

A STUDY ON FLEXURAL BEHAVIOR OF PRESTRESSED CONCRETE SIMPLE BEAMS WITH EXTERNAL TENDONS UNDER STATIC LOADS

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Using a verified finite element algorithm, non-linear analysis of externally prestressed concrete beams was carried out to investigate the flexural behavior of such beams under static loads. The behavior of such beams is represented by; change of tendon's eccentricity, load deflection diagram, the relations of load and stress in external tendons, effective depth of beam, and bond strain reduction factor. The parameters affecting the behavior of such beams include: span to depth ratio, distance between loads to span ratio, distance between deviators to span ratio, the ratio of prestressing index and reinforcement index ρ/ρ_c concrete compressive strength and tensile and compressive reinforcement ratios. The strain variation in an external tendon was investigated on the basis of the deformation compatibility of beam. The proposed method for the numerical analysis can satisfactorily predict the behavior of externally prestressed concrete beams up to the ultimate loading stage. The stress increase in an external tendon depends mainly on the overall deformation of beam. The behavior of externally prestressed concrete beams is different than that of bonded prestressed beams due to the second order effect of tendon's eccentricity. This point is clearly investigated in the current study. From the obtained results, a model was proposed to estimate both of the bond strain reduction factor (R) and effective depth reduction factor (K) considering the effect of the parameters affecting the behavior of such beams. Based on numerical analyses, a model was proposed to calculate the flexural strength of externally prestressed concrete beams considering the parameters of the study.

KEYWORDS: Externally prestressed beam, external tendon, deviators, flexural strength, bond strain reduction factor, effective depth

NOTATION

A_{ps}	cross sectional area of external tendon	d_{pu}	effective depth of external tendon at ultimate stage (mid-span)
A_s, A'_s	area of tension and compression reinforcement, respectively	d_{ps}	effective depth of external tendon at initial stage (mid-span)

d_{pe}	effective depth of external tendon at initial stage (ends)	L_t	length of tendon between end anchorages
d_s	effective depth of tension steel	L_p	distance between loading points
d'_s	effective depth of compression steel	L/d_{ps}	span-to depth ratio,
f'_c	concrete compressive strength	L_p/L	distance between point loads to span ratio
f_{pe}	stress in tendon at effective prestressing stage	S_d/L	distance between deviators to span ratio
f_{py}	yielding strength of prestressing tendon	S_d	distance between deviators
f_{pu}	nominal ultimate strength of prestressing tendon	ω_s	reinforcing index of tension steel

INTRODUCTION

Externally prestressed concrete (PC) beams are widely used nowadays – especially for bridges - because of its great merits such as simplicity of construction, cost effectiveness and construction time savings [1-8]. This type of structures could be applied not only to new structures but also to those, which need to be repaired or strengthened. External prestressing is defined as prestress tendon introduced by the high strength cable, which is placed outside the cross section and attached to the beam at some deviator points along the beam [1-3]. The prestressing force is transferred to the beam through the end anchorages and deviators. Although various advantages of external prestressing have been reported, some questions concerning the behavior of externally prestressed concrete beams at ultimate are often arisen in the design practice. The complication involves the overall analysis of the whole structure deformation (member – dependant) and the external tendon's eccentricity is some of the problems facing the analysis of such type of structures. One of major problems concerning the beams prestressed with external tendons is in calculating the tendon stress beyond the effective prestress. In the case of beams prestressed with bonded tendons, since the tendon strain is assumed to be the same as the concrete strain, the calculation of tendon strain under the applied load is a problem related only to a section of maximum moment, i.e., the increase of tendon strain is section-dependent [1-3]

This is totally different in the case of beams prestressed with external tendons. Since the tendon is unbonded, the tendon freely moves in the relation of beam deformation. Therefore, the tendon strain is basically different from the concrete strain at every cross section, i.e., the tendons strain cannot be determined from the local strain compatibility between the concrete and the cable. For the calculation of cable strain, it is necessary to formulate the global deformation compatibility of beam between the extreme ends. The strain variation in an external tendon should be considered to be a function of the overall deformation of beam. This means that the strain change in the tendons is member-dependent, and is influenced by the initial tendon profile, span to depth ratio, deflected shape of the structure, friction at the deviators, the initial

condition of beam, etc. [1 -9]. This makes the analysis of a beam with external tendons more complicated, and proper modeling of the overall deformation of beam becomes necessary. Figure 1 illustrates the typical deflected shape and the change of tendon's eccentricity in externally PC beam. Figure 2 illustrates the details of externally PC simple beam showing the beam, deviator section and the end section of the beam. Figure 3 illustrates the details of reinforcement.

When the behavior of externally prestressed concrete beams was investigated, many researchers attempted to calculate the increase of tendon stress either by using their formulations with some parameters involved for the certain cases [1-14] or by using equations, which are provided in the codes for unbonded beams.

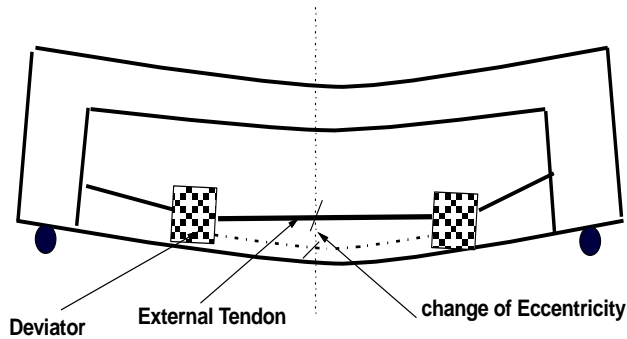


Fig. 1 Typical externally Prestressed Beam

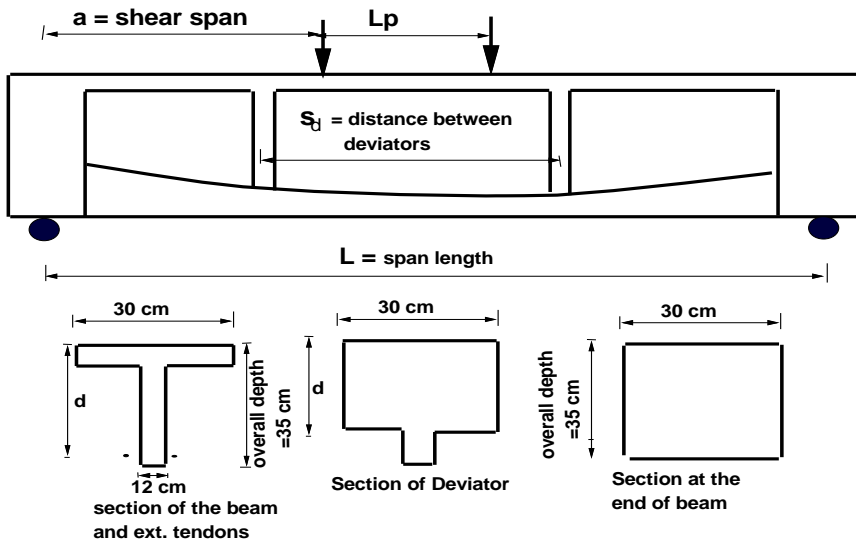


Fig. 2 Details of externally PC simple beam

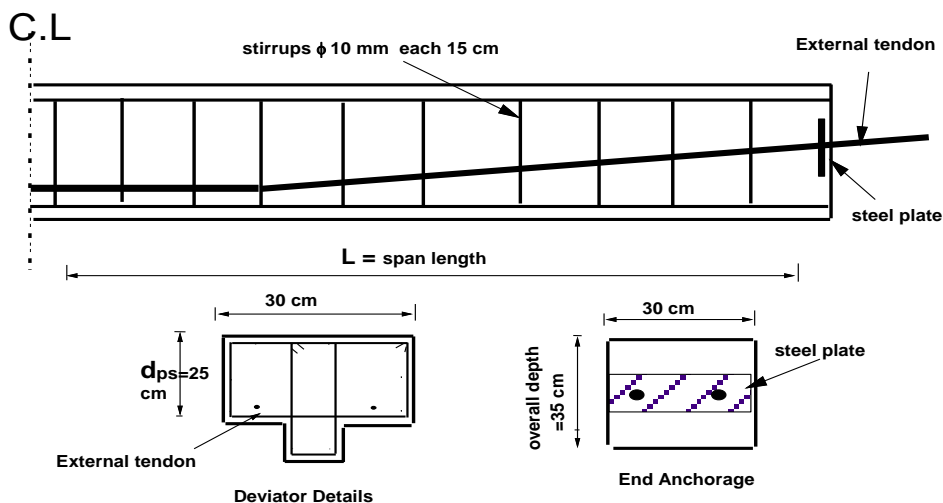


Fig. 3 Details of reinforcement

While some researchers assumed that the strain variation in a tendon is uniform over its entire length, and tried to calculate the tendon strain by adopted assumption, which stated that the total elongation of a tendon must be equal to the total elongation of concrete at the tendon level [1-14]. Since the prestressing force transfers to the concrete beam through the deviator points and anchorage ends, the tendon friction obviously exists at the deviators, resulting in a different level of strain increase between the two successive tendon segments. However due to the complicated calculation of cable strain, almost all analytical approaches did not consider friction at the deviators into account because of its unknown extent. For simplicity in the calculation of cable strain, two extreme cases are usually considered, namely, free slip (no friction) and perfectly fixed (no movement) at the deviator. In the first case, the tendon moves freely through the deviators without any restraint, and the cable is treated as an internally unbonded cable. The tendon strain is assumed to be constant over its entire length regardless of the friction at the deviators. The characteristic behavior of externally prestressed concrete beams at the ultimate state are a research topic, which has yet to be well understood in any depth.

Research Significance

The current study is carried out to investigate the structural behavior of externally PC beams under flexural loads, which is dominant on such a long span structure. A numerical analysis model, which was previously developed in Japan, was utilized to investigate the flexural behavior of externally PC beams throughout the loading up to ultimate state. The model has been adopted from the fundamental assumptions used for flexural members (beam theory).

Non-linear Analysis algorithm

A previously developed nonlinear model was adopted in the analysis [1-5]. The following assumptions were used: Plane sections still remain plane after bending; only flexural deformations are considered and shear deformations are neglected; nonlinearity of materials (concrete, reinforcement and prestressing tendons) are considered through constitutive equations as shown in Fig. 4 (a, b and c) and external tendons are free to slip at deviator points or the increase of strain in external tendon is uniformly distributed along the whole length. The last assumption implies the concept of deformation compatibility which means that the deformation of concrete located at prestressing tendon level equals to that of elongation of tendons. The model was modified [1, 2] to consider the shear deformations. It consists of three main parts; data input; computational loop and output of the results. Fig. 5 illustrates the model of PC beam used in the analysis. The program includes the following iterative loops:

1- **Force Equilibrium** (Loop 1): Compression forces in concrete:

$$C_{conc.} = \sum_{x_n=0}^{x_n=c} \sigma_c(x_n) b \Delta x - \sum_{x_n=h_f}^{x_n=c} \sigma_c(x_n) (b - b_w) \Delta x \tag{1}$$

Tensile forces in reinforcement and prestressing tendons

$$T_s = A_s \sigma_s - A_s' \sigma_s' + A_{ps} \sigma_{ps} \tag{2}$$

Where, $\sigma_c(x_n), \sigma_s, \sigma_{ps}$ are stress of concrete, reinforcement and prestressing tendon respectively, which are calculated from constitutive relations. The other items of equations 1 and 2 are illustrated in Fig. 6 (a, b, and c).

2- **Moment Equilibrium** (Loop 2): In this step, the calculation involves the moment equilibrium of all integration points. The equilibrium of internal forces will be repeated in iterative loop 1 by assuming the compressive concrete strain at the extreme fiber $\epsilon_c(i)$ and strain distribution. The moment of internal force $M_i(i^{th} = 1, 2, \dots)$ can be simultaneously obtained and the iterations continue till the difference between any moment and the previous moment is less than 1%.

3- **Compatibility of Deformation** (loop 3): Once the equilibrium of forces and moment is achieved for all integration points, the deformation of concrete at tendon level δ_c is calculated by numerical integration of the average strain increase in concrete at tendon level, $\Delta\epsilon_{c,ps}$, between end anchorages, $\delta_c = \sum \Delta\epsilon_{c,ps} \Delta L$. The total elongation of external tendons δ_f is obtained from the strain increase in tendon $\Delta\epsilon_{ps}$, that is: $\delta_f = \sum \Delta\epsilon_{ps} L_t$ as shown in Fig. 7.

4- **Evaluation of Change of External Tendon's Eccentricity**: The term of change of external tendon's eccentricity is defined in the external prestressing system when the tendons are arranged outside the cross section of the beam. When the beam of external tendons is subjected to bending, the tendon will not follow the beam deflection except at deviator points. The large deflection of the beam, particularly at the failure, could induce second order effect on flexural strength of the beam. The effect of change of external tendon's eccentricity is considered in the analysis for accurate estimation of beam. Fig. 8 illustrates the evaluation of effective depth of external tendons to consider

such change of tendon’s eccentricity. The value of deflection of tendon d_{ps} is given as follows:

a) For $Lx < L1L$

$$d_{ps} = d_{pe} + \frac{(d_{pm} - d_{pe}) * Lx}{L1L} - \Delta_i \tag{3}$$

b) For $L1L < Lx(i) < L1L + L2$

$$d_{ps} = d_{pm} + \Delta_1 + \frac{(d_1 - d_2) * (Lx - L1L)}{L2} - \Delta_i \tag{4}$$

3) For $Lx(i) > L1L + L2$

$$d_{ps} = d_{pc} + \frac{(d_{pm} - d_{pe} + \Delta_2) * (Lx - Lx)}{L1R} - \Delta_i \tag{5}$$

Where, d_{pe} , d_{pc} , d_{pm} are effective depth of tendon at end supports, deviator and middle span respectively. The other items are defined in Fig. 8. Appendix A illustrates the flow chart diagram of the program.

In previous studies [1-4], a non-linear finite element program together with the displacement control method had been developed to obtain the entire behavior of externally prestressed concrete beams up to the ultimate limit state [1-4]. The program used a stepwise analysis and deformation control to trace the nonlinear response of prestressed concrete beams with external tendons. A complete description of the model can be found in references [1-4]. The accuracy of the model was verified experimentally [3-4] in Structural Material Laboratory of Saitama University, Japan [3-5] at which the author participated in carrying out the experimental tests for several beams strengthened with external tendons.

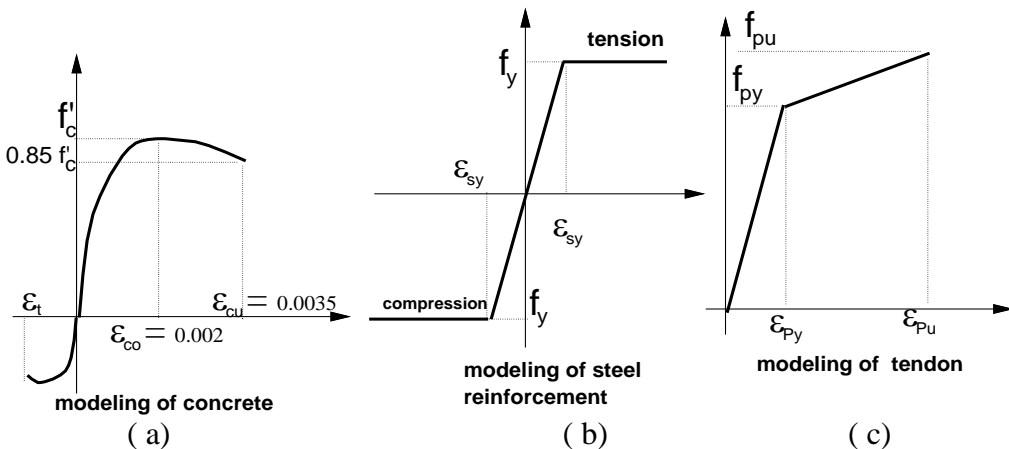


Fig. 4 (a, b, c) Modeling of Concrete, steel reinforcement and tendon

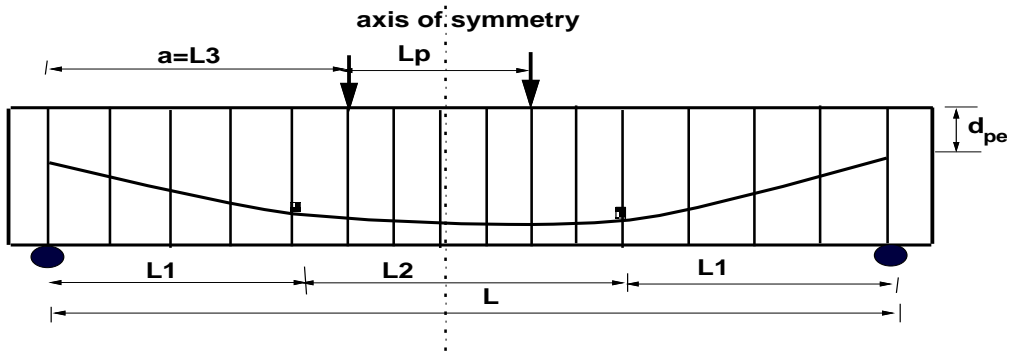


Fig. 5 Model of PC beam

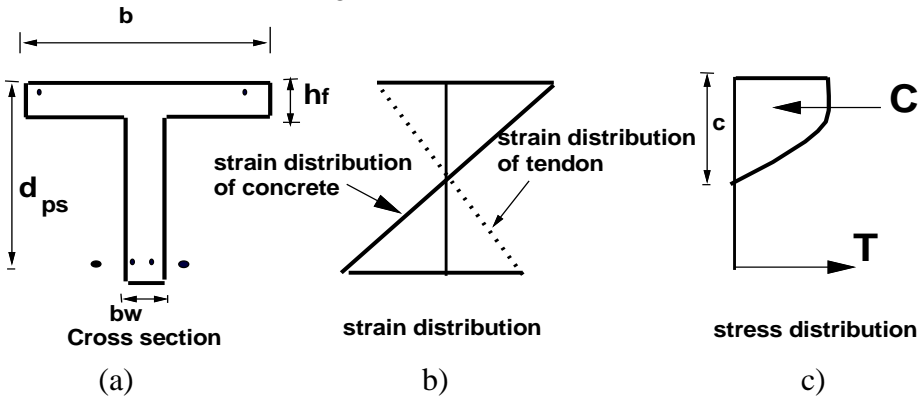


Fig. 6 Cross section, strain and stress distribution

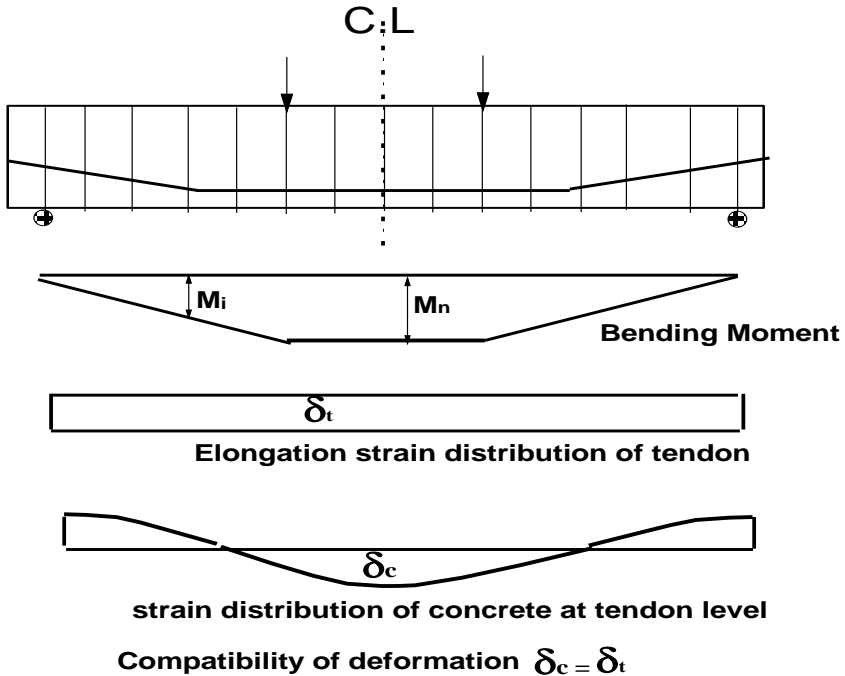


Fig. 7 Moment Distribution and Compatibility of Deformation

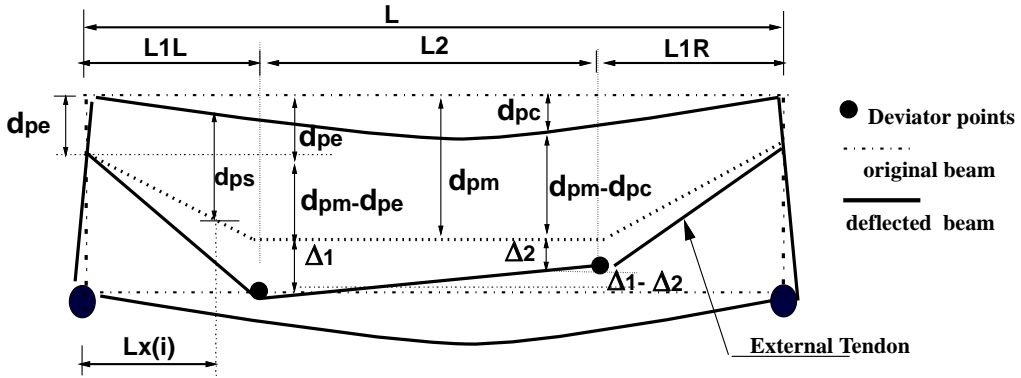


Fig. 8 Evaluation of effective depth of external tendons

Previous Equations to Estimate the Stress in External Tendons

All the existing equations give estimation for the stress in external tendon but they neglected the second order effect of tendon’s eccentricity effect.

1- **ACI Building Code.** The ACI suggested the following equation [15]:

$$f_{ps} = f_{pe} + 105 \tag{6}$$

Where f_{pe} is the effective prestressing stress and f_{ps} is the stress of tendon

2- **Canadian Code** [16] suggested the following equation:

$$f_{ps} = f_{pe} + \frac{500(d_{ps} - c_y)}{L_e}, \quad c_y = \frac{\Phi_{ps} A_{ps} f_{ps} + \Phi_s A_s f_y}{0.85\Phi_c \beta_1 f'_c b} \tag{7}$$

Where Φ_{ps} , Φ_s and Φ_c are resistance factors for the prestressing steel, nonprestressing steel and concrete respectively.

3- **European code** [17] suggested the following equation:

$$f_p = f_{pe} + \Delta f_{ps} < f_{py}, \quad \Delta f_{ps} = E_{ps} \left(\frac{\Delta L}{L} \right) \tag{8}$$

Where $\Delta L = d_{ps} / 17$ and L is the length of the tendon between end anchorages.

4. **Other researchers** suggested several equations. Warwaruk et al [18] suggested the following equation from experimental tests:

$$f_{ps} = f_{pe} + (207 + (\rho_{ps} / f'_c) \times 10^{12}) \tag{9}$$

Mattock et al [19] modified the ACI code equation as follows:

$$f_{ps} = f_{pe} + 700 + 1.4(f'_c / \rho_{ps}) \tag{10}$$

Where $f_{pe} \leq 0.6f_{pu}$, ρ_{ps} is the prestressing steel ratio, f'_c is the strength of concrete and f_{pu} is the nominal tensile strength of tendon. Pannel et al [20] investigated the effect of span to depth ratio L/d_{ps} on flexural strength and they proposed the following:

$$f_{ps} = f_{pe} + \gamma_s \left(1 - \frac{c}{d_{ps}} \right), \quad c = \frac{\mu_p A_{ps} + A_s f_y - A'_s - C_f}{0.85\beta_1 f'_c b_w + \gamma_s \frac{A_s}{d_{ps}}} \tag{11}$$

$$C_f = 0.85f'_c(b - b_w)h_f, \quad \gamma_s = 10.5E_{ps} \frac{\epsilon_{cu}}{L/d_{ps}}, \quad \mu_p = f_{pe} + \gamma_s \quad (12)$$

If $\beta_1 c < h_f$ then $C_f=0.0$, and $b_w=b$. E_{ps} is Young's Modulus of prestressing steel, L/d_{ps} is the span to depth ratio, b is the width, h_f is the flange thickness, A_{ps} is the area of prestressing steel, A_s and A'_s are area of tension and compression steel respectively. Du and Tao [21] suggested the following equation based on experimental study:

$$f_{ps} = f_{pe} + 786 - 1920 \left(\frac{A_s f_y + A_{ps} f_{pe}}{bd_s f'_c} \right). \quad (13)$$

Provided that: $(A_s f_y + A_{ps} f_{pe}) / bd_s f'_c \leq 0.3$, $0.55f_{py} \leq f_{pe} \leq 0.65f_{py}$, $f_{ps} \leq f_{py}$

Where f_{ps} , f_y are yield stress of prestressing and nonprestressing tensile steel, d_s is the distance from extreme concrete compressive fiber to centroid of tensile steel.

Harajli and Kani [22] proposed the following equation:

$$f_{ps} = f_{pe} + \gamma_o f_{pu} \left(1.0 + 3.0 \left[\frac{A_{ps} f_{pe} + A_s f_y}{bd_{ps} f'_c} \right] \right), \quad \gamma_o = \frac{L_1}{L_2} \left(0.12 + \frac{0.25}{L/d_{ps}} \right) \quad (14)$$

Where, $(A_{ps} f_{pe} + A_s f_y) / bd_{ps} f'_c$ is not more than 0.23, L_1 is the loaded span length, L_2 is the total length of the beam and L/d_{ps} is the span to depth ratio.

Naaman and Alkhairi [17] proposed the following simple equation:

$$f_{ps} = f_{pe} + E_{ps} \Omega_u \epsilon_{cu} \left(\frac{d_{ps}}{c} - 1 \right) \quad (15)$$

Where $\Omega_u = \frac{2.6}{L/d_{ps}}$ for one-point loading, $\Omega_u = \frac{5.4}{L/d_{ps}}$ for two-point loading ϵ_{cu} is the

concrete strain, c is the depth from extreme concrete compressive fiber to neutral axis and L/d_{ps} is the span to depth ratio.

Factors affecting the flexural strength of Externally PC beam

Based on the previous studies, the parameters affecting the flexural behavior of externally prestressed concrete beams are summarized as follows:

- a) **Span to depth ratio [L/d_{ps}]**. Previous studies [1-5] showed that (L/d_{ps}) ratio has significant effect on the flexural strength.
- b) **Distance between loads to span ratio [Lp/L]**. It is 0.0 for one-point loading to 0.333 for two points loading. This ratio has an effect on the flexural strength of beams [1-5]
- c) **Distance between deviators – to span ratio [S_d/L]**. It was shown that [S_d/L] has a significant influence on flexural strength of beams [1-13]
- d) **The ratio between prestressing index and reinforcement index ω**. The effect of the amount of ordinary reinforcement is considered using ω_s and ω_s' indices for tensile and compressive reinforcement respectively. The effect of the prestressing tendons is considered using ω_{pe} index, as follows [1, 2]:

$$\omega_s = \frac{A_s f_y}{f'_c b d_s}, \quad \omega'_s = \frac{A'_s f_y}{f'_c b d_s}, \quad \omega_{pe} = \frac{A_{ps} f_{pe}}{f'_c b d_{ps}}$$

$$\omega = \omega_{pe} + \frac{d_s}{d_{ps}} (\omega_s + \omega'_s) = \frac{A_{ps} f_{pe} + (A_s - A'_s) f_y}{f'_c b d_{ps}}$$

All the above parameters are non-dimensional values.

e) **Concrete compressive strength** f'_c , and

f) **Tensile reinforcement** and compressive reinforcement ratios μ, μ' %

CASES OF STUDY

Table 1 illustrates the details and variables of the analyzed beams. The effective prestressing f_{pe} was 50 % and 80 % of the ultimate prestressing, f_{pu} . Tensile reinforcement ratios ρ_s were 0.4 and 1.8 % and compression reinforcement ρ'_s was 0.25 %. Grade of steel reinforcement is 36/52. Figure 9 illustrates the details of the analyzed beams. Mechanical properties of the external tendons are illustrated in Table 2.

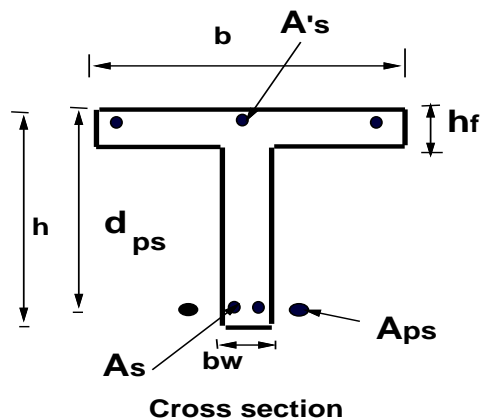


Fig. 9 Details of the analyzed beams

RESULTS AND ANALYSIS

1- Change of Tendon's Eccentricity

Figure 10 illustrates the relation between load and change of tendon's eccentricity as a ratio of the effective depth (d_{ps}) for beams with different number and distance of deviators. As the number of deviators increases, the change of tendon's eccentricity decreases. As distance between deviators increases, change of eccentricity increases.

Table 1: Details and variables of the analyzed beams

No.	Description	Number of Deviators	Distance of deviators S_d (m)	Area of tendons	Concrete Grade kg/cm^2
1	Effect of distance between deviators (S_d) and number of deviators	2	1.8	$2\phi 18$	400
2		2	3.0	$2\phi 18$	400
3		3	1.8	$2\phi 18$	400
4		3	3.0	$2\phi 18$	400
5	To study the effect of area of tendons and grade of concrete	2	1.8	$3\phi 18$	400
6		2	1.8	$2\phi 18$	600
7		2	1.8	$3\phi 18$	600
8	To study the effect of span to depth ratio, L/d_{ps}	2	8	$3\phi 18$	400
9		2	12	$3\phi 18$	400
10		2	20	$3\phi 18$	400
11	Effect of distance between load L_p/L	2	0.0	$3\phi 18$	400
12		2	0.33	$3\phi 18$	400

Table 2 Mechanical Properties of the prestressing tendons

Type	Sectional area (cm^2)	Yield load (t)	Ultimate load (t)
Ext. tendons	1.54	21.4	25.0
	2.54	24.9	39.4

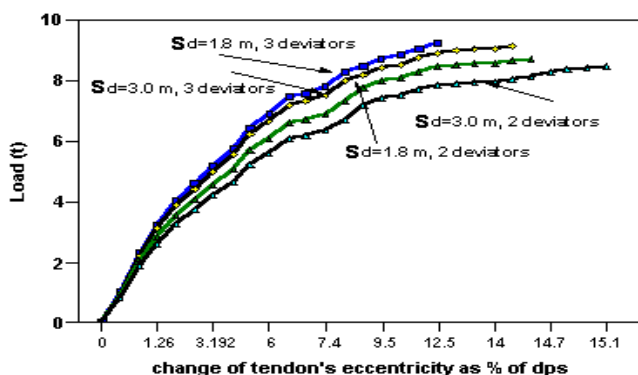


Fig. 10 Relation between the load and the change of tendon's eccentricity

2-Load Deflection Diagram

Figure 11 illustrates the load deflection diagrams for beams with different number and distance of deviators. As number of deviators increases, both of ultimate load, maximum deflection and ductility increase. As distance between deviators increases, both of ultimate load, maximum deflection and ductility decrease. Fig.12 illustrates the

effect of the prestressing force (as a ratio of ultimate force) on load deflection diagram. As the prestressing force is high, maximum deflection and ultimate load increase significantly.

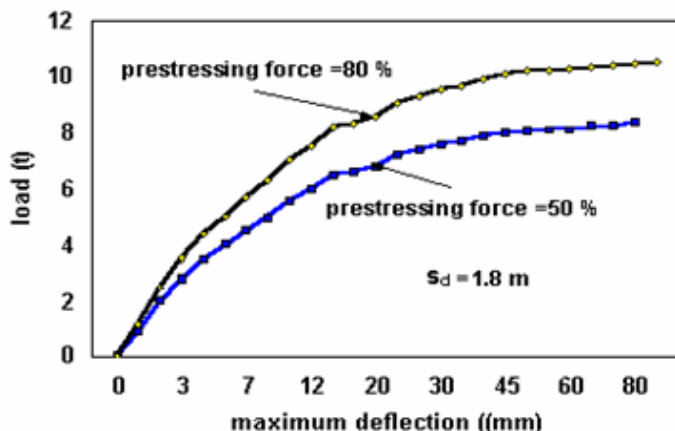


Fig. 11 Load – maximum deflection diagrams

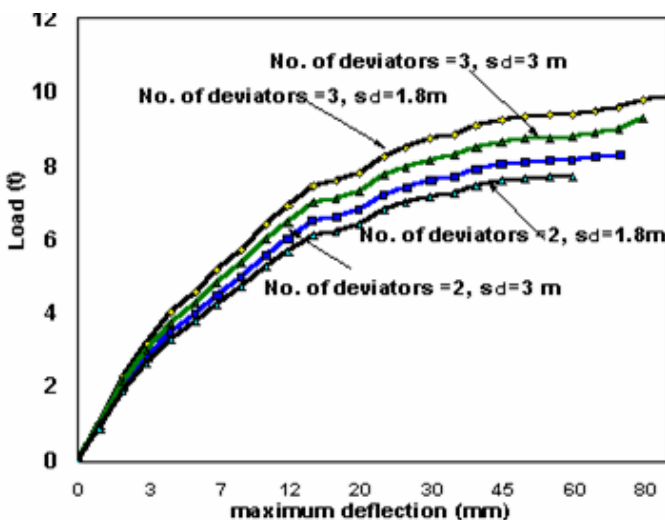


Fig. 12 Effect of the prestressing force on load – maximum deflection diagram

3- The relations of load and stress in external tendons

As the number of deviators increases, the external tendon resists higher stress. Also, as the distance between the deviators is small, the tendon carries higher stress. This is clear from Fig.13, in which the relations between the stress in external tendon and the carrying load are plotted for beams with different numbers of deviators and different distances between deviators. The given figures are plotted for prestressing force equals 80 % of the ultimate prestressing. Similar results are obtained for other ratios of prestressing force.

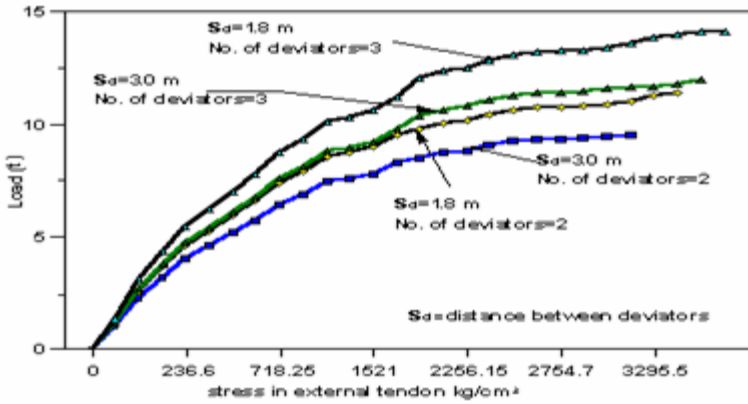


Fig. 13 Relations between the stress in external tendon and the carrying load

4- Effective Depth of Externally Prestressed Beam

It is not clear from the previous studies how to evaluate the effective depth of the cross section of flexural beams with external tendon. For this reason, it is not easy to evaluate the flexural strength of such beams. This is due to the effect of tendon's eccentricity as shown in Fig.14. Here, we will try to estimate the value of effective depth of external tendon, d_{pu} , as a function of the initial effective depth of the external tendon, d_{ps} . We introduce the factor K , which is defined as effective depth reduction factor as follows:

$$K = (d_{pu}) / (d_{ps}) \text{ or } d_{pu} = K \cdot d_{ps} \tag{16}$$

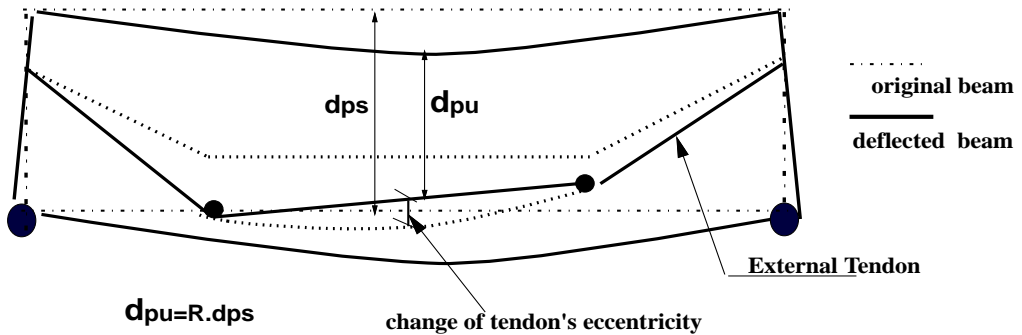


Fig. 14 Tendon's Eccentricity

From the obtained results, the factor K is obtained for different cases. Fig.15 illustrates the relations of effective depth reduction factor (K) and the distance between deviators S_d which taken as a ratio of the span L . As the distance between deviators is big, the reduction factor K is small and vice versa depending on the span to depth ratio L/d_{ps} . From curve fitting, the relations of Fig.15 are represented by the following equations:

$$K = 0.953 - 0.602(S_d / L) \quad \text{for } L/d_{ps} = 15 \text{ With correlation factor } R=0.977$$

$$K = 0.962 - 0.536(S_d / L) \quad \text{for } L/d_{ps} = 20 \quad R=0.978$$

$$K = 0.986 - 0.515(S_d / L) \quad \text{for } L/d_{ps} = 25 \quad R=0.99$$

$$K = 1.025 - 0.503(S_d / L) \quad \text{for } L/d_{ps} = 30 \quad R=0.978$$

$$K = 1.041 - 0.476(S_d / L) \quad \text{for } L/d_{ps} = 35 \quad R=0.977$$

The above equations are represented by one equation as follows:

$$K = (0.874 + 0.00478(L/d_{ps})) - [0.669 - 0.0057(L/d_{ps})]\{S_d / L\}, R=0.97 \quad (17)$$

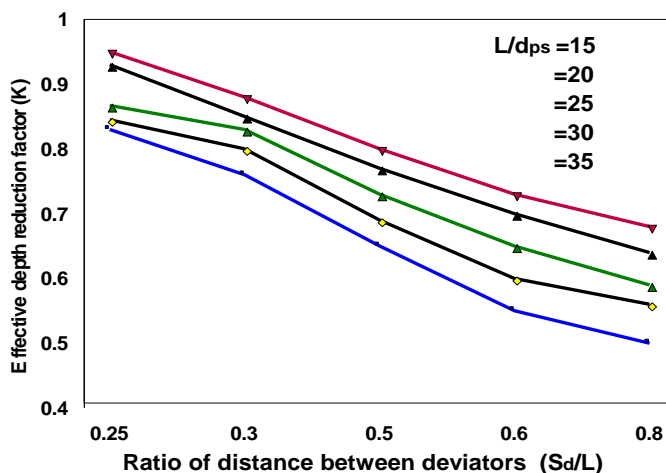


Fig. 15 Evaluation of the Effective Depth of PC beam

5- Bond Strain Reduction Factor

If the prestressing tendons are bonded with concrete, so complete bond is assumed. However, for externally prestressed beams, the tendons are bonded to beams at only the end anchorages and at deviators. For such reason, partial bond is assumed. This is introduced through bond strain reduction factor (**R**). This factor is evaluated as a function of (L/d_{ps}), which is the ratio of the beam span to effective depth ratio. Fig. 16 illustrates the relations of the factor R and (L/d_{ps}) ratios for different cases of (S_d/L) ratio, where S_d is the distance between deviators and L is the span of the beam. From curve fitting, the shown relations are represented by the following equations:

$$R = 0.371 - 0.0078(L/d_{ps}) \quad \text{for } S_d / L = 0.33 \quad R=0.98$$

$$R = 0.399 - 0.00767(L/d_{ps}) \quad \text{for } S_d / L = 0.5 \quad R=0.973$$

$$R = 0.42 - 0.00732(L/d_{ps}) \quad \text{for } S_d / L = 0.7 \quad R=0.973$$

The above equations are written in one equation as:

$$R = [0.33 + 0.13(S_d / L)] - [0.0083 - 0.0014(S_d / L)]\{L/d_{ps}\}, R=0.989 \quad (18)$$

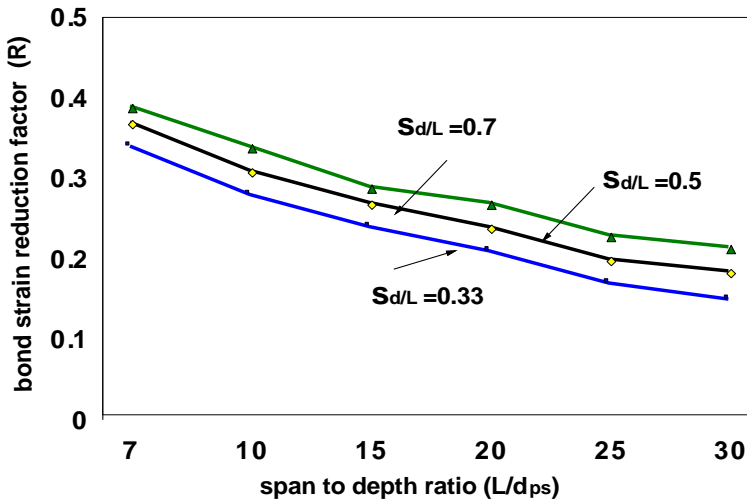


Fig. 16 Relations of the factor R and (L/d_{ps}) ratios for different (S_d/L) ratios

Figure 17 illustrates the relations of the factor R and (L/d_{ps}) ratios for different cases of (L_p/L) ratio, where L_p is the distance between loads. From curve fitting, the shown relations are represented by the following equations:

$$R = 0.0604 - 0.0040(L/d_{ps}) \quad \text{for } L_p/L = 0.0 \quad R=0.90$$

$$R = 0.1654 - 0.0020(L/d_{ps}) \quad \text{for } L_p/L = 0.1 \quad R=0.94$$

$$R = 0.2463 - 0.0033(L/d_{ps}) \quad \text{for } L_p/L = 0.2 \quad R=0.986$$

$$R = 0.3171 - 0.0054(L/d_{ps}) \quad \text{for } L_p/L = 0.333 \quad R=0.986$$

The above equations are written in one equation as:

$$R = [0.0761 + 0.76(L_p/L)] - [0.000453 + 0.015(L_p/L)](L/d_{ps}), R=0.99 \quad (19)$$

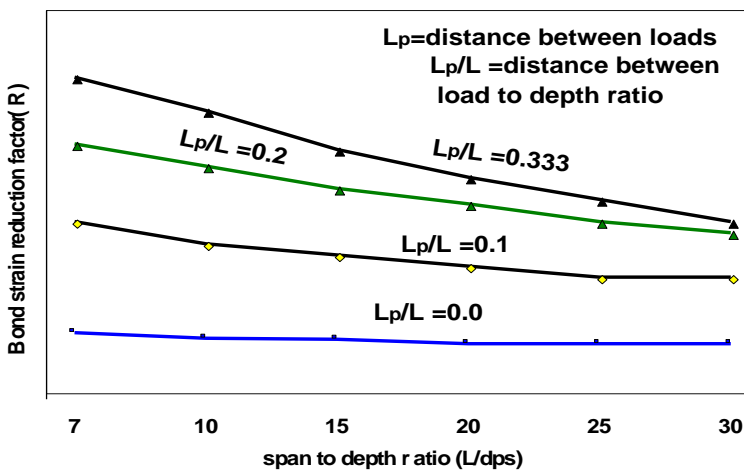


Fig. 17 Relations of the factor R and (L/d_{ps}) ratios for different (L_p/L) ratios

PROPOSED MODEL FOR FLEXURAL STRENGTH OF EXTERNALLY PRESTRESSED FLEXURAL BEAMS

Based on ACI Code for analysis of unbonded prestressed beam, the following method is proposed to calculate the flexural strength of externally prestressed concrete beams

1- Calculate the effective depth d_{ps} and strain reduction factor R . To calculate the effective depth, the reduction factor K is calculated using Equation (17). Equations 18 and 19 are used to calculate the bond strain reduction factor.

2- Referring to Fig. 18, and from equilibrium of forces at the critical section, we determine the values of c and f_{ps} as follows:

$$A_{ps}f_{ps} + A_s f_y + A'_s f_y = 0.85f'_c b h_f + 0.85f'_c (b - b_w) c \quad (20)$$

$$f_{ps} = f_{pe} + E_{ps} R \varepsilon_{cu} \left(\frac{d_{pu}}{c} - 1 \right) \leq f_{py} \quad (21)$$

Solving the above equations, we get

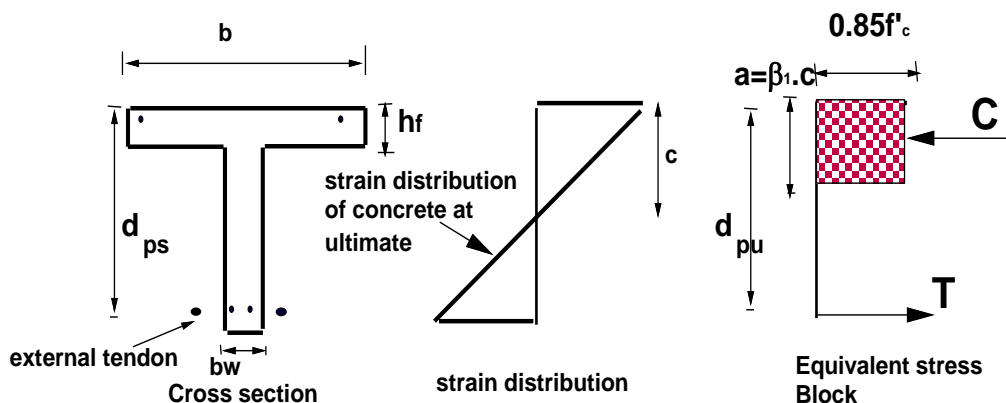


Fig. 18 Equilibrium of Forces of Externally Prestressed Beam

$$c = \frac{-B1 + \sqrt{(B1)^2 - 4A1C1}}{2A1} \quad (22)$$

$$A1 = 0.85f'_c b_w \beta_1,$$

$$B1 = A_{ps} (E_{ps} \varepsilon_{cu} R - f_{pe}) - (A_s - A'_s) f_y = 0.85f'_c (b - b_w) h_f$$

$$C1 = -A_{ps} E_{ps} \varepsilon_{cu} R d_{pu}$$

β_1 is a coefficient of equivalent stress block; ε_{cu} is the ultimate strain of concrete ($0.003 \approx 0.004$), f_{pe} is the effective prestressing (50–70%) of f_{pu}

3- Calculate the moment of resistance by taking moment at the center of compression force of concrete as follows:

$$M_u = A_{ps} f_{ps} \left(d_{pu} - \frac{\beta_1 c}{2} \right) + A_s f_y \left(d_s - \frac{\beta_1 c}{2} \right) + A'_s f_y \left(d'_s - \frac{\beta_1 c}{2} \right) \quad (23)$$

Conclusions and Recommendations

Using a finite element algorithm, non-linear analysis of externally prestressed concrete beams was carried out to investigate the flexural behavior of such beams under static loads. The accuracy of the used model was verified experimentally in Japan. The behavior of such beams is represented by; Change of Tendon's Eccentricity, Load Deflection Diagram, The relations of load and stress in external tendons, Effective Depth of Beam, and Bond Strain Reduction Factor. The strain variation in an external tendon was investigated on the basis of the deformation compatibility of beam. The following conclusions can be obtained:

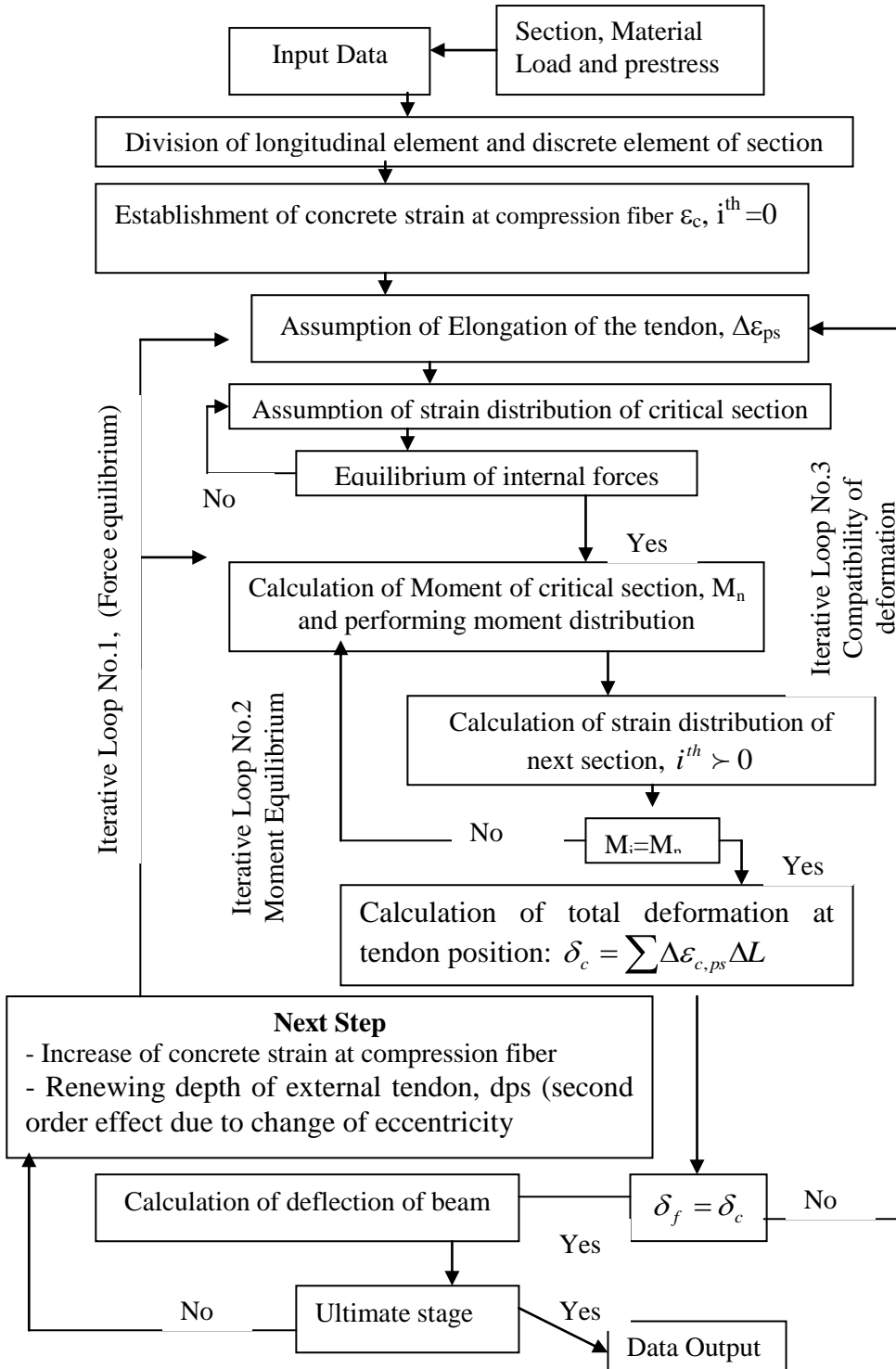
1. The proposed method for the numerical analysis can satisfactorily predict the behavior of externally prestressed concrete beams up to the ultimate loading stage.
2. The parameters affecting the behavior of such beams are: span to depth ratio $[L/d_{ps}]$, distance between loads to span ratio $[L_p/L]$, distance between deviators to span ratio $[S_d/L]$, the ratio of prestressing index and reinforcement index μ , concrete compressive strength f'_c and tensile and compressive steel ratios μ, μ' %
3. The stress increase in an external tendon depends mainly on the overall deformation of beam.
4. The behavior of externally prestressed concrete beams is different than that of bonded prestressed beams due to the second order effect of tendon's eccentricity. This point is clearly investigated in the current study.
5. From the obtained results, a model was proposed to estimate the both of the bond strain reduction factor (R) [Equation 19 in the text] and effective depth reduction factor (K) [Equations 17 and 18 in the text] considering the effect of the mentioned parameters.
6. Based on numerical analyses, a model was proposed to calculate the flexural strength of externally prestressed concrete beams considering its parameters
7. The study should be extended to analyze the continuous beams and the beams with large eccentricity.

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Appendix A Flow Chart of the Finite Element Program

دراسة سلوك الانحناء للكمرات الخرسانية بسيطة الارتكاز والسابقة للإجهاد بشدادات خارجية تحت تأثير الأحمال الاستاتيكية

في هذا البحث تم استخدام نموذج لاخطى باستخدام نظرية العناصر المحددة لدراسة سلوك الانحناء للكمرات الخرسانية بسيطة الارتكاز والسابقة للإجهاد بشدادات خارجية والمعرضة لأحمال استاتيكية. لقد تم التحقق من دقة النموذج المستخدم معمليا في اليابان. لقد ثبت أن النموذج المقترح يعطى نتائج معبرة عن سلوك هذه الكمرات من بدء التحميل حتى أقصى حمل. تم تمثيل سلوك هذه الكمرات بالعناصر التالية:

- 1- التغيير في مكان الشداد الخارجى مع تغير الحمل tendon' eccentricity
- 2- منحنى الحمل والتشكل deflection-Load
- 3- منحنى الحمل والإجهاد المتولد فى الشداد الخارجى Load – stress in tendon
- 4- العمق الفعال لقطاع الكمره effective depth
- 5- معامل نقص انفعال الالتصاق بين الكمره والشداد Bond strain reduction factor

لقد تم دراسة تأثير العوامل التالية على عناصر السلوك السابقة وهي:

- 1- نسبة البحر الى العمق span to depth ratio
- 2- نسبة المسافة بين الحمل الى بحر الكمره Distance between loads to depth ratio
- 3- نسبة المسافة بين نقاط تثبيت الشدادات الى بحر الكمره Distance between deviators to depth
- 4- نسبة عامل القوة سابقة الإجهاد Ratio of prestressing index
- 5- مقاومة أو رتبة الخرسانة Concrete compressive strength
- 6- نسب حديد التسليح فى الشد والضغط Compression and tension steel reinforcement

لقد أثبتت النتائج أن الإجهاد في الشداد الخارجى يتوقف على تشكل الكمره الكلى وليس على انفعال القطاع ولذلك تم دراسة إجهاد الشداد على هذا الأساس. لقد تبين أن سلوك الكمرات سابقة الإجهاد بشدادات خارجية يختلف عن سلوك الكمرات سابقة الإجهاد بشدادات داخلية ملتصقة وذلك لعدم استمرارية تشكل الشداد مع تشكل الكمره. من نتائج الدراسة تم استنتاج نماذج ومعادلات لحساب كل من الآتي معتبرا العوامل والمتغيرات السابق ذكرها:

- 1- العمق الفعال للكمرات السابقة للإجهاد بشدادات خارجية
 - 2- معامل نقص انفعال الالتصاق بين الكمره والشداد
 - 3- مقاومة الانحناء للكمرات السابقة للإجهاد بشدادات خارجية Flexural Strength
- يوصى بتطبيق هذه المعادلات فى تصميم هذه الكمرات كما يوصى بامتداد الدراسة لتشمل الكمرات المستمرة والكمرات التى لها تشكلات كبيرة جدا