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A PRECISE ANALYSIS AND VISUALIZATION OF PWM INVERTERS OUTPUT VOLTAGE THD USING COMPUTER ALGEBRA

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In this paper a comprehensive study of THD of PWM inverters output voltage is illustrated by advanced technique using computer algebra systems. First, the target function is explained using mathematical analytical methods, and then these very complex equations are simplified by manipulating trigonometric identities and mathematical formulas. By using CAS (Computer Algebra System) MATHEMATICA - which is one of the most popular and powerful software - its symbolical and numerical manipulation capabilities to calculate the exact THD and produce it in a simple and fast convergent formula is presented. Second, a simulation package is established using MATHEMATICA to visualize output voltage THD. Finally, this package is utilized to analyze the effect of changing carrier duty cycle on output voltage THD of SPWM inverters. In addition a comparison study with [18] is presented, consequently a result that the work in [18] is fake.

KEYWORDS: *Total Harmonic Distortion (THD), PWM inverters, Computer Algebra, Mathematica*

1. INTRODUCTION

In recent years, switch-mode DC-to-AC power inverters are used in AC motor drive and uninterruptible AC power supplies (UPS) etc. [1-3].In most cases, low distortion sinusoidal output voltage waveforms are required with controllable magnitude and frequency [4]. The load of the aforementioned is either critical or sensitive. Therefore, good performance and high quality of the switch mode DC to AC inverters are mandatory [5].The main task of using PWM as a control signal for power inverters is to provide the inverter by the accurate switching signals to generate the desired inverters output voltage characteristics. One of the most important characteristics is the Total Harmonic Distortion (THD) which will be concerned in this paper.

Computer algebra programs represent one class of platforms for computer-aided exploration of material. Their symbolic manipulation abilities are sophisticated and extensible, and they offer many ways to visualize formulas. Such programs could become better teaching tools if they were coupled with a true hypertext program [6]. One group has tried this combination for the teaching of electromagnetics [7]. An alternative is to build a custom user-interface to a computer algebra program, but this usually requires much programming. A promising class of software is evolving that would allow a user to do this with a minimal amount of programming [8]. MATHEMATICA is a powerful computer algebra system developed by Wolfram

Research [9]. It is a very high level programming language, adaptive to various types of courses in mathematics. It includes arbitrary precision and exact numerical computation, symbolic computation, graphics, sound, hyperlinked documentation and interprocess communication - all integrated together in one easy-to-use package [10].

A study of duty cycle variation of the carrier in PWM effect on THD is investigated by modeling the problem functions and, analyzing the frequency response and calculating the THD by using mathematical simplifications using some special functions and its properties to get closed form of few terms summation instead of infinite summation as the ore definition.

2. PROBLEM FORMULATION

In this section our problem is a single phase full bridge inverter circuit as shown in Figure 1.

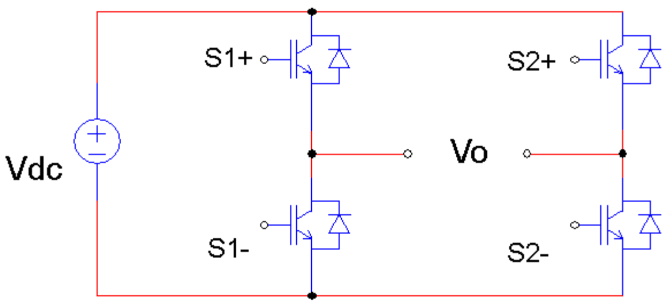


Figure 1. Single Phase full bridge voltage source inverter

This circuit can be driven for square wave operation (i.e. to produce square wave output voltage wave shape.) as the first principle of inverters. In the square wave operation the output voltage and its harmonics characteristics is shown in Figure2.

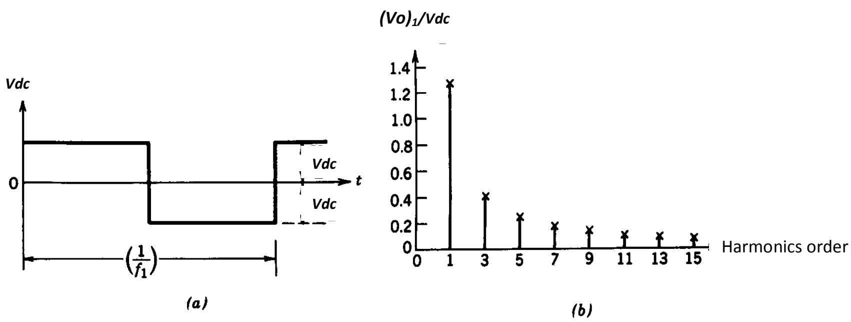


Figure 2. Square wave switching and its harmonics spectrum
(a) Inverter output voltage in time domain
(b) Inverter output voltage in frequency domain

From Figure 2, it is evident that the output voltage has a significant harmonics values especially at low frequencies which need very complex filters to obtain clean sinusoidal output. From this point of interest, we are in need for another control technique to produce low harmonics values especially at low frequencies. One of these switching techniques is the Pulse Width Modulation (PWM), in which a sinusoidal control signal at the desired fundamental frequency (f_1) is compared with a triangular waveform at a switching frequency (f_s) which establishes the frequency with which the inverter switches are switched as shown in Figure 3 [11].

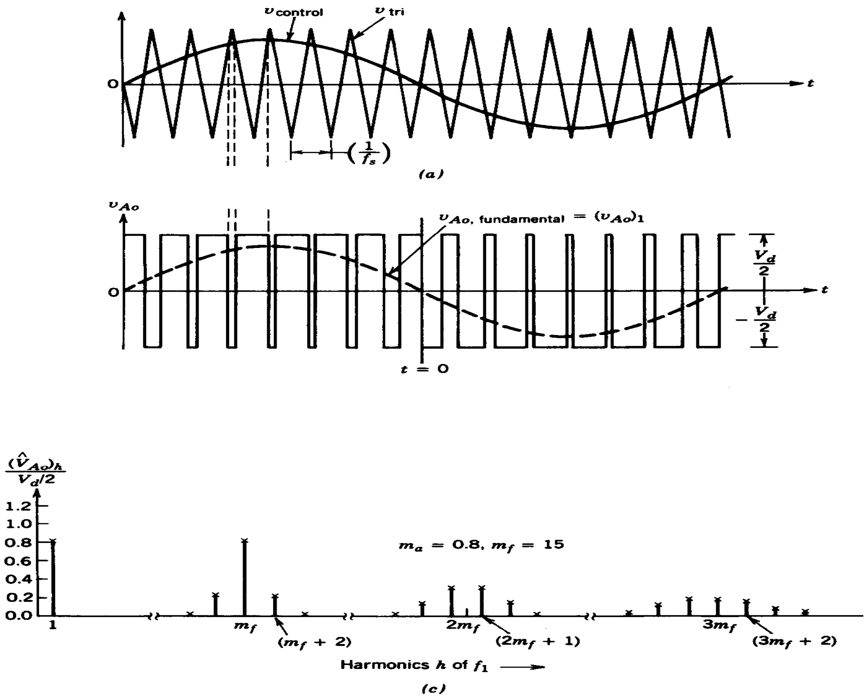


Figure 3 PWM with bipolar switching scheme

(a) Time domain of control sinusoidal frequency and triangular carrier frequency
(b) Output voltage waveform and its fundamental component
(c) Harmonics spectrum of the output voltage

The amplitude modulation ratio m_a is defined as, $m_a = \frac{\hat{V}_{control}}{\hat{V}_{tri}}$, where $\hat{V}_{control}$ is the peak amplitude of the control signal, and $\hat{V}_{tri}$ is the peak amplitude of the triangular signal. The frequency modulation ratio m_f is defined as, $m_f = f_c / f_1$

One another switching scheme is the PWM with Unipolar Voltage Switching in which the two legs of the full bridge inverter of Figure 1 are not switched simultaneously, as in the previous PWM scheme. Here, the legs S1 and S2 of the full bridge inverter are controlled separately by comparing v_{tri} with $v_{control}$ and $-v_{control}$, respectively. As shown in Figure 4

(a) Time domain of control sinusoidal frequency and triangular carrier frequency
(b) Switching signals of first leg S1
(c) Switching signals of second leg S2
(d) Output voltage waveform and its fundamental component
(e) Harmonics spectrum of the output voltage

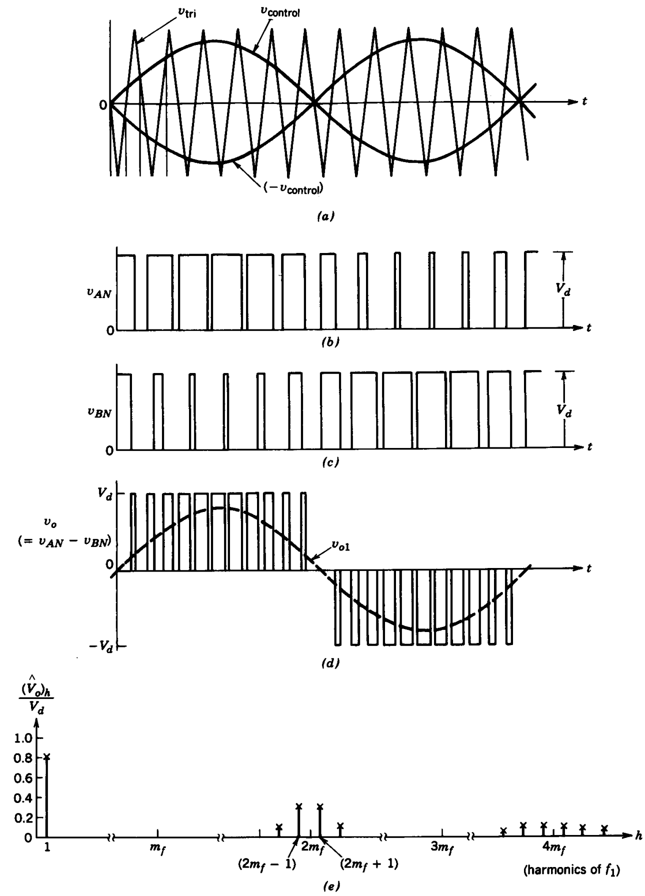


Figure 4 Unipolar PWM switching scheme

From Figure 4, it is obvious that the harmonic spectrum of the unipolar switching scheme is better where the significant harmonics are at higher frequencies than that at the bipolar switching scheme (i.e. the first significant harmonics appear at m_f - 2 -1 while the first significant harmonics appeared at m_f -2 at bipolar PWM case)

As a simplified analog implementation of this type of PWM modulation scheme (i.e. Unipolar PWM) we can construct it by using operational amplifier (OpAmp) as a comparator for generating PWM pulses as shown in Figure 5.

From the characteristics of ideal comparator OpAmp, the output voltage is summarized from [12] as follows

$$V_o = \begin{cases} V_{cc} & \text{if } v_{control} > v_{tri} \\ -V_{cc} & \text{if } v_{control} < v_{tri} \end{cases} \quad (1)$$

A mathematical description of the harmonics of these carrier-based methods is difficult because the intersection moments are not linearly or equally spread over the reference cycle [13].

By focusing in the circuit of the full bridge inverter shown in Figure 1 at Unipolar switching scheme, we can deduce the output voltage in the following simplified equation

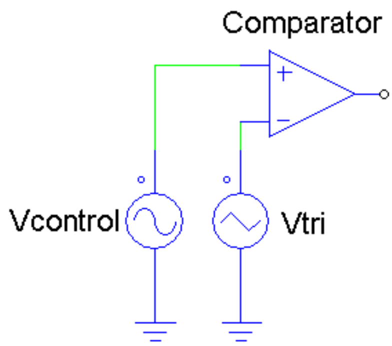


Figure 5 Analog implementation of PWM generation using comparator OpAmp

$$v_o = V_d(sign(v_{control} - v_{tri}) - sign(-v_{control} - v_{tri})) \quad (2)$$

Where, $sign(v_{control} - v_{tri})$ is a simplified form of comparator output voltage instead of using piecewise form.

In this study we will deduce an exact closed form for calculating THD by applying a Fourier series expansion to calculate harmonics components which is defined in [14]as follows:

$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \quad (3)$$

Where

$$a_0 = \frac{1}{1/f}\int_0^T v(t)\,dt \quad (4)$$

$$a_n = \frac{2}{1/f}\int_0^T v(t)\cos(n\omega_0 t)\,dt \quad (5)$$

$$b_n = \frac{2}{1/f}\int_0^T v(t)\sin(n\omega_0 t)\,dt \quad (6)$$

Calculating the a_n and b_n (the Fourier series coefficients) is obtained by direct substitution of the output voltage given by equation (2) into equations (5) and (6). It is very difficult to find a closed form of this type of equations. So we will try to clarify this problem in the following section.

3. REDUCTION STEPS

First, defining the main two signals in the PWM generation process

$$v_{control} = m_a \sin(2\pi t) \quad (7)$$

$$v_{tri} = trianglewave(2\pi f_c t, f, duty\,cycle) \quad (8)$$

The triangle wave can be expressed as follows:

$$trianglewave(t, f, d) = -1 + \begin{cases} \frac{-2(tf + \lfloor tf \rfloor)}{d}, & \frac{tf - \lfloor tf \rfloor}{f} < \frac{d}{f} \\ \frac{-2(1 - tf + \lfloor tf \rfloor)}{d-1} & otherwise \end{cases} \quad (9)$$

Where d is the duty cycle of the triangle wave (i.e. the ratio between rising time and the period) and f is the frequency in which triangle wave is repeated. In our case f is replaced with f_c.

We will define another two functions in order to calculate the THD exactly as follows

$$s1(m_a, d, f_c) = roots(sign(v_{control} - v_{tri}) = 0) \tag{10}$$

$$s2(m_a, d, f_c) = roots(sign(-v_{control} - v_{tri}) = 0) \tag{11}$$

where s1 and s2 are all roots of its functions over one fundamental period of $v_{control}$ After calculating s1 and s2 we can redefine output voltage in equation (2) as follows

$$v_o = \sum_{i=1}^{2f_c} (-1)^i u(t - s1_i) + (-1)^{i+1} u(t - s2_i) \tag{12}$$

Where, u is the unit step function.

Now, we are interested in calculating Fourier series coefficients a_n and b_n using the equation (12) and applying it in equations (5) and (6) we can get

$$a_n = 2f\left(\int_0^{1/f}\left(\sum_{i=1}^{2f_c}(-1)^i u(t - s1_i) + (-1)^{i+1} u(t - s2_i)\right)\cos(\omega_n t)\,dt\right) \tag{13.a}$$

$$a_n = 2f\left(\int_0^{1/f}\left(\sum_{i=1}^{2f_c}(-1)^i u(t - s1_i) + (-1)^{i+1} u(t - s2_i)\right)\cos(2\pi f n t)\,dt\right) \tag{13.b}$$

For more simplifications we will consider that the control signal frequency is unity and the carrier frequency is the frequency modulation ratio (i.e. $f_1 = 1$ so, $m_f = f_s$). In fact this consideration will not affect on any PWM generation process or harmonics calculations because we are dealing into scale. So a_n can be rewritten as following

$$a_n = 2\left(\int_0^1\left(\sum_{i=1}^{2f_c}(-1)^i u(t - s1_i) + (-1)^{i+1} u(t - s2_i)\right)\cos(2\pi n t)\,dt\right) \tag{13.c}$$

By manipulating this equation symbolically

$$a_n = \frac{1}{n\pi}\left(\sum_{i=1}^{2f_c}(-1)^{i+1}\sin(2\pi n s1_i) + (-1)^i \sin(2\pi n s2_i)\right) \tag{13.d}$$

Similarly we can get b_n as follows:

$$b_n = \frac{1}{n\pi}\left(\sum_{i=1}^{2f_c}(-1)^i\cos(2\pi n s1_i) + (-1)^{i+1}\cos(2\pi n s2_i)\right) \tag{14}$$

From the definition of voltage total harmonic distortion (THD) in [15] which equals

$$THD = \frac{\sqrt{\sum_{i=2}^{\infty} v_i^2}}{v_1} \tag{15.a}$$

where, v_i is the harmonic voltage value of harmonic order i, and v_1 is the fundamental voltage value.

By applying Fourier series coefficients into equation (15.a) we will get

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty}(a_n^2 + b_n^2)}}{\sqrt{a_1^2 + b_1^2}} \tag{15.b}$$

Before substituting the simplified Fourier series coefficients given in equations (13.d) and (14) into equation (15.b) calculation of a_n^2 and b_n^2 in simplified form is illustrated as follows:

$$a_n^2 = \left(\frac{1}{n\pi}\left(\sum_{i=1}^{2fc}(-1)^{i+1}\sin(2\pi n s1_i) + (-1)^i \sin(2\pi n s2_i)\right)\right)^2 \tag{16.a}$$

By using the property of multinomial in [16] $(\sum_i^m x_i)^2 = \sum_{i=1}^m \sum_{j=1}^m x_i x_j$ we get

$$a_n^2 = \sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{(n\pi)^2}\Big((-1)^{i+j}(-\sin(2\pi n s1_i) + \sin(2\pi n s2_i))(-\sin(2\pi n s1_j) + \sin(2\pi n s2_j))\Big) \tag{16.b}$$

and,

$$b_n^2 = \sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{(n\pi)^2}\Big((-1)^{i+j}(\cos(2\pi n s1_i) - \cos(2\pi n s2_i))(\cos(2\pi n s1_j) - \cos(2\pi n s2_j))\Big) \tag{16.c}$$

so,

$$\begin{aligned} a_n^2 + b_n^2 = \sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{(n\pi)^2}\Big((-1)^{i+j}(-\sin(2\pi n s1_i) &+ \sin(2\pi n s2_i))(-\sin(2\pi n s1_j) + \sin(2\pi n s2_j)) \\ &+ (-1)^{i+j}(\cos(2\pi n s1_i) - \cos(2\pi n s2_i))(\cos(2\pi n s1_j) \\ &- \cos(2\pi n s2_j))\Big) \end{aligned} \tag{16.d}$$

Now, we are in face to compute this term $\sum_{n=2}^{\infty}(a_n^2 + b_n^2)$. From the duality property of summation (i.e. $\sum_{i=1}^{m}\sum_{j=1}^{n} x_i x_j = \sum_{j=1}^{n}\sum_{i=1}^{m} x_i x_j$) so

$$\begin{aligned} \sum_{n=2}^{\infty}(a_n^2 + b_n^2) = &\sum_{n=2}^{\infty}\sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{(n\pi)^2}\Big((-1)^{i+j}(-\sin(2\pi n s1_i) + \\ &\sin(2\pi n s2_i))(-\sin(2\pi n s1_j) + \sin(2\pi n s2_j)) + \\ &(-1)^{i+j}(\cos(2\pi n s1_i) - \cos(2\pi n s2_i))(\cos(2\pi n s1_j) - \\ &\cos(2\pi n s2_j))\Big) \end{aligned} \tag{17.a}$$

Then

$$\begin{aligned} \sum_{n=2}^{\infty}(a_n^2 + b_n^2) = &\sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\sum_{n=2}^{\infty}\frac{1}{(n\pi)^2}\Big((-1)^{i+j}(-\sin(2\pi n s1_i) + \\ &\sin(2\pi n s2_i))(-\sin(2\pi n s1_j) + \sin(2\pi n s2_j)) + \\ &(-1)^{i+j}(\cos(2\pi n s1_i) - \cos(2\pi n s2_i))(\cos(2\pi n s1_j) - \\ &\cos(2\pi n s2_j))\Big) \end{aligned} \tag{17.b}$$

Now, using the power of Computer Algebra Systems as MATHEMATICA for simplifying the most inner summation of equation (17.b) which sums terms from 2 to ∞ yields the following

$$\sum_{n=2}^{\infty}\left(a_n{}^2+b_n{}^2\right)=\sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{2\pi^2}(-1)^{i+j}\Big(polylog\left(2,e^{-I2\pi\left(s1_i-s1_j\right)}\right)+ polylog\left(2,e^{I2\pi\left(s1_i-s1_j\right)}\right)-polylog\left(2,e^{-I2\pi\left(s1_j-s2_i\right)}\right)- polylog\left(2,e^{I2\pi\left(s1_j-s2_i\right)}\right)\Big)-polylog\left(2,e^{-I2\pi\left(s1_i-s2_j\right)}\right)- polylog\left(2,e^{I2\pi\left(s1_i-s2_j\right)}\right)+polylog\left(2,e^{-I2\pi\left(s2_i-s2_j\right)}\right)+ polylog\left(2,e^{I2\pi\left(s2_i-s2_j\right)}\right)-8\cos\left(\pi\left(s1_i-s1_j+s2_i-s2_j\right)\right)\ \sin\left(\pi\left(s1_i-s2_i\right)\right)\sin\left(\pi\left(s1_j-s2_j\right)\right) \tag{18}$$

Note: polylog is the polylogarithm function which is defined in [17] as

$$polylog(n,z)=\sum_{k=1}^{\infty}\frac{z^k}{k^n}\quad,\ |z|<1 \tag{19}$$

And by using the following identity property of $polylog$ function

$$polylog(2,x)+polylog\left(2,\frac{1}{x}\right)=-\frac{\pi^2}{6}-\frac{1}{2}(\ln(-x))^2 \tag{20.a}$$

Substituting in the above formula for $x=e^{I2\pi x}$ yields

$$polylog(2,e^{I2\pi x})+polylog(2,e^{-I2\pi x})=-\frac{1}{6}(\pi^2+3(\ln(-e^{I2\pi x}))^2) \tag{20.b}$$

By mapping the above equation in the form of real and imaginary parts separation results in

$$polylog(2,e^{I2\pi x})+polylog(2,e^{-I2\pi x})=-\frac{\pi^2}{6}+\frac{1}{2}(Arg(-e^{I2\pi x}))^2 \tag{20.c}$$

Manipulating the above equation using special function series expansion and applying some algebraic simplifications the following simplified function is obtained.

$$polylog(2,e^{I2\pi x})+polylog(2,e^{-I2\pi x})=\frac{\pi^2}{3}(1-6\,x\,sign(x)+6\,x^2) \tag{20.d}$$

Now we'll name the simplified function as following

$$poly2(x)=\ polylog(2,e^{I2\pi x})+polylog(2,e^{-I2\pi x}) \tag{20.e}$$

$$poly2(x)=\frac{\pi^2}{3}(1-6\,x\,sign(x)+6\,x^2) \tag{20.f}$$

Finally by substituting the above equation into equation (16) we can deduce the final simplified exact closed form of the THD equation as follows:

$$THD=\frac{\sqrt{\sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{2\pi^2}(poly2(x1)-poly2(x2)-poly2(x3)+poly2(x4)-8\cos\left(\pi\left(s1_i-s1_j+s2_i-s2_j\right)\right)\sin\left(\pi(s1_i-s2_i)\right)\sin\left(\pi\left(s1_j-s2_j\right)\right)}}{\sqrt{\sum_{i=1}^{2fc}\sum_{j=1}^{2fc}\frac{1}{(n\pi)^2}\left((-1)^{i+j}(-\sin(2\pi s1_i)+\sin(2\pi s2_i))\left(-\sin\left(2\pi s1_j\right)+\sin(2\pi s2_j)\right)+(-1)^{i+j}(\cos(2\pi s1_i)-\cos(2\pi s2_i))\left(\cos\left(2\pi s1_j\right)-\cos(2\pi s2_j)\right)\right)}} \tag{21}$$

Where, $x1, x2, x3, x4$ equal $s1_i-s1_j, s1_j-s2_i$, $s1_i-s2_j, s2_i-s2_j$ respectively.

4. The Proposed Simulation Package

There are many advantages of using Mathematica to simulate PWM inverter output voltage THD. First, we can visualize the effect of changing system parameters online which is not available in traditional simulation software. This feature can be used for fine tuning of system parameters. Second, the simulation is much faster and more accurate than other traditional simulation software. In this section we will make use of the power of Mathematica in fast and exact processing of equations and our closed form equation (21) therefore, we can visualize the effect of changing PWM parameters in time and frequency domains all online in one sheet which is not the case in traditional simulation package.

In fact, Mathematica can simulate the system from its not simplified equation (15.a) but it is clear that equation (21) has finite terms for summation and can be calculated exactly and rapidly on the opposite at equation (15.a) which has an infinite summation terms. And this is verified using one of a rugged computer algebra system (Mathematica) where the following example is done for calculating THD by the two equations (15.a) and (21) and calculating time consumed in seconds for executing these calculation as following Mathematica commands

SPWM[s1[9/10,0/10,1,200],s2[9/10,0/10,1,200]]//Timing

{1277.04,0.644025}

SPWM1[s1[9/10,0/10,1,200],s2[9/10,0/10,1,200]]//Timing

{25.569,0.644025}

The above four lines are copied from Mathematica workspace where the first and third lines represent input commands for calculating THD of SPWM inverter output voltage using equation (15) and (17) respectively with the following parameters: $m_a = 9/10$, $m_f = 200$ and $d = 0$

and also for calculating time consumed during this calculation.

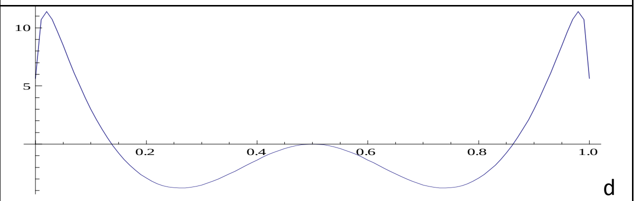
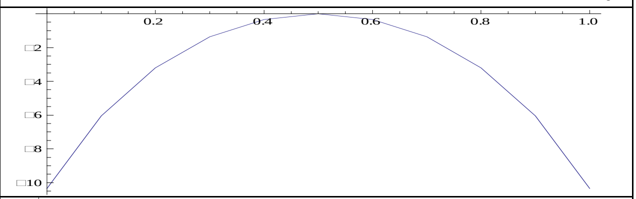
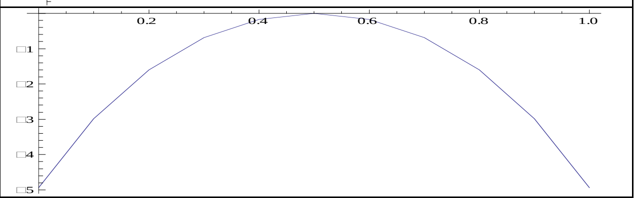
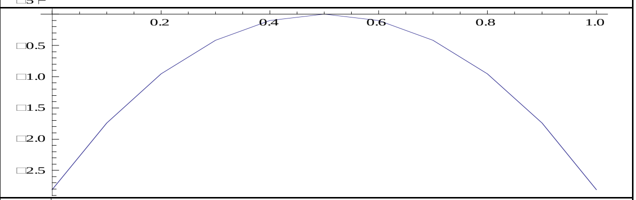
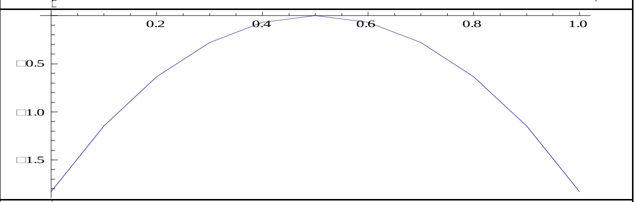
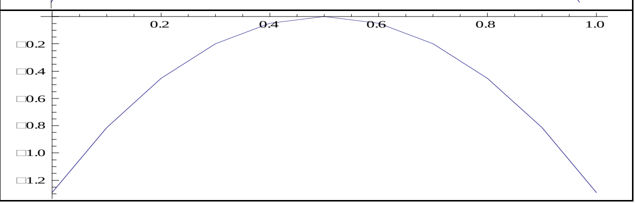
The second and fourth lines represent the results of first and second commands in a list form with the first element representing time consumed during calculation and the second element representing the result of THD. It is clear that the result of calculating THD are the same but time consumed using equation (15) is extremely long (1277.04 seconds) as compared with that consumed using equation (17) (25.569 seconds) only. In other words, in this example it is clear that we saved consumed time for calculating THD by ratio 1:50 by using our simplified closed form for calculating THD of SPWM inverter output voltage.

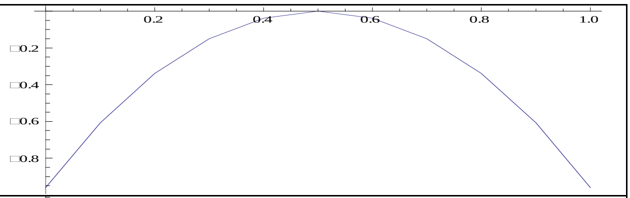
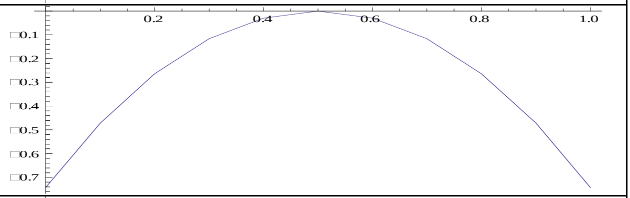
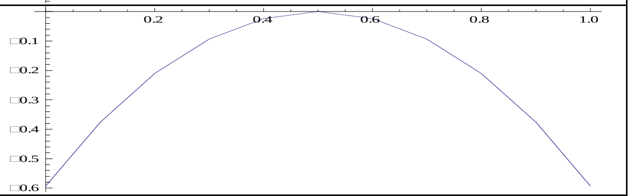
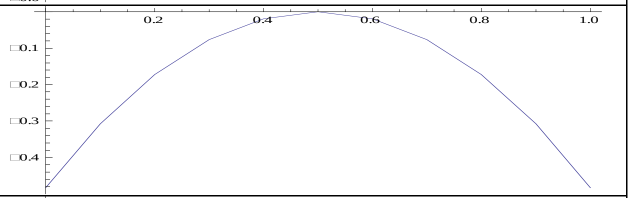
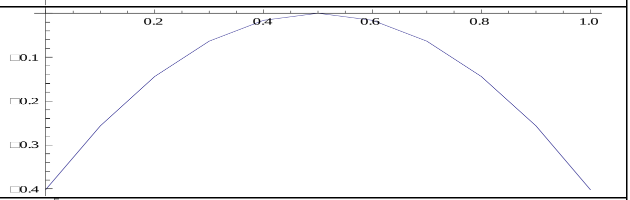
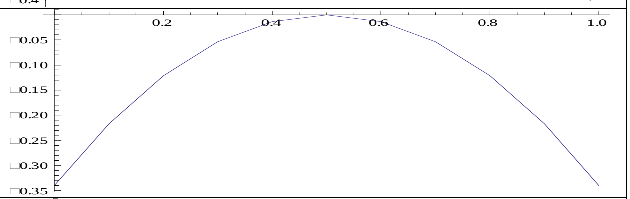
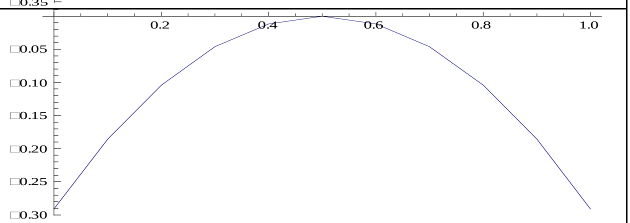
5. SIMULATION PACKAGE APPLICATION EXAMPLE

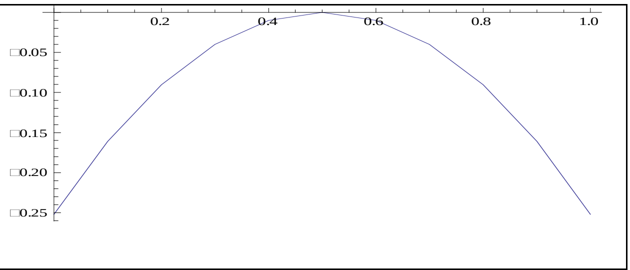
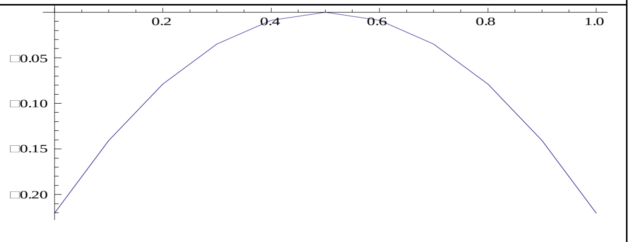
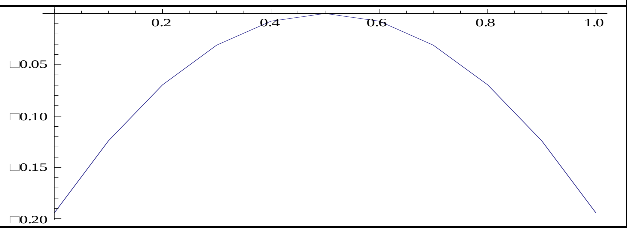
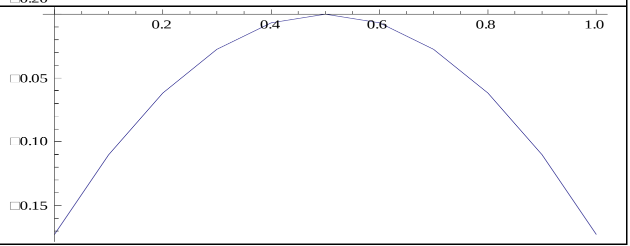
In this section we will study the effect of changing the carrier duty cycle on the inverter output voltage THD. By calculating equations (10) and (11) at varying carrier duty cycle from 0 (Right-aligned triangle waveform) to 1 (Left- aligned triangle waveform). Take care that the default carrier has duty cycle equals ½ (Center aligned triangle waveform), so we will calculate the percentage difference due to duty cycle variation from default.

The following table summarizes the effect of changing carrier duty cycle (d) over THD at different frequency modulation index for single phase PWM inverter. The x-axis represent duty cycle (d) and y-axis represents percentage change of THD

Table 1. Effect of changing carrier duty cycle on PWM inverter output voltage THD

No.	m_f	Max THD reduction %	d Satisfies Max. reduction	Figure (THD Vs variable d from 0 to 1)
1	3	2.7 %	0.25 , 0.75	
2	5	10%	0,1	
3	7	5%	0,1	
4	9	2.6%	0,1	
5	11	1.8%	0,1	
6	13	1.25%	0,1	

7	15	0.9%	0,1	
8	17	0.8%	0,1	
9	19	0.6%	0,1	
10	21	0.5%	0,1	
11	23	0.4%	0,1	
12	25	0.35%	0,1	
13	27	0.3%	0,1	

14	29	0.25%	0,1	
15	31	0.22%	0,1	
16	33	0.19%	0,1	
17	35	0.17%	0,1	

From Table1, it is clear that the maximum reduction in THD was 10% at $\mathbf{m_f = 5}$, and duty cycle is 0 or 1. Moreover the reduction value decreases as m_f increases and the best duty cycle for maximum reduction at all values of $\boldsymbol{m_f}$ is 0 or 1 (i.e. at aligned right or aligned left triangle carrier waveform)

One may refuse the deductions from Table1 especially the evaluation of THD isn't evaluated in wide range space of combined variables ($\boldsymbol{d,m_f}$) , so for more confident and reliable work, three dimensional plots are executed in order to figure out the effect of changing carrier duty cycle on THD as shown in the following two figures

Figure 6 shows a three dimensional plot of the THD function of variables $\mathbf{d}$ and $\mathbf{m_f}$ where x-axis represents $\mathbf{d}$, y-axis represents $\mathbf{m_f}$ and, z-axis represents THD. From Figure 6, we can calculate maximum change of THD as follows:

Max. Change of THD = $\frac{0.650-0.640}{0.650} * 100 = 1.53\%$

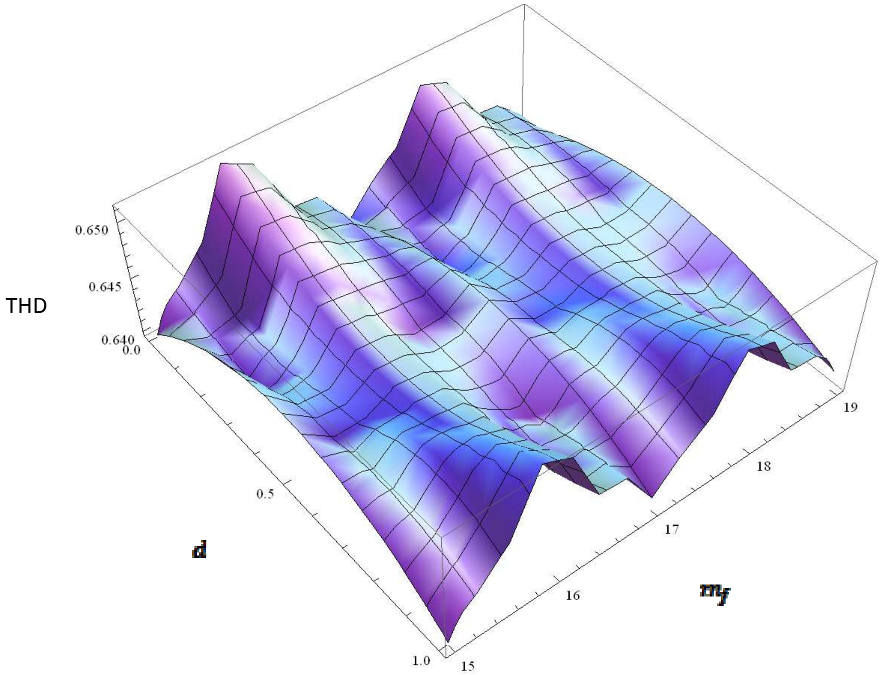


Figure 6. A three dimensional plot of THD versus $\mathbf{d}$ and $\mathbf{m_f}$

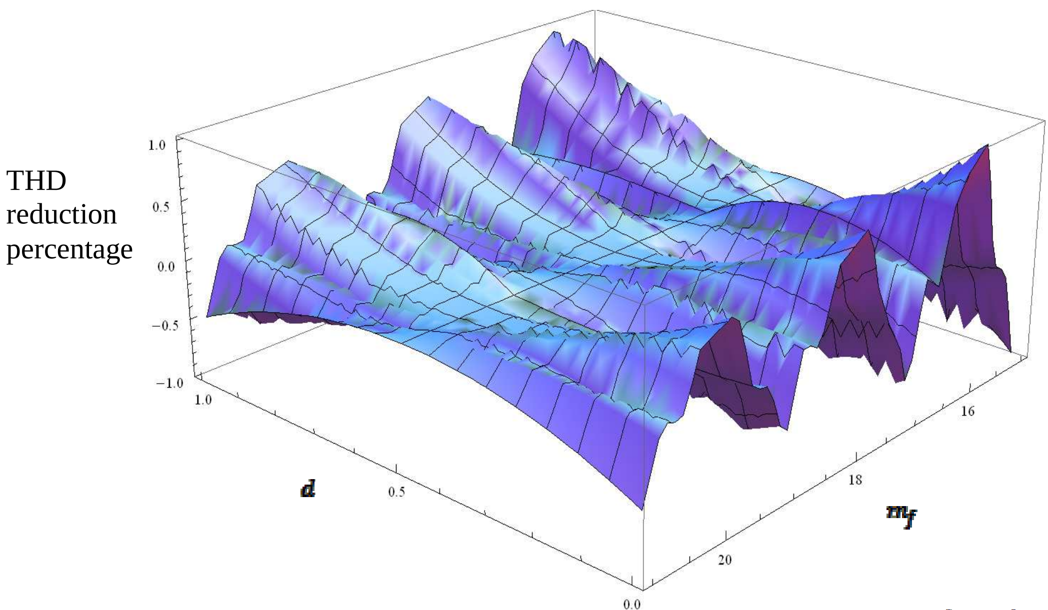


Figure 7. A three dimensional plot of THD reduction percentage by varying $(\mathbf{d}, \mathbf{m_f})$

The definition of THD reduction percentage is a given as follows:

$$\text{THD reduction percentage}(\mathbf{d}, \mathbf{m_f}) = \frac{THD(0.5, m_f) - THD(d, m_f)}{THD(0.5, m_f)}\ \% \qquad (22)$$

Which represents the effect of changing carrier duty cycle from symmetrical one (at $\mathbf{d = 0.5}$) on THD in a percentage form.

Figure 7 shows a three dimensional plot of the THD reduction percentage function of variables $\mathbf{d}$ and $\mathbf{m_f}$ where x-axis represents $\mathbf{d}$, y-axis represents $\mathbf{m_f}$ and, z-axis represents THD reduction percentage.

From Figures 6 and 7, it is obvious that maximum THD reduction is less than 1%, and also we can deduce that $\boldsymbol{m_f}$ increases as THD reduction decreases. In addition the lowest THD occurs at $\boldsymbol{d = 0 \text{ or } d = 1}$(left or right aligned carrier signal).

From the above discussion it is figured out that the varying carrier duty cycle has not significant THD reduction especially at high carrier frequencies. Unfortunately, this figures out results were obtained in [18] that “a simple but efficient modulation approach based on the optimization of the shape of the triangular voltage waveform is proposed. The results have shown that the total harmonic distortion of the optimized voltage waveform decreases gradually and thus helps to improve the power quality during the conversion. ” and “The optimized THD at the switching frequencies of 5 and 10 kHz surprisingly increased at the modulation indices being larger than 0.7. It should be noted that at these switching frequencies, the THD difference between the symmetric and optimized triangular waveforms increased by almost up to 29% around the modulation index of 0.7 and this is quite meaningful and encouraging outcomes.” **are fake**. Because lowest THD occurs at left or right aligned carrier wave shape not at certain value of α (duty cycle) as mentioned in [17] and THD reduction by 29% cannot be occurred by this technique.

From the Table 1 it is obvious that the carrier duty cycle variation has not significant change for THD especially at a high carrier frequency.

6. CONCLUSION

A precise analysis of PWM inverter output voltage THD is proposed in this paper. By modeling the problem and simplifying its equations a fast convergent closed form is obtained and, using Mathematica, which is a powerful integrated tool for numerical computation, symbolic mathematics, optimization, and visualization and is built on top of its own splendid functional programming language [19]. Therefore, we have a solid framework to simulate PWM inverter output voltage in time and frequency domain. By employing our closed form in Mathematica, the simulation speed became extremely faster. So, we established our simulation package relying on exact symbolical calculation avoiding numerical manipulation errors. In other words any other numerical manipulation can be failed in the gross error problem. In fact these numerical calculation errors can be accumulated to get fake results especially when dealing with very large amount of numbers with high precision values.

Indeed some close results to the fake results of [18] were obtained when we simulate the system by using traditional simulation software. By focusing to blowout the mystery of these results we got from simulation software were due to accumulated sampling errors and rounding errors and mathematical operations rounding errors. By taking into account of these errors and increasing calculation precision and decreasing sampling intervals we got more efficient results. But by modeling the system and using the most powerful CAS (Mathematica) software, we got exact results which show the real behavior of THD depending on varying carrier duty cycle and carrier frequency.

Also, THD reduction by changing carrier duty cycle technique is not an efficient method especially at high carrier frequency signal and didn't give valuable results.

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تحليل وتوضيح دقيق لاجمالى تشويه التوافقيات للجهد الخارج من عواكس تعديل عرض النبضة

في هذه المقالة تم تقديم دراسة شاملة عن إجمالى تشويه التوافقيات للجهد الخارج من عواكس تعديل عرض النبضة باستخدام جبر الحاسوب. أولا تم شرح دالة الهدف باستخدام طرق تحليلية رياضية وبتبسيط هذه المعادلات باستخدام مسلمات حساب المثلثات والصيغ الرياضية وباستخدام احد برامج جبر الحاسوب (MATHEMATICA). وبتوظيف امكانياتة في معالجة المعادلات الرياضية الرقمية لحساب اجمالى تشويه التوافقيات وعرض المعادلة الناتجة في صورة معادلة بسيطة وسريعة الحل. ثانيا تم عمل برنامج محاكاة باستخدام برنامج جبر الحاسوب (MATHEMATICA) لعرض اجمالى تشويه التوافقيات للجهد الخارج من عواكس تعديل عرض النبضة. أخيرا تم استخدام هذا البرنامج لدراسة تأثير تغيير عرض الموجة الحاملة على اجمالى تشويه التوافقيات للجهد الخارج من عواكس تعديل عرض النبضة، بالإضافة إلى ذلك تمت مقارنة هذه الدراسة بمرجع [18] وبالتالي تم استنتاج أن نتائج هذا المرجع خاطئة.