated from (14).
6. Calculate the error between the output of fuzzy model and the output of the actual nonlinear plant: $e(\hat{\Theta}) = \hat{y}(t) - y(t)$.
7. Update the parameter matrix $\hat{\Theta}_{nk\times 2n}$ according to Eq. (21).
8. Return to step 3.

The two parameters which should be known before identification are the order n of the plant and the pole λ of the filter. These two parameters are determined based on the prior information of the plant. The requirement of $\lambda(s)$ is not very restrict. Besides being a stable Hurwitz polynomial, $1/\lambda(s)$ should cover the bandwidth of interest. If the exact order n of the plant is not known, an approximate value around n of each local linear plant still ensures the match accuracy of the fuzzy model. The reason is that the parameters of each linear local system are tuned to the "optimal" values to make the output of the fuzzy model to match the output of the plant.

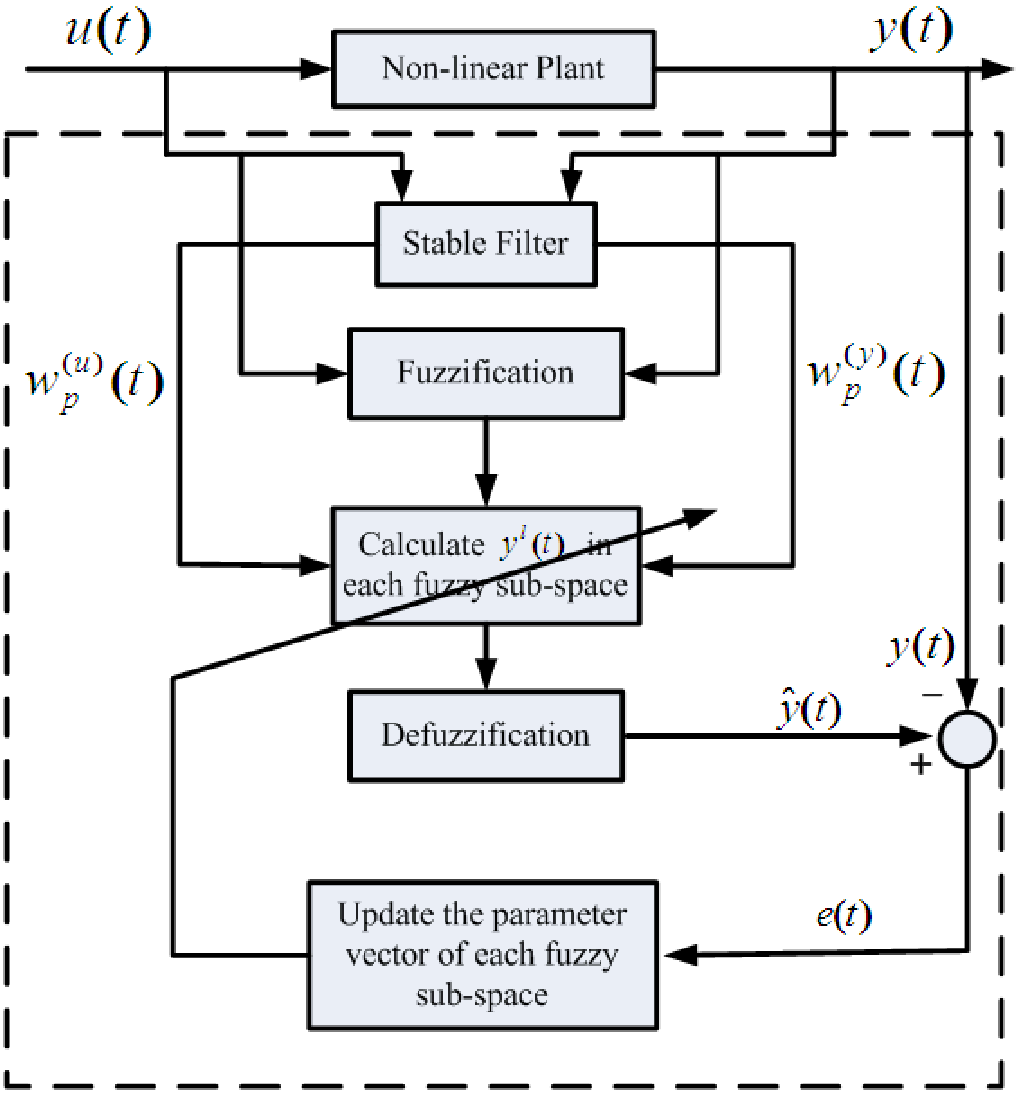

Fig. 3. Fuzzy on-line identification implementation

6. SIMULATION TESTS OF ONE-LINK ROBOT

In this Section, the following one link robot is used to demonstrate the proposed fuzzy on-line identification algorithm. As second order systems, robots can be viewed as highly nonlinear systems. Figure 4 shows a diagrammatic sketch for one-link robot whose dynamic equation of motion can be described as [1]:

$$(I + k_r^2 I_m)\ddot{q} + F\dot{q} + mgl \sin(q) = u$$
$$u = 10 \cos(q)$$

where I is the load mass moment of inertia, l and m are its length and mass, $K_r > 1$ is the gear ratio, I_m is the motor mass moment of inertia and F is the friction torque. The parameter values are listed in Table 1.

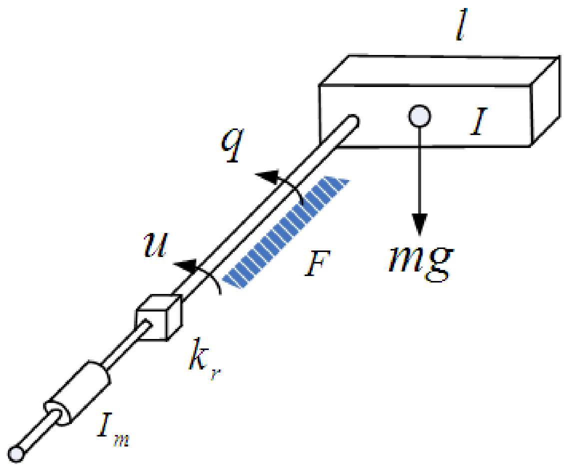

Fig. 4. One-link robot.

Parameter	Value	Value after 50 sec
m mass (kg)	1.0	15.0
l length (m)	0.6	No change
F friction coefficient $N.m.\sec/\ rad$	10.0	No change
K_r reduction ratio	20.0	No change
I inertia around c.g. ($kg.m^2$)	5.0	20.0
I_m inertia of the motor ($kg.m^2$)	0.1	No change

Table 1: Parameters of the robot arm.

The system is initialized with $q(0) = 0$. To check the ability of the proposed algorithm, it is assumed that load parameters changed to the values listed in column 3 after 50 seconds. Simulation results are shown in Figs. 5-9. For this nonlinear system, the poles of the filter are selected as $\lambda = .05$. The order of the local linear system to be identified is selected as $n = 1$ and the number of fuzzy subspace is selected as 15. So that the dimension of the parameter matrix to be identified is 225×2. The parameter matrix is set to null at the beginning of the identification. One may notice that the error (Fig. 6) between the output of the actual nonlinear system and the output of the fuzzy model is relatively large during the first two periods. After two periods, the error begins to converge.

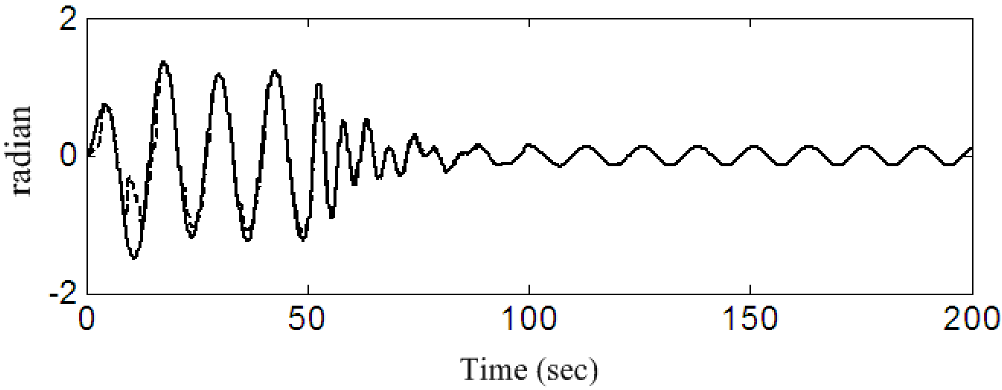

Fig. 5. Actual output (solid line) and the estimated output (dashed line).

Since the order of the local linear system is set as $n = 1$, the model in each fuzzy rule is as follows:

$$R^l \textbf{ : if } \textit{ input } u(t) \textbf{ is } A^l \textbf{ and } \textit{ output } y(t) \textbf{ is } B^l \textbf{ then } y^l(t) = b_1 U(t) + a_1 Y(t)$$

Figure 7 and 8 show respectively the time history of b_1 and a_1 in the 1^{st} , 30^{th} and 225^{th} local linear systems. Figure 9 shows the output surfaces of the adaptive fuzzy system at different stages of the identification process.

It follows that the proposed fuzzy on-line identification method can identify the system on-line.

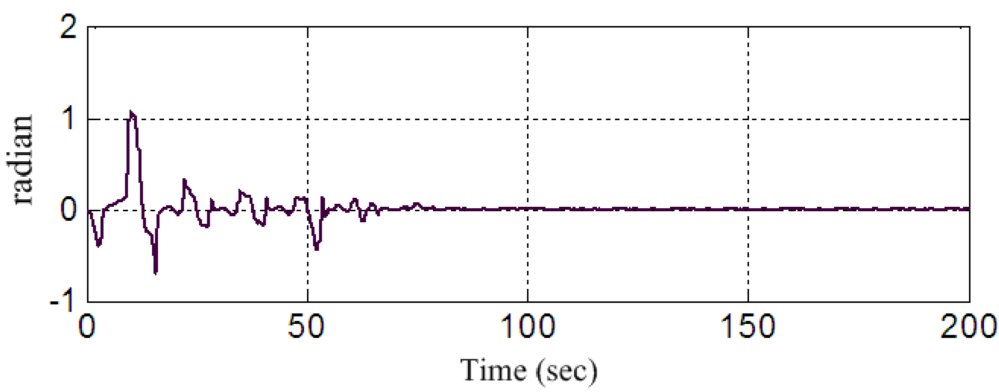

Fig. 6. Identification error

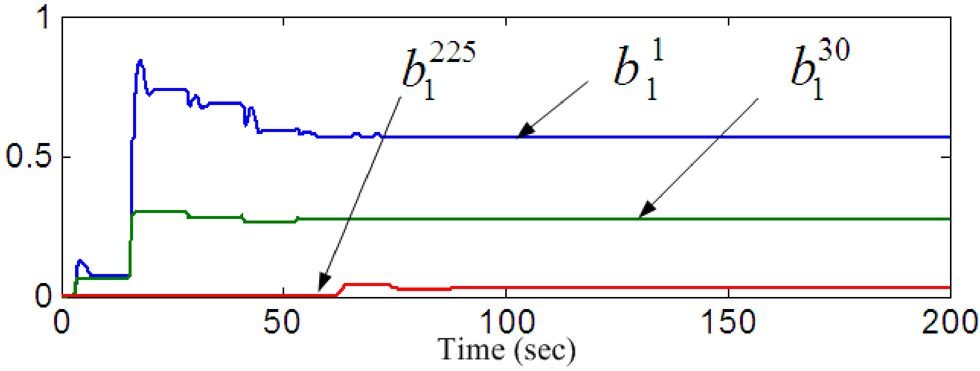

Fig. 7. Time history of some parameters of the parameter matrix (coefficients of the output)

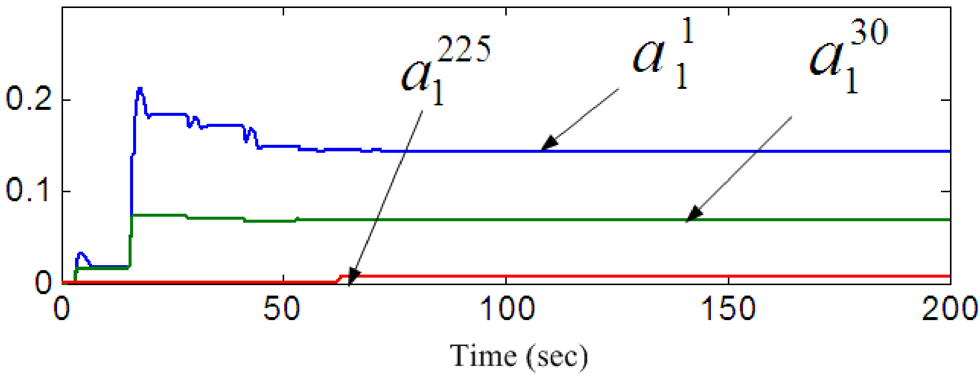

Fig. 8. Time history of some parameters of the parameter matrix (coefficients of the input)

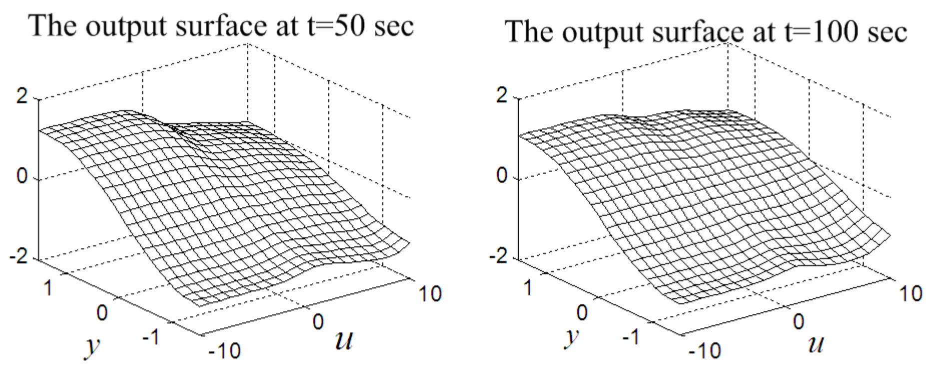

Fig. 9. The output surfaces of the fuzzy identifier at different stages.

7. CONCLUSIONS

In this paper, an adaptive fuzzy identification method for SISO nonlinear system has been proposed. The key feature of the algorithm is the integration of conventional on-line identification with fuzzy logic theory. The identification process is entirely designed in continuous time domain in contrast to other works in literature which are designed in discrete time domain, i.e. off-line identification. Simulation results of one link robot have demonstrated the capabilities of this method and have shown that the algorithm can effectively match the time varying nonlinear systems.

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تمييز هلامى متأقلم للأنظمه اللاخطيه أحاديه المدخل والمخرج

تأتى أهمية تمييز الأنظمه الهندسيه منً صعوبة عمل نموذج رياضى لها لأسباب عده من بينها:

- صعوبه الفهم الكامل لعمل هذه الأنظمه,
- عدم أمكانية تحديد عناصرها بدقه,
- صعوبه ايجاد النموذج الرياضى خاصة للأنظمه المركبه.

فى هذه الدراسه يتناول البحث كيفية تمييز الأنظمه اللاخطيه أحاديه المدخل والمخرج (SISO) بأستعمال منظومه هلاميه متأقلمه لاتعتمد على أى معرفه مسبقه للنموذج الرياضى (المعادلات التفاضليه الزمنيه). الطريقه المقترحه فى هذه الدراسه تستعمل البيانات الناتجه أثناء التشغيل لأستنتاج منظومه هلاميه تقريبيه لوصف العلاقه بين المدخل والمخرج للمنظومه الهندسيه. بكلمات أخرى, الطريقه المقترحه تعتمد على ايجاد نماذج رياضيه خطيه هلاميه تصف العلاقه بين مدخل المنظومه الهندسيه ومخرجها.
تم تطبيق الدراسه على ذراع آلى أحادى الوصله فأثبتت نتائج المحاكاه الرياضيه قدره فائقه للمنظومه الهلاميه على وصف حركه الذراع الآلى.