Journal of Engineering Sciences, Assiut University, Vol. 41 No1 pp.113-135 - January 2013

Passive earth pressure against retaining wall using log-spiral arc

AbdelAziz Ahmed Ali senoon
Associate Professor, Civil Engineering Dept., Faculty of Eng., Assiut University, Assiut , Egypt.
email:asenoon2000@yahoo.ca

(Received November 1, 2012 Accepted November 29, 2012)

Abstract

Passive earth pressure against retaining wall depends on a number of factors such as, soil friction angle φ, soil wall friction angle δ, backfill angle (ground surface inclination behind wall β), inclination of wall face on horizontal α, and surface of rupture. Several theories have been developed to overcome this problem, i. e., determination of the coefficient of passive earth pressure using the plane surface of rupture. One of the important parameter which affect the coefficient of the passive earth pressure is the surface of rupture. In the present paper, formulation is proposed for calculating coefficient of passive earth pressure on a rigid retaining wall undergoing horizontal translation based on surface of rupture consisting of log-spiral and linear segments assisted by computer program (MATLAB program). The present study is compared with coulomb's results. The comparisons of that the present study predicted values of earth pressure are much less than those of coulomb's values specially if δ≥ 0.3 φ. These results agree well with another research.

In order to facilitate the calculation of coefficient of passive earth pressure, using the proposed equations, a modified coefficient of passive earth pressure is provided. It is a function of (φ, δ, β, α).

Keywords: Passive earth pressure, retaining wall, surface of rupture, log-spiral

1. Introduction

Retaining structures are vital geotechnical structures; because the topography of earth rupture surface is a combination of plain, sloppy and undulating terrain. The retaining wall has traditionally been applied to free-standing walls which resist thrust of the bank of earth as well as providing soil stability of a change of ground elevation. The design philosophy of the wall deals with the magnitude and distribution of the lateral pressure between soil mass and wall.

Estimation of passive earth pressure acting on the rigid retaining wall is very important in the design of many geotechnical engineering structures; particularly retaining wall. Passive earth pressure calculations in geotechnical analysis are usually performed with the aid of Rankine [24] or Coulomb [4] theories of earth pressure based on uniform soil properties. These traditional earth pressure theories are derived from equations of equilibrium along on an assumed pla.nner failure surface passing through the soil mass. Both assume that the distribution of the passive earth pressure exerted against the wall is triangular. However, the distribution of the earth pressure on the face of rough wall depends on the wall movement (rotation about top, rotation about bottom and horizontal translation) and is nonlinear. This is different from the assumption made by both Rankine and Coulomb.

Coulomb’s theory is more versatile in accommodating complex configurations of backfills and loading conditions as well as frictional effects between wall and backfill. However, both theoretical and experimental studies have shown that the Coulomb assumption of plane surface sliding is not perfectly valid when the wall is rough, especially in the passive case when interface friction is more than 1/3 of internal soil friction angle. The curvature of the failure surface behind the wall needs to be taken into account. Hence, Coulomb’s theory leads to a large overestimation of the passive earth pressure.

Rankine’s theory is applicable for the calculation of the earth pressure on a perfectly smooth and vertical wall, but most retaining walls are far from frictionless soil structure interface.

The passive earth pressure problem has been widely treated in the text books, literature and articles [1-22]. Theoretical procedures for evaluating the earth pressure using different approaches (the limit equilibrium method [11] and [8], the slip line method [5], [15] , [22] and [14] , the upper and lower bound theorems of limit analysis [23] and numerical computation.

Rupa and Pise, [19] used a circular arc due to arching effect for determining the passive earth pressure coefficient. Janbu [13] used a method of slices with bearing capacity factors to calculate passive pressure resultants. These different approaches generally confirm the accuracy of the Log Spiral Theory [5] for a wide range of the internal soil friction and the soil–structure interface friction angle. Similarly, Martin [10] and Benmebarek et al. [17] who used FLAC2D numerical analysis to evaluate passive earth pressures have found fairly close agreement with Log Spiral Theory. In spite of recent published methods, the tendency today in practice is to use the values given by Caquot and Kérisel [5] and Kérisel and Absi [15].

Many studies have investigated the capacity and load-deflection relationships for walls under passive conditions using finite element and finite difference methods. Duncan and Mokwa [7] reviewed the results of many of these studies, and reported that they have generally found the log-spiral surface accurately reflect the computed failure surface from the models. Moreover, they found that log-spiral solutions for passive capacity are much more compatible with the results of element modeling than the Coulomb model. Smith and Griffiths [21] used the finite element method to estimate the earth pressure using an elastic-perfectly Mohr-Coulomb constitutive model with stress redistribution achieved iteratively using a reduced integration elasto- viscoplasticity algorithm.

In order to appreciate the accuracy of the present analysis, the theoretical approach of Coulomb and others are used for comparison.

1.1. Coefficient of passive earth pressure

Lateral earth pressure is the pressure that soil exerts in the horizontal plane. To describe the pressure a soil will exert a lateral earth pressure coefficient, K,. This coefficient is the ratio of horizontal pressure to vertical pressure (K= σ_h/σ_v). It is used in geotechnical engineering analysis depending on the characteristics of its applications. There are many theories for predictions of lateral earth pressure, some are empirically based and some are analytically derived. In this section, we will discuss the theories for the passive earth pressure only.

1.2. Coulomb's theory [4]

Coulomb (1776) first studied the problem of the lateral earth pressure on the retaining structures. He used limit equilibrium theory, which considers the failing soil block as a free body in order to determine the limiting horizontal earth pressure. His theory treats the soil as isotropic and accounts for both internal friction at the wall-soil interface (friction angle δ)

The coefficient of passive earth pressure based on Coulomb's theory is:

$$K_{pC} = \frac{sin^2(\alpha-\varphi)}{sin^2(\alpha)\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\varphi+\delta)\sin(\varphi+\beta)}{\sin(\alpha+\delta)\sin(\alpha+\beta)}}\right]^2} \quad (1)$$

Where:

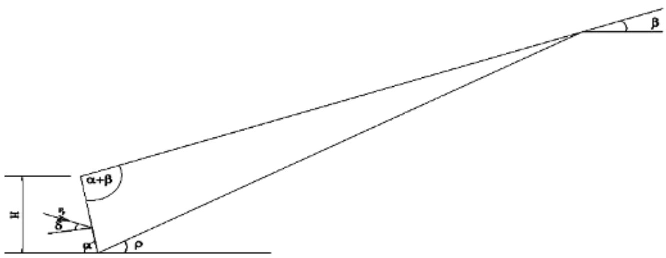
K_{pc} = the coefficient of the passive earth pressure based on Coulomb's theory

β = angle between backfill surface line and a horizontal line

φ = friction angle of the backfill soil

α = angle between a horizontal line and the back face of the wall

δ = angle of wall friction



1.3. Rankine's theory

Rankine's method (1857) of evaluating passive pressure is a special case of the conditions considered by Coulomb. In particular, Rankine assumes that there is no friction at the wall-soil interface (δ = 0). The coefficient of Rankine's passive earth pressure can be computed as:

$$K_{pR} = \cos\beta\,\frac{\cos\beta + \sqrt{cos^2(\beta) - cos^2(\varphi)}}{\cos\beta - \sqrt{cos^2(\beta) - cos^2(\varphi)}} \quad (2)$$

When the embankment slope angle β equal zero, $K_{pR} = tan^2\left(45 + \varphi/2\right)$.

1.4. Properties of logarithmic spiral

The equation of the logarithmic spiral [6] is generally used in solving problems in soil mechanics in the form:

$$r = r_o\, e^{\theta \tan\varphi} \qquad (3)$$

Where r = radius of the spiral
r_o =starting radius at θ=0.0
φ = angle of friction of soil
θ = angle between r and r_o

the basic parameters of a logarithmic spiral are shown in Fig(2)., in which O is the center of the spiral. The area of the sector OAB is given by

$$A = \int_0^{\theta} \frac{1}{2} r (r d\theta) \qquad (4)$$

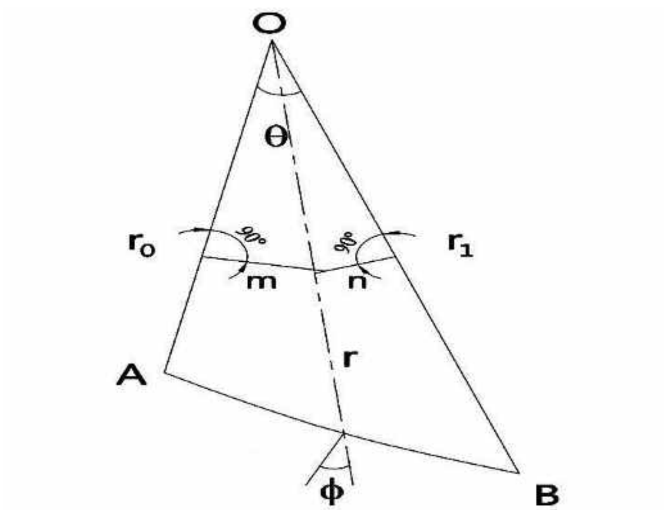


Figure 2 General parameters of a logarithmic spiral (after Das [6])

Substituting the values of r from Eq.(3) into Eq.(4) , we get

$$A = \int_0^{\theta_1} \frac{1}{2} r_o^2 e^{2\theta \tan\emptyset} d\theta = \frac{r_1^2 - r_o^2}{4 \tan\varphi} \qquad (5)$$

The location of the centroid can be defined by the distances $\bar{m}$ and $\bar{n}$ in Figure (2) measured from OA and OB respectively, and can be given by the following equations (Hijab, 1956):

$$\bar{m}=\frac{4}{3}r_o\ \frac{tan\varphi}{(9\ tan^2\varphi+1)}\left[\frac{\left(\frac{r_1}{r_o}\right)^3(3\tan\varphi\ \sin\theta-\cos\theta)+1}{\left(\frac{r_1}{r_o}\right)^2-1}\right] \quad (6)$$

$$\bar{n}=\frac{4}{3}\ r_o\ \frac{\tan\emptyset}{(9\ tan^2\ \emptyset+1)}\left[\frac{\left(\frac{r_1}{r_o}\right)^3-3\tan\emptyset\sin\theta-\cos\varphi}{\left(\frac{r_1}{r_o}\right)^2-1}\right] \quad (7)$$

Another important property of the logarithmic spiral defined by equation (3) is that any radial line makes an angle φ with the normal to the curve drawn at the point where the radial and spiral lines intersect. This basis is particularly useful in solving problem related to lateral earth pressure.

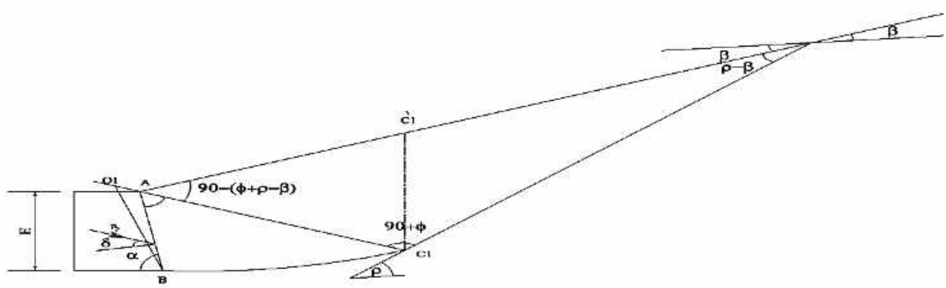
2. Procedure for determination of passive earth pressure (cohesion less backfill)

Figure (3a) shows the curved failure plane in the granular backfill of a retaining wall of height H. The shear strength of the granular backfill is expressed as $\tau_f\ = \sigma\ \tan\varphi$. The curved lower portion BC_1 of the failure wedge is an arc of logarithmic spiral defined by Eq.(3) The center of the log spiral lies on the line C_1A (not necessarily within the limits of the points(C_1 and A). The upper portion C_1D is a straight line that makes an angle (ρ) with the horizontal. (ρ) defined by the following equation.

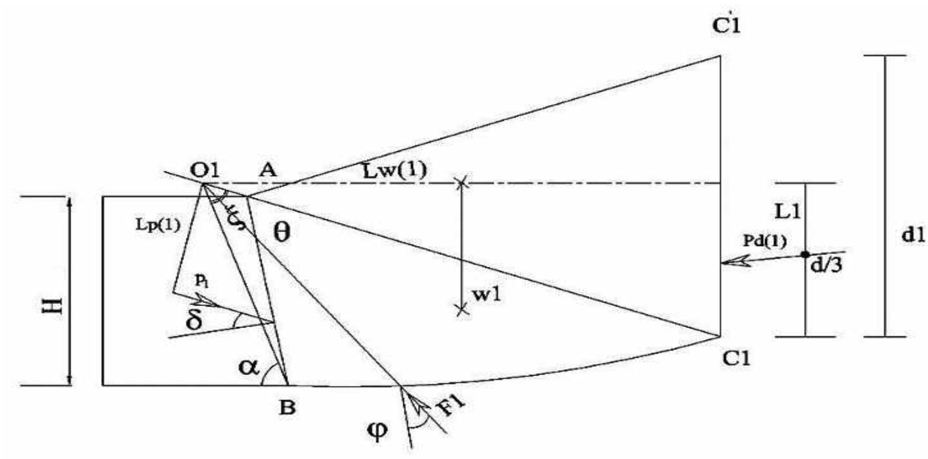
$$\rho = \omega - \varphi \quad (8)$$

Where ω as follows:

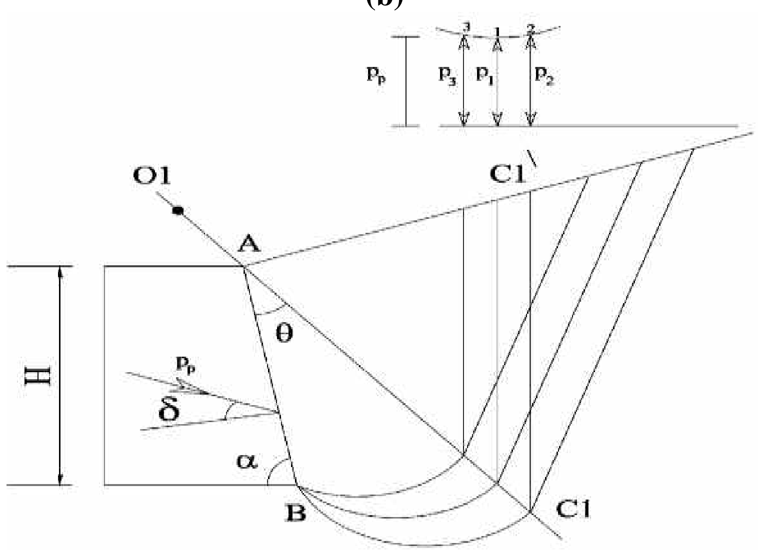
$$\tan\omega = \frac{\tan(\varphi+\beta)+\sqrt{\tan(\varphi+\beta)[\tan(\emptyset+\beta)+\cot(\varphi+\alpha)][1+\tan(\delta-\alpha)\cot(\varphi+\alpha)]}}{1+\tan(\delta-\alpha)[\tan(\varphi+\beta)+\cot(\varphi+\alpha)]} \quad (9)$$



(a)



(b)



(c)

Figure 3 Passive earth pressure against retaining wall with curved failure surface

The soil in zone AC_1D is in Rankine's passive state. Figure(3) shows the procedure for evaluating the passive resistance by trail wedges (Terzaghi and Peck, 1967). The retaining wall is first drawn to scale as shown in Figure(3a). The line C_1A is drawn in such a way that it makes an angle of (ρ-β) with the surface of the backfill. BC_1D_1 is trials wedge in which $BC_1$is the arc of a logarithmic spiral according to the equation Eq. (3). O_1 is the center of the spiral (note: $O_1B = r_o$ and $O_1C_1 = r_1$ and angle BO_1C_1 = angle between two radial lines of spiral, Figure 3b. Now let us consider the stability of the soil mass ABC_1C_1' (Figure (3b). For equilibrium the following forces per unit length of the wall are to be considered:

1- Weight of soil in zone ABC_1C_1'= W_1 = (γ) (area of ABC_1C_1')(1)

2 -The vertical face, C_1C_1' , is the zone of Rankine's passive state; hence, the force acting on this face is

$$P_{d(1)} = \frac{1}{2}\gamma(d_1)^2 tan^2\left(45+\frac{\emptyset}{2}\right) \tag{10}$$

Where $d_1 = C_1C_1'$, $P_{d(1)}$ acts parallel to the ground surface at a distance of $d_1/3$ measured vertically upward from C_1

3- F_1 is the resultant of the shear and normal forces that act along the surface of sliding BC_1. At any point on the curve, according to the property of the logarithmic spiral, a radial line makes an angle φ with the normal. Because the resultant, F_1 makes an angle φ with the normal to the spiral at its point of application, its line of application will coincide with a radial line and will pass through the point O_1.

4- P_1 is the passive force per unit length of the wall. It acts at distance of H/3measured vertically from the bottom of the wall. The direction of the force P_1 is inclined at an angle δ with the normal drawn to the back face of the wall.

Now, taking the moment of W_1, $P_{d(1)}$, F_1 and P_1 about the point O_1 for equilibrium, we have

$$W_1\left[l_{w(1)}\right] + P_{d(1)}[l_1] + F_1\ [0] = P_1\left[l_{p(1)}\right] \tag{11}$$

$$P_1 = \frac{1}{l_{p(1)}}\left[W_1 l_{w(1)} + P_{d(1)} l_1\right] \tag{12}$$

Where $l_{w(1)}, l_1$ and $l_{p(1)}$are moment arms for the forces W_1 ,$P_{d(1)}$ and P_1, respectively.

The preceding procedure for finding the trial passive force per unit length of the wall is repeated for several trial wedges such as those shown in Figure (3c). Let P_1, P2, P_3,,…..P_n be the forces that corresponding to trial wedges 1, 2, 3, ……, n. The lowest point of the smooth curve defines the actual minimum passive forces, P_p, per unit length of the wall. The coefficient of the passive earth pressure $K_p= 2P_p/\gamma H^2$.

It is worthwhile mentioning here that when we did not get a clear minimum coefficient of passive earth pressure, take $k_{p(}$min.) corresponding the angle BO_1C between $O_1B = r_o$ and $O_1C_1 = r_1$ equal to (ρ - β) ,where ρ inclination angle of tangent at C_1on the horizontal and β inclination of the ground surface

3. Main goal of the present work

The main goal of the present work is the transfer of the shown case of passive earth pressure against rigid retaining wall using surface of rupture consisting of log-spiral curve and linear segments as depicted in Figure(3) into group of equations that can be solved easily by computer with high accuracy.

3.1. Parameters used in the program

Wall geometry: height of the wall, H, inclination of the back wall on the horizontal, α, =90°, 80° and 70°

Ground surface slope of the backfill β = (0, 0.2, 0.4, 0.6 and 0.8) ϕ

Soil properties: angle of internal friction, ϕ, =5, 10, 15, 20, 25, 30, 35, 40 and 45

Friction between wall and soil δ = (0, 0.2, 0.4, 0.6, 0.8 and 1) ϕ

3.2. Procedure of calculations

1- For a constant $\alpha = 90^0$; φ is changed nine times as mentioned above and the corresponding minimum coefficient of passive earth pressure was found as discussed before by computer program (MATLAB program).
2- The value δ is changed six times and step No. 1 was repeated.
3- The value β is changed five times and steps No. 1 and 2 were repeated.
4- For $\alpha = 90^0$, 80^0 and 70^0 degree steps No. 1, 2 and 3 were repeated.
5- Results for steps No. 1, 2, 3 and 4 are shown in Table 1, 2 and 3

Table 1 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 90^0$

φ	β =0.0					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.218	1.225	1.233	1.233	1.240	1.247
10	1.495	1.510	1.527	1.547	1.575	1.598
15	1.811	1.862	1.918	1.971	2.039	2.109
20	2.224	2.310	2.428	2.556	2.709	2.892
25	2.712	2.893	3.120	3.395	3.740	4.175
30	3.319	3.672	4.100	4.661	5.429	6.425
35	4.120	4.712	5.532	6.703	8.450	10.417
40	5.140	6.168	7.746	10.301	14.089	18.047
45	6.484	8.305	11.427	17.381	25.307	34.026
φ	β =0.2					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.255	1.252	1.259	1.266	1.273	1.273
10	1.567	1.594	1.609	1.628	1.656	1.679
15	1.987	2.022	2.080	2.143	2.213	2.286
20	2.519	2.624	2.748	2.883	3.056	3.260
25	3.208	3.427	3.695	4.012	4.414	4.924
30	4.156	4.564	5.108	5.824	6.771	7.977
35	5.458	6.280	7.369	8.940	11.238	13.722
40	7.379	8.919	11.202	14.970	20.097	25.462
45	10.203	13.274	18.356	27.648	39.359	51.956

Table 1 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 90^0$ (continuous)

φ	β =0.4					
	Δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.282	1.288	1.284	1.291	1.297	1.304
10	1.653	1.664	1.691	1.706	1.734	1.755
15	2.132	2.201	2.241	2.306	2.378	2.458
20	2.813	2.922	3.070	3.222	3.403	3.620
25	3.743	4.007	4.295	4.658	5.113	5.690
30	5.098	5.589	6.237	7.091	8.233	9.642
35	7.088	8.126	9.561	11.566	14.446	17.479
40	10.262	12.459	15.636	20.834	27.556	34.460
45	15.681	20.272	28.161	41.812	58.329	75.369
φ	β =0.6					
	Δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.306	1.312	1.317	1.313	1.319	1.324
10	1.720	1.745	1.755	1.781	1.796	1.824
15	2.295	2.342	2.397	2.458	2.539	2.616
20	3.114	3.246	3.370	3.541	3.724	3.948
25	4.323	4.592	4.888	5.293	5.782	6.415
30	6.111	6.688	7.433	8.384	9.703	11.289
35	8.978	10.311	11.962	14.419	17.869	21.400
40	14.053	16.804	20.921	27.694	36.103	44.495
45	23.270	29.626	41.012	60.019	82.058	103.672
φ	β =0.8					
	Δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.327	1.332	1.336	1.340	1.335	1.339
10	1.779	1.801	1.822	1.830	1.854	1.868
15	2.448	2.485	2.530	2.583	2.646	2.729
20	3.397	3.517	3.627	3.791	3.981	4.196
25	4.838	5.094	5.414	5.816	6.324	6.973
30	7.134	7.696	8.511	9.528	10.934	12.604
35	11.009	12.400	14.249	17.039	20.899	24.723
40	18.046	21.292	26.249	34.531	44.296	53.707
45	32.072	40.413	55.714	80.184	107.414	132.636

Table 2 Coefficient of passive earth pressure using log-spiral curve failure surface at α = 80^0

φ	β =0.0					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.253	1.253	1.2538	1.2548	1.256	1.2575
10	1.568	1.569	1.5641	1.5763	1.5776	1.5908
15	1.850	1.876	1.8977	1.9319	1.9644	2.0035
20	2.218	2.257	2.3294	2.4063	2.4993	2.6101
25	2.624	2.750	2.8837	3.0558	3.2707	3.5347
30	3.136	3.361	3.6346	3.9872	4.4435	5.0564
35	3.792	4.158	4.6703	5.3675	6.3587	7.7397
40	4.569	5.218	6.1662	7.565	9.8053	12.6693
45	5.561	6.673	8.4238	11.373	16.5068	22.5112
φ	β =0.2					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.286	1.287	1.287	1.288	1.289	1.290
10	1.648	1.661	1.655	1.661	1.668	1.677
15	2.030	2.058	2.079	2.115	2.148	2.188
20	2.525	2.574	2.653	2.735	2.843	2.963
25	3.133	3.278	3.440	3.643	3.894	4.201
30	3.949	4.225	4.582	5.023	5.591	6.348
35	5.040	5.575	6.278	7.226	8.544	10.360
40	6.600	7.591	8.985	11.037	14.284	18.265
45	8.809	10.687	13.584	18.384	26.352	35.386
φ	β =0.4					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.318	1.318	1.318	1.319	1.319	1.320
10	1.667	1.664	1.678	1.693	1.709	1.718
15	2.042	2.079	2.123	2.183	2.237	2.298
20	2.540	2.635	2.749	2.882	3.031	3.215
25	3.160	3.366	3.625	3.944	4.325	4.814
30	3.969	4.407	4.949	5.668	6.638	7.798
35	5.075	5.915	7.078	8.773	11.128	13.520
40	6.608	8.256	10.824	15.078	20.217	25.471
45	8.876	12.191	18.524	28.704	40.596	53.300

Table 2 Coefficient of passive earth pressure using log-spiral curve failure surface at α = 80^0 (continuous)

φ	β =0.6					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.3472	1.347	1.3469	1.3469	1.3471	1.3475
10	1.7065	1.708	1.7263	1.7445	1.7541	1.7746
15	2.1263	2.1745	2.228	2.287	2.3448	2.4195
20	2.6778	2.8046	2.9358	3.0932	3.2796	3.4969
25	3.4052	3.6671	3.9936	4.3817	4.8789	5.4941
30	4.3918	4.9466	5.6697	6.6465	7.9508	9.3019
35	5.7766	6.9429	8.6207	11.164	14.0912	16.9258
40	7.863	10.3493	14.6983	20.709	27.1968	33.8051
45	11.2451	17.0734	28.4959	42.516	58.8074	75.5015
	β =0.8					
	δ					
φ	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.3745	1.3734	1.3724	1.3713	1.3704	1.3697
10	1.739	1.7433	1.7635	1.7832	1.7941	1.8156
15	2.1967	2.2526	2.3006	2.3679	2.4341	2.5116
20	2.7967	2.9325	3.091	3.2676	3.4801	3.7326
25	3.606	3.9211	4.2943	4.775	5.3872	6.0744
30	4.739	5.433	6.3547	7.6607	9.1817	10.6155
35	6.4319	7.9737	10.4122	13.6467	16.9049	20.0532
40	9.1701	12.9573	19.4407	26.598	34.3447	41.8913
45	14.1789	25.3723	40.2277	58.3373	78.8112	82.6926

Table 3 Coefficient of passive earth pressure using log-spiral curve failure surface at α = 70⁰

φ	β =0.0					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.265	1.265	1.265	1.266	1.268	1.269
10	1.523	1.522	1.525	1.530	1.536	1.544
15	1.862	1.861	1.868	1.881	1.899	1.923
20	2.321	2.267	2.294	2.333	2.407	2.460
25	2.681	2.679	2.771	2.883	2.993	3.160
30	3.133	3.214	3.368	3.586	3.874	4.247
35	3.611	3.850	4.197	4.621	5.218	6.041
40	4.333	4.713	5.269	6.130	7.367	9.255
45	5.161	5.731	6.820	8.481	11.244	15.414
φ	β =0.2					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.300	1.300	1.300	1.301	1.303	1.304
10	1.614	1.615	1.618	1.622	1.629	1.637
15	2.047	2.049	2.057	2.072	2.091	2.115
20	2.469	2.509	2.560	2.605	2.680	2.758
25	2.974	3.033	3.160	3.317	3.495	3.722
30	3.546	3.750	3.999	4.334	4.757	5.317
35	4.259	4.647	5.168	5.873	6.838	8.192
40	5.175	5.876	6.913	8.377	10.653	13.681
45	6.394	7.657	9.610	12.844	18.417	25.022
φ	β =0.4					
	δ					
	0	0.2 φ	0.4 φ	0.6 φ	0.8 φ	φ
5	1.332	1.332	1.333	1.334	1.335	1.337
10	1.703	1.705	1.708	1.713	1.719	1.727
15	2.204	2.188	2.207	2.229	2.256	2.288
20	2.643	2.691	2.777	2.854	2.949	3.062
25	3.229	3.363	3.538	3.753	4.003	4.318
30	3.945	4.265	4.647	5.124	5.743	6.567
35	4.902	5.508	6.293	7.368	8.903	10.838
40	6.143	7.287	8.944	11.442	15.268	19.448
45	7.916	10.086	13.616	20.060	28.988	38.690

4. Analysis and Discussions

The discussions illustrate the effect of the parameters study on the coefficient of passive earth pressure. The main investigated parameters are:-

- Angle of internal friction of soil
- Interface friction angle between soil and wall
- Ground surface slope
- Inclination of back surface

A comparison was made between the results of present work and some researches using different surface failure, to evaluate the coefficient of the passive earth pressure.

The deduced formula for calculation k_p corresponding to Coulomb's coefficient (k_{pc}).

4.1 Relation between φ and K_p

The relation between φ and K_p is plotted and shown Figs (4,5), it is clear that with increasing φ the value of K_p increases, and K_p increasing with the increase of δ for constant value of β. Figs (4 and 5) have the same trend for the given values of β = (0.0, 0.8) φ

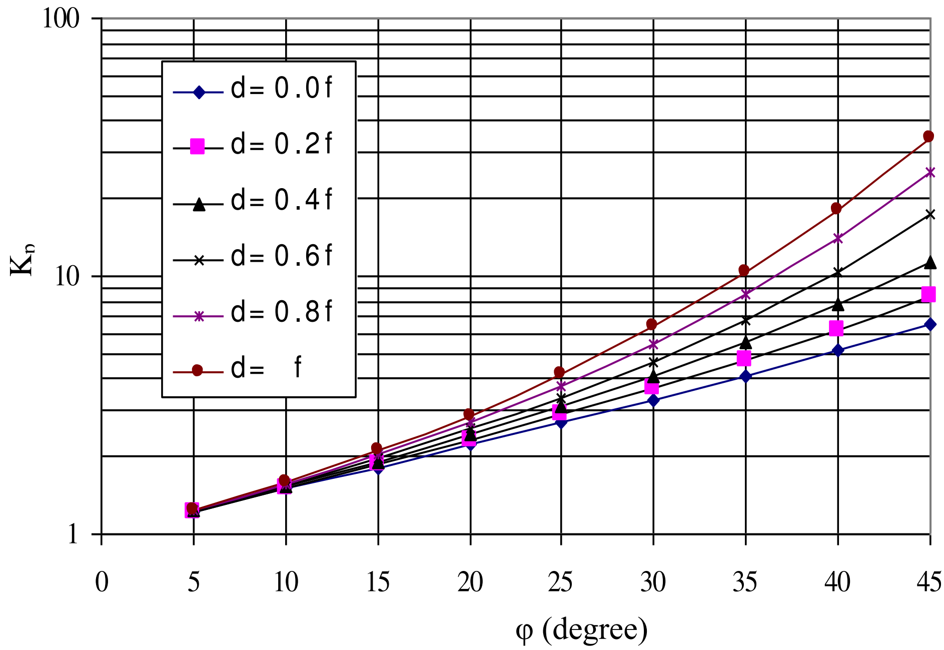


Figure 4 K_p versus φ at β = 0.0 φ and α = 90

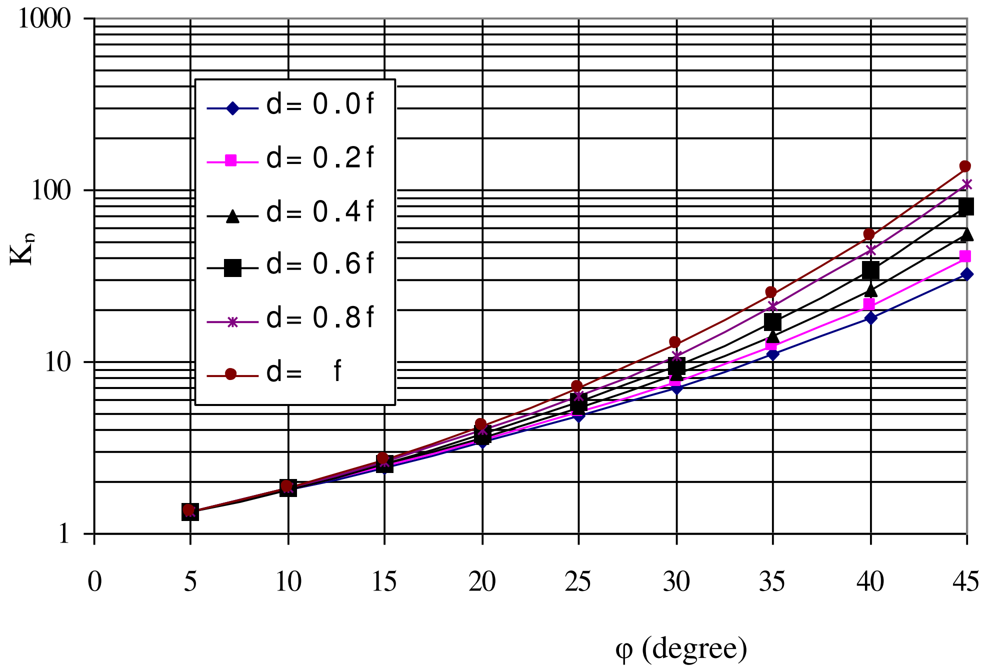


Figure 5 K_p versus φ at β = 0.8 φ and α = 90

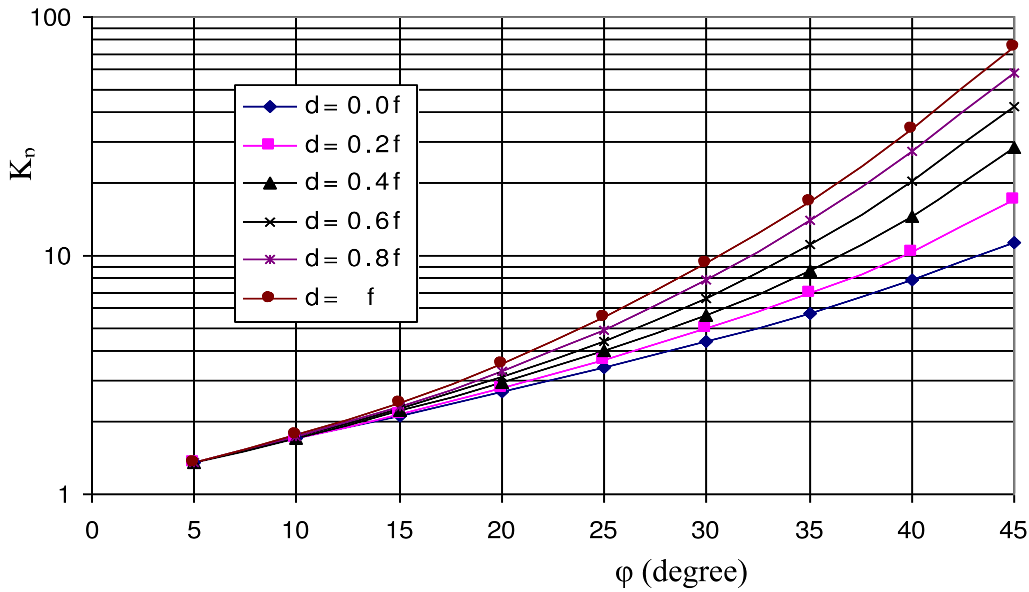


Figure 6 K_p versus φ at β = 0.8 φ and α = 80

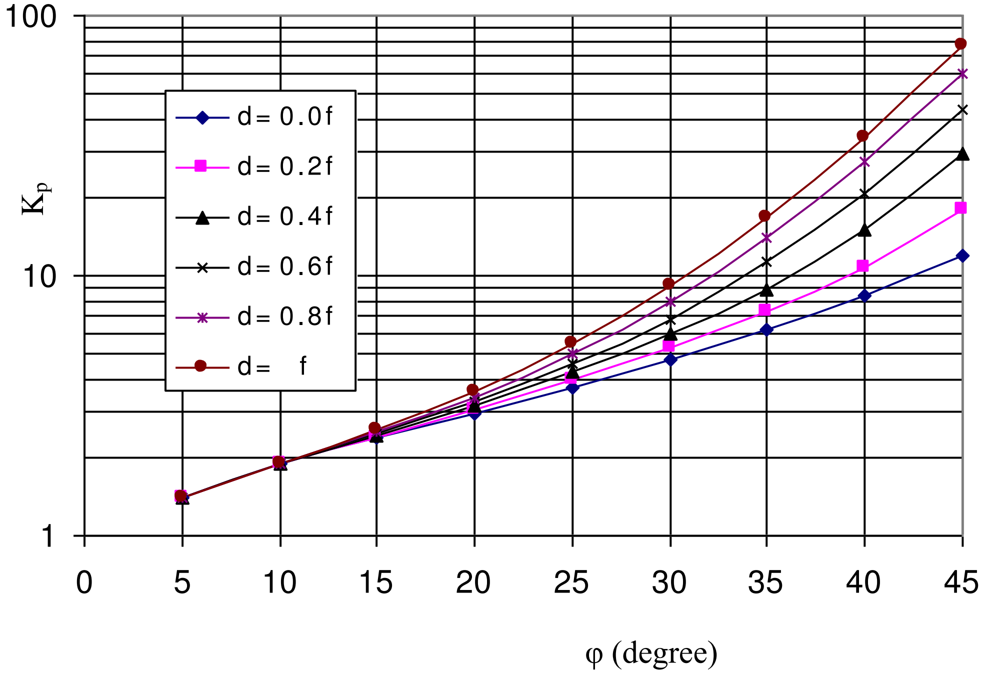


Figure 7 K_p versus φ at β = 0.8 φ and α = 70^0

Figures (5 to 7) show the relation between K_p and φ at β=0.8 φ for different values of α. It is evident that K_p decreases with decreasing α.

4.2 Ground surface slope β

The relation between K_p and β is plotted and shown Fig (8), it is clear that with increasing β the value of K_p increases, and decreases with decreasing α for constant value of δ. Figs (8) have the same trend for the given values of δ = (0.0, 0.2, 0.6, 0.8 and 1) φ.

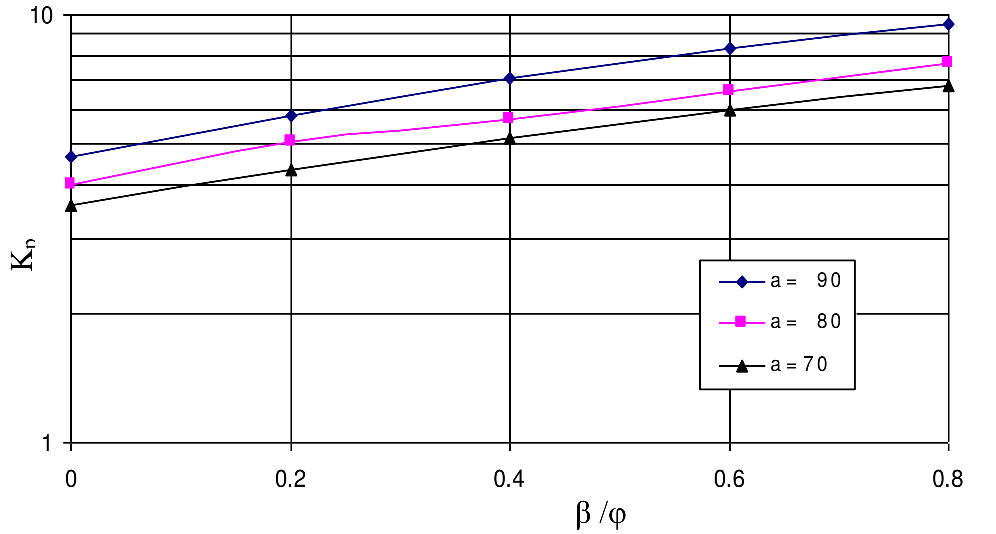


Figure 8 K_p versus β/ φ at $\varphi = 30^0$, δ = 0.6 φ

4.3 Interface angle of internal friction between wall and soil δ

The relation between K_p and δ is plotted and shown in Fig (9), it is clear that with increasing δ the value of K_p increases, and decreases with decreasing of α for constant value of β. Figure (8) has the same trend for the given values of β = (0.0, 0.2, and 0.8) of φ.

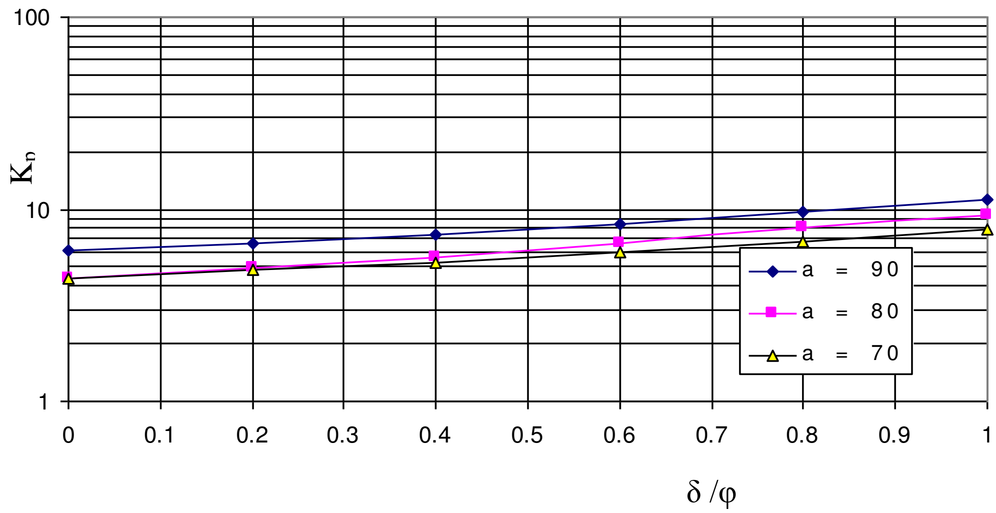


Figure 9 K_p versus δ / φ at $\varphi = 30^0$, β = 0.6 φ

4.4 Inclination of the back wall face α

The relation between K_p and α is plotted in Fig (10). It is clear that with increasing α the value of K_p increases, and increases with increasing δ for constant value of β. Figure (8) has the same trend for the given values of β = (0.0, 0.2, and 0.8) φ.

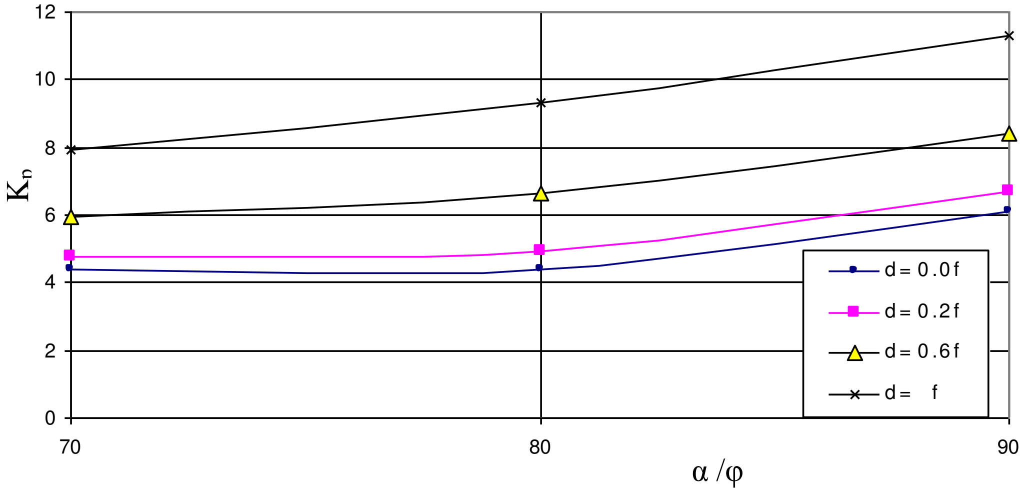


Figure 10 K_p versus α/φ at φ = 30^0, β = 0.6 φ

5. The deduced formula for calculation of K_p corresponding K_{pc} (Columb's coefficient)

Where the magnitude of friction is low so the angle (δ) is small, the rupture surface is approximately planner. As the angle δ increases, however, the lower zone failure wedge becomes curved for values of, (δ > φ/3), up to about one-third of φ. But, as δ becomes larger, the error in the computed K_p increasingly greater, whereby the actual passive is less than the computed value (using Eq. (1)). For larger δ, analysis of force resulting from passive pressure should be based on a curved surface of rupture. When φ <20^o, the difference between planner and curve surface failure little and may be neglected. In this section, we will try found the relation between k_p and K_{pc} for (δ > φ/3, φ>20^o) with different another study parameters.

Based on data recorded in Tables 1, 2 and 3, the values of K_{pc} (Columb's coefficient) are computed using Eq. (1). The relation between K_p / K_{pc} for

Different values of φ at certain δ, β and α may be represented by the following expression:-

$$K_p / K_{pc} = -a \tan(\varphi) + b$$

Where a and b are coefficients obtained by regression formula depending on δ, α and β are listed in Tables 4 and 5 respectively.

Table 4 Coefficient a

α = 90°				
β /φ	δ /φ			
	0.4	0.6	0.8	1.0
0.0	0.37	0.647	1.136	1.456
0.2	0.638	1.024	1.294	1.63
0.4	1.035	1.283	1.61	1.907
0.6	0.766	1.062	1.287	1.594
0.8	1.578	1.826	1.859	2.319
α = 80°				
0.0	0.173	0.378	0.639	1.07
0.2	0.419	0671	1.068	1.402
0.4	0.713	1.08	1.401	1.668
0.6	1.102	1.409	1.659	1.893
0.8	1.422	1.652	1.868	2.044
α = 70°				
0.0	0.065	0.219	0.405	0.676
0.2	0.262	0.447	0.697	1.093
0.4	0.491	0.734	1.104	1.441
0.6	0.788	1.127	1.455	1.677
0.8	1.171	1.47	1.676	1.746

6. Application of the Program and Comparison with Others

Some examples were solved using program and are compared with the references given in Figs. (11-14). Figure(11) shows the K_p versus φ at α =90^0 , β/ φ = 0.0, δ / φ =0.6 using different method. It is clear that where the magnitude of friction is low so that the angle (δ) is small K_p is the same for different methods. After that, clear difference is noticed between planner surface and log-spiral surface failure methods.

Table 5 Coefficient b

$\alpha = 90^o$				
β/φ	δ/φ			
	0.4	0.6	0.8	1.0
0.0	1.132	1.20	1.386	1.449
0.2	1.187	1.293	1.33	1.395
0.4	1.302	1.323	1.380	1.419
0.6	1.288	1.323	1.325	1.369
0.8	1.354	1.366	1.285	1.392
$\alpha = 80^o$				
0.0	1.127	1.163	1.220	1.361
0.2	1.204	1.247	1.364	1.436
0.4	1.285	1.378	1.440	1.469
0.6	1.40	1.447	1.465	1.474
0.8	1.456	1.458	1.454	1.439
$\alpha = 70^o$				
0.0	1.177	1.176	1.198	1.263
0.2	1.241	1.247	1.291	1.408
0.4	1.303	1.331	1.428	1.501
0.6	1.385	1.454	1.518	1.523
0.8	1.495	1.53	1.523	1.454

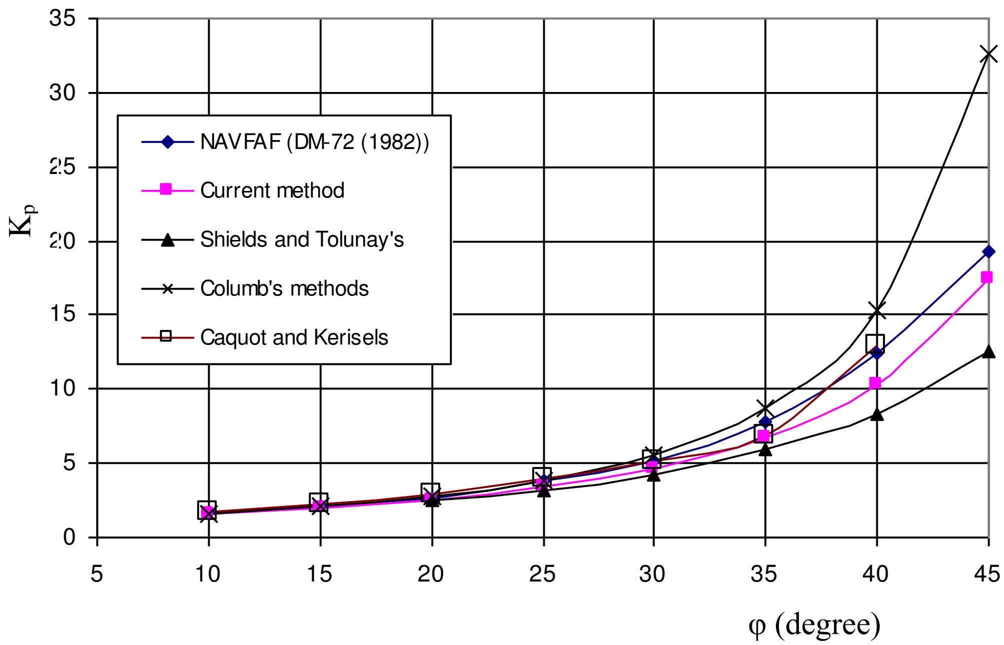


Figure 11 K_p versus φ at $\alpha = 90^0$, $\beta/\varphi = 0.0$, $\delta/\varphi = 0.6$ using different method

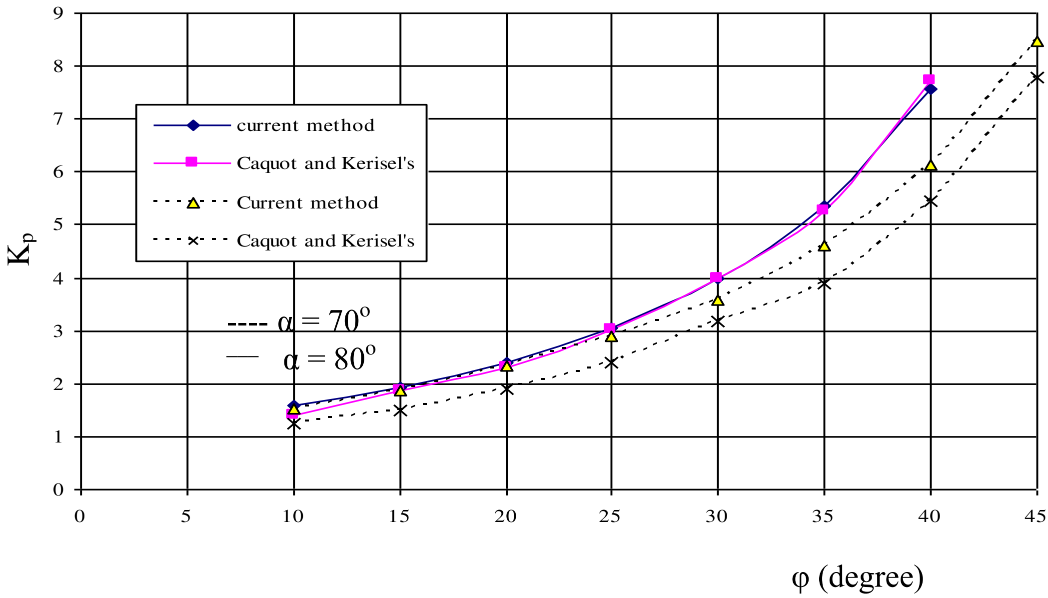


Figure 12 K_p versus φ at α =80⁰, 70°, β/ φ = 0.0, δ / φ =0.6 using different method

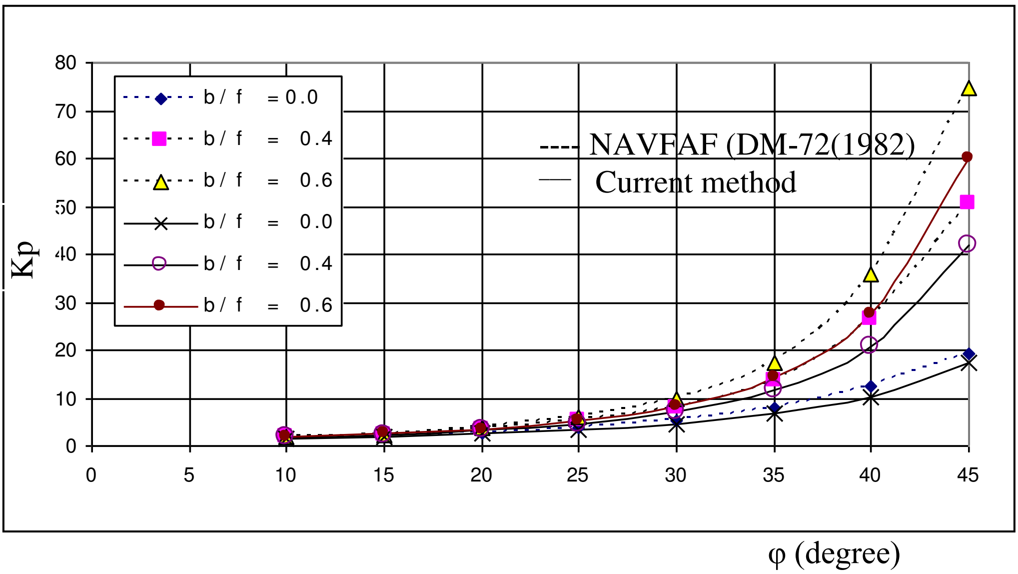


Figure 13 K_p versus φ at α =90°, δ / φ =1.0 using different method

7. Conclusions

The main conclusions of the present study can be drawn as follows:-

- Coefficient of the passive earth increases with the increasing angle of internal friction of soil.
- Coefficient of the passive earth increases with increasing δ /φ.
- Coefficient of the passive earth increases with increasing β/φ.
- Coefficient of the passive earth decreases with decreasing α.
- Where the magnitude of friction is low so the angle (δ) is small, the rupture surface is approximately planner. As the angle δ increases, however, the lower zone failure wedge becomes curved for values of, (δ > φ/3). But as δ becomes larger, the error in the computed K_p increasingly greater, whereby the actual passive is less than the computed value (using Columb's theory)). For larger δ, analysis of force resulting from passive pressure should be base on a curved surface of rupture. When φ <20^o, the difference between planner and curved surface failure is small and may be neglected.

8. References

[1] Amr Radwan, **Fundamentals of Soil Mechanics**, (2006), Electronic version.

[2] Arpad Kezdi and Laszlo Rethati, (1980) " **Handbook of Soil Mechanic**s " Vol.

[3] Akademiai, Kiada, Budapest and Elsevier Scientific Publishing Company, Amsterdam Printed in Hungary.

[4] Chandrakant S. Desai nad John T. Christion (1977) "**Numerical methods in geotechnical engineering"** McGraw Book Company-New York.

[5] Coulomb CA. Essai sur une application des règles des maximas et minimas à quelques problèmes de statique relatifs à l'architecture. Mém. acad. roy. pres. divers savanta, vol. 7, 1776, Paris [in French]

[6] Caquot, A.and Kérisel, ‚J. Tables for the calculation of earth pressure, active pressure and bearing capacity of foundations, Gauthier-Villard, Paris (1948).

[7] Das., B. M. **Principles of Geotechnical Engineering.,** Books/ Cole Engineering Division , Monterey, California (2001)

[8] Duncan, J. M. and Mokwa, R. L. (2001), **Passive earth pressure:** Theories and tests, ASCE J. Geotech. Geoenviro. Engng 127, No. 3, 248-257

[9] D.-Y. Zhu, Q.-H. Qian and C.F. Lee, Active and passive critical slip fields for cohesionless soils and calculation of lateral earth pressures. *Géotechnique*, **51** 5 (2001).

[10] El-shafay, U. M. **Soil Mechanics Part 2**, Dar El-Rateb Universities, Beirut 1990.
Arabic version

[11] G.R. Martin and L. Nad Yan, Modelling passive earth pressure for bridge abutments. Earthquake-induced movements and seismic remediation of existing foundations and abutments. *Geotech Spec Publ*, **55** (1995), pp. 1–16.

[12] H. Rahardjo and D.G. Fredlund, General limit equilibrium method for lateral earth forces. *Can Geotech J*, **21** 1 (1984).

[13] Iqbal H. Khan, "A Text Book of Geotechnical Engineering", New Delhi-110001, (1998).

[14] Janbu, N. (1957). "Earth pressure and bearing capacity calculations by generalized procedure of slices," *Proc. 4th Int. Conf. on Soil Mechanics and Foundation Eng*., Vol. 2, pp. 207-213.

[15] J. Graham, Calculation of earth pressure in sand. *Can Geot J*, **8** 4 (1971).

[16] Kérisel J, Absi E. Tables de poussée et de butée des terres. 3rd ed. Presses de l'École Nationale des Ponts et Chaussées, Paris, 1990

[17] Mc Carthy David F., **" Essential of soil mechanics and Foundations"** Basic Geotechnics, Seventh Edition, Pearson Prentice Hall Upper Saddle Rivers, New Jersey, Columbus, Ohio, Copyright 2007

[18] N. Benmebarek, S. Benmebarek, R. Kastner and A.H. Soubra, Passive and active earth pressures in the presence of groundwater flow. *Géotechnique, The Institution of Civil Engineers, London*, **56** 3 (2006), pp. 149–158.

[19] R. M. El-Hansy, Solved **Problems in Soil Mechanics**, Dar El-Rateb Universities, Beirut ,1990.

[20] Rupa, S. D. and Pise, P.J. (2008)., **Effect of Arching on Passive Earth Pressure Coefficient.,** The 12th International Conference of International Association of Computer Method and Advances in Geomechanics, IACMAG,1-6 October 2008 Goa, India

[21] Shields, D.H. and Tolunay, A.Z. (1973). "Passive pressure coefficients by method of slices," *J. Soil Mech. and Found. Div*., ASCE, 99(12), 1043–1053.

[22] Simth, I. M. and Griffiths, D. V. (2004)**, Programming the Finite Element Method.,** 4th Edition , New York , John Wiley and Sons.

[23] V.V. Sokolovski, Statics of granular media, Pergamon Press, New York (1965).

[24] W.F. Chen, Limit analysis and soil plasticity, Elsevier, Amsterdam (1975)

[25] W.J.M. Rankine, On the stability of loose earth, Philosophical Trans Royal Soc, London (1857).

ضغط الأتربة السالب على الحوائط الساندة باستخدام منحنى الانهيار اللوغاريتمى

المنشات الساندة تعتبر من المنشآت الجيوتقنية المهمة لان طبوغرافية سطح الأرض تكون خليط بين المستوية و مائلة و متعرجة وتستخدم الحوائط الساندة لكي تسند الأتربة و تعطى اتزان للتربة عند تغير المناسيب و تصميم هذه الحوائط يعتمد على قيمة و شكل توزيع الضغوط الجانبية بين الحائط و التربة. حساب ضغط الأتربة السالب على الحوائط الساندة يكون مهم في عديد من المنشات الجيوتقنية . ضغط الأتربة السالب يعتمد على عدة عوامل مثل ، زاوية الاحتكاك الداخلي للتربة ، زاوية الاحتكاك بين سطح الحائط الساند و التربة الملاصق للتربة ، زاوية ميل سطح الأرض خلف الحائط الساند وكذلك زاوية ميل وجهه الحائط الساند و كذلك سطح الانهيار المفروض لإيجاد معامل الضغط الجانبي السالب على الحائط . أكثر من نظرية استخدمت للتغلب على هذه المشكلة لكي تحدد معامل الضغط الجانبي للتربة باستخدام سطح الانهيار المستوى. ومن أهم العوامل التي تؤثر على ضغط الأتربة السالب سطح الانهيار المفروض. في هذا البحث تم حساب معامل ضغط التربة السالب على الحوائط الجاسئة تحت الحركة الأفقية معتمدا على سطح الانهيار مكون من جزءين جزء من منحنى قوس لوغاريتمى عند قاع الحائط و جزء مستقيم يمس المنحنى اللوغاريتمى ويمتد حتى يتقاطع مع سطح الأرض و ذلك باستخدام برنامج كمبيوتر (ماتلاب).

تم مقارنة نتائج هذا البحث مع النتائج المماثلة المستنتجة باستخدام نظرة كولومب و النتائج المستنتجة باستخدام البرنامج و أوضحت النتائج أن معامل ضغط الأتربة السالب اقل بكثير من المستنتج باستخدام نظرية كولوب و خاصة إذا كانت زاوية الاحتكاك بين التربة و ظهر الحائط اقل أو اكبر او يساوى 0.3 زاوية الاحتكاك الداخلي للتربة و تم استنتاج معادلة تربط بين القيم المستنتجة وقيم كولوب و بمقارنة النتائج المستنتجة باستخدام البرنامج الخاص بالبحث مع نتائج أبحاث آخرين وجدت معها توافق تام.