

## **SENSING COVERAGE AND OPTIMAL POWER ASSIGNMENT IN WIRELESS SENSOR NETWORKS**

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### **Abstract**

*This paper addressed the sensing coverage problem in wireless sensor networks. A sensing coverage range estimation scheme is presented and analytically validated. In presence of channel fading, we have also proposed a Lagrange based, optimal power assignment algorithm that guarantees a predefined probability of detection over the sensor-to-fusion center communication channels at a given probability of false alarm. Analytic results reflect the effectiveness of the proposed algorithms which tries to make the network operational point falls in the feasible SNR region at the lowest (optimal) transmission energy consumption.*

**Keywords:** Wireless sensor networks; Sensing coverage; Constraint optimization; Adaptive power assignment

## **1. INTRODUCTION**

Wireless sensor networks (WSNs) have the potential to influence our daily lives to a great extent and have many potential civil and military applications including object tracking, intrusion detection, environment and health related applications [1], [2]. WSNs face various design, operational, and management challenges such as low processing power and bandwidth, limited battery life, and short radio ranges. Nodes in a WSN are performing two demanding tasks simultaneously: sensing the environment and communicating with each other to transfer useful information. In WSN, network coverage is an important issue. It means how well an area of interest is being monitored by a network. Usually, a node has a limited sensing range. Any event is said to be detectable if at least one node lies within its observable range. In the coverage algorithms, the commonly adopted sensing model is the Boolean sensing model which assumes that a sensor can cover a disk area centered at itself with a radius equal to its sensing range. However, sensing range is environment-dependent and always changed by obstacles. If we consider the effect of path loss and absorption caused by obstacles, the sensor cannot maintain its sensing range unless it is working on a completely flat and obstacle-free area. In this paper, we consider the probability sensing model which resolves such a problem by adapting the path loss model to estimate the sensing ability on different directions. On the other hand, energy efficiency is another critical design factor in WSNs, because the sensor nodes are usually of low cost and designed with strict restrictions on power consumption. Previous research works on WSNs range

from general theoretic analysis, to proposing optimization solutions for the detection process [3]–[5]. However, these publications mostly neglect the effects of fading in the communication channel, which is an important issue in real environments and ignoring it may cause significant degradation of performance for any designed detection process.

In this paper, we consider the problem of determining the coverage provided by sensors using a realistic coverage model. An optimal power assignment strategy is presented in order to optimize the detection performance in terms of the probability of errors in WSNs.

The remainder of this paper is organized as follows. Related research work is provided in Section 2. Section 3 presents the analytical formulations for the probabilistic coverage algorithm and the optimal power assignment model. In section 4, numerical results are presented and Section 5 concludes the paper.

## 2. RELATED WORK

The coverage problem based on the disc sensing model has been well studied [6], [7], [8]; in such model, an object inside (outside) a sensor's sensing disc is detected with probability one (zero). Despite its simplicity for analysis, many researchers consider alternative sensing models in order to better understand and characterize sensor measurements which are usually affected by noise and vary with the distance between the sensor and the object. In [9]–[13], [14], the exposure model or the probabilistic sensing model has been adopted to analyze coverage and detection problems in sensor networks. For the purpose of energy conservation, it was shown in [15], that when the network is subjected to a joint power constraint, having identical sensor nodes (i.e. all nodes using the same transmission scheme) is asymptotically optimal for binary decentralized detection. Efficient node power allocation to achieve a required performance has been considered in [16], [17]–[19]. In [19], the optimal power assignment problem was addressed with amplify-and forward processing at local nodes. It was shown that such an analog forwarding scheme is optimal in the single sensor case by Shannon's separation principle. It was also shown that optimal power scheduling improves the mean squared error performance by a large margin compared to that achieved by a uniform power allocation scheme. The minimum energy, decentralized estimation with correlated data was addressed in [18]. They exploited knowledge of the noise covariance matrix to select quantization levels at nodes that minimized the power, while meeting a target mean-squared error.

## 3. SENSING MODEL AND AREA COVERAGE

Since the detection of a target is inherently stochastic due to the noise in sensor measurements. The detection performance is usually characterized by two metrics, namely, the false alarm probability,  $P_f$ , and probability of detection,  $P_d$ .  $P_f$  is the probability of making a positive decision when no target is present, and  $P_d$  is the probability that a present target is correctly detected. In stochastic detection, although the detection probability can be improved by setting lower detection thresholds, the fidelity of detection results may be unacceptable because of high false alarm rates.

Therefore,  $P_d$  together with  $P_f$  characterize the sensing quality provided by the network. We introduce the concept called  $(\alpha, \beta)$ -coverage that quantifies the fraction of the area wherein  $P_f$  and  $P_d$  are bounded by  $\alpha$  and  $\beta$ , respectively. That is  $P_f \leq \alpha$ ,  $P_d \geq \beta$ .

### 3.1 Coverage Under Probabilistic Model

Consider an isotropic signal source model with path loss factor  $a$ . This model is general and captures cases such as a moving armored vehicle in a battlefield, or a source of a radioactive material [20]–[23]. The path loss factor,  $a$ , will depend on the type of signal considered. Thus, the received signal strength at a distance,  $d$ , away from the target is given by,

$$P(d) = \frac{P_o}{d^a} \tag{1}$$

where  $P_o$  is the signal strength measured at 1 meter from the location of the source. Now, consider a sensor with observations that are independent, after appropriate sampling and processing is given by,

$$y_i = \begin{cases} w_i & \text{when target is absent, } H_0 \\ \sqrt{P(d)} + w_i & \text{when target is present, } H_1 \end{cases} \tag{2}$$

where  $i = 1, \dots, N$ , and  $w_i$  is independent observation noise, is  $N(0, \sigma_w^2)$ . Thus, the target detection problem is formulated as a binary hypothesis testing problem with the following hypotheses:

$$H_0 : \text{with pdf, } p(y/H_0),$$

$$H_1 : \text{with pdf, } p(y/H_1),$$

Where,

$$p(y/H_0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} \tag{3}$$

and

$$p(y/H_1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\sqrt{P(d)})^2}{2\sigma^2}} \tag{4}$$

The likelihood ratio is, therefore defined as,

$$\Lambda(y) = \frac{p(y/H_1)}{p(y/H_0)} \tag{5}$$

Now, in order for  $P_f = \alpha$  and  $P_d = \beta$  to hold true, the threshold,  $\eta$ , should satisfy [24],

$$\eta = \frac{1-\beta}{\alpha}$$

With  $\varepsilon(\alpha, \beta) = P(\Lambda(y) \geq \eta/H_1)$

$$= Q\left(\left|\frac{\log(\eta)\sigma}{\sqrt{P(d)}} - \frac{\sqrt{P(d)}}{2\sigma}\right.\right) \quad (6)$$

where  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$

Solving equation (6) for the sensing range  $d$ , we have,

$$d = \frac{\sqrt{P(d)}}{2\mu\sigma} \left( \sqrt{v^2 + 2\mu} - v \right) \quad (7)$$

where,  $v = Q(\alpha, \beta)^{-1}$  is the inverse function of  $Q(\cdot)$ , and  
 $\mu = \sigma \log(\eta)$

If the target is more than  $d$  meters from the sensor, the detection performance requirements, i.e.,  $\alpha$  and  $\beta$ , cannot be satisfied by setting any detection threshold. From (7), the sensing range of a sensor varies with the user requirements (i.e.,  $\alpha$  and  $\beta$ ) and

the signal to noise ratio,  $SNR = \frac{P(d)}{\sigma^2}$ . This confirms the intuition that a sensor can detect a farther target if the noise level is relatively lower (i.e., a greater SNR). Moreover, assume that  $d$  is the sensing/coverage radius given by Equ. (7). Any event in that area will be detected by any arbitrary sensor if it is within  $d$  distance from the target. Thus, for an area with  $N$  sensors randomly deployed, the probability that the event will be detected by at least one of the  $N$  nodes is equal to the coverage probability  $P_n$ , [25]

$$P_n = 1 - (1 - P_n)^N \quad (8)$$

Equation (8) above, allows us to extend the coverage of random networks derived under classical disc model  $(\alpha, \beta)$ -coverage. A location is regarded as being covered if it is within at least one sensor's sensing range. Accordingly, the area of the union of all sensors' sensing ranges is regarded as being covered by the network. Specifically, the coverage of a network deployed according to a Poisson point process of density  $\rho$  is given by [25],

$$C = 1 - e^{-\rho \pi d^2} \quad (9)$$

Therefore, if the sensing range,  $d$ , is chosen by (7). Equ. (9) computes the coverage of a random network under the probabilistic model.

### 3.2 Detection in Presence of Channel Fading

In WSNs, sensors collect observations generated under specific hypothesis. After receiving its observations, each sensor makes a hard (binary) decision, send it to the fusion center. Decisions at local sensors are transmitted over wireless channels that are assumed to undergo independent fading. In this respect, flat Rayleigh fading channels between local sensors and the fusion center is considered. We assume that the effect of fading channel is simplified as a real scalar multiplication given that the transmitted signal is binary. In the development of fusion rules, the amplitude of the fading channel is considered as a (possibly unknown) constant during the transmission of a single local decision with additive white Gaussian noise.

In this section, we present a power control strategy which can guarantee, theoretically, error-free communications. In traditional transmission scenarios, this means that the system operational point lies in the feasible signal-to-noise ratio (SNR) region. We propose a power control strategy, which tends to balance (i.e., equalize) the link SNRs and is optimal as the sources are independent. The algorithm we propose in this section improves the global probability of bit-error by compensating the effects of fading in the communication channel through updating the effective sensor SNR.

### 3.3 Optimal Power Assignment Algorithm

Assume that the received signal at the fusion center is given by,

$$\mathbf{r} = |\mathcal{H}|\sqrt{P(d)} + \mathbf{w} \tag{10}$$

where,  $\sqrt{P(d)}$  denotes the transmitted signal,  $\mathcal{H}$  is the path gain (with the fading amplitude between the sensor and the fusion center) and  $\mathbf{w}$  is additive white Gaussian noise with standard deviation  $\sigma$ . The SNR, at the fusion center is therefore,

$$\gamma = \frac{|\mathcal{H}|^2 P(d)}{\sigma^2} \tag{11}$$

In the following, we derive an optimal power control strategy scheme that minimizes the power spent by sensor subjected to the thresholds  $P_f \leq \alpha, P_d \geq \beta$  (i.e.,  $\varepsilon(\alpha, \beta)$  in Equ. (6)). Here  $\varepsilon(\alpha, \beta)$  is the required bit error at the fusion center. This is, in effect, a constrained optimization problem, can be formulated as follows,

$$\begin{aligned} \text{Min. } & \sqrt{P(d)} \text{ such that} \\ & \varepsilon(\alpha, \beta) \leq Q \left( \left| \frac{\log(\eta)\sigma}{\mathcal{H}\sqrt{P(d)}} - \frac{\mathcal{H}\sqrt{P(d)}}{2\sigma} \right| \right), \end{aligned} \tag{12}$$

$$\sqrt{P(d)} \geq 0,$$

the inequality in problem (12) above can be rewritten as follows,

$$\varphi \leq \left| \frac{\log(\eta)\sigma}{\mathcal{H}\sqrt{P(d)}} - \frac{\mathcal{H}\sqrt{P(d)}}{2\sigma} \right|,$$

where we defined  $\varphi = Q^{-1}(\varepsilon(\alpha, \beta))$ , the optimization problem (12) can thus be rewritten as follows,

$$\text{Min. } \sqrt{P(d)} \quad \text{such that} \quad (13)$$

$$\varphi \leq \left| \frac{\log(\eta)\sigma}{\mathcal{H}\sqrt{P(d)}} - \frac{\mathcal{H}\sqrt{P(d)}}{2\sigma} \right|,$$

$$\sqrt{P(d)} \geq 0$$

Now, the optimization problem (13) can be formulated using the Lagrange constrained optimization approach as follows. Assume the following objective function, F,

$$F = \sqrt{P(d)} + \Gamma \left( \varphi \leq \left| \frac{\log(\eta)\sigma}{\mathcal{H}\sqrt{P(d)}} - \frac{\mathcal{H}\sqrt{P(d)}}{2\sigma} \right| \right) \quad (14)$$

where  $\Gamma$  is the Lagrange multiplier. Thus, the unconstrained optimization problem (14) can be solved. It can be shown that the optimal solution for (14) is,

$$\sqrt{P(d)}_{\text{opt}} = \frac{d\sqrt{(\sigma\varphi)^2 + 2\sigma\eta} - d\sigma\varphi}{\mathcal{H}} \quad (15)$$

where  $\eta = \log(\eta)\sigma$ . Equation (15) gives the minimum (sensor) transmission power necessary to balance the effects of channel fading amplitudes and noise. Substituting (15) into (11) gives the target SNR,

$$\sqrt{\text{SNR}_{\text{trg}}} = d\sqrt{\varphi^2 + 2\eta/\sigma} - d\varphi \quad (16)$$

Our optimal power adaptation strategy consists of two steps. First, a target SNR of the link between the sensor and the fusion center is computed (Eq. (16)) based on the sensor location, d. In the second step, optimal transmission power (Eq. (15)) is updated to the sensor node.

#### 4. VALIDATION RESULTS

In this section, the performance of the proposed coverage and optimal power assignment models are validated through numerical examples. As shown in Figure (1), the sensor's coverage d increases with the SNR according to Eqs. (11, 16). This conforms to the intuition that a sensor can detect a farther target if the noise level is lower. For instance, the sensing range, d, is about 5m if  $\alpha = 2\%$ ,  $\beta = 93\%$ , SNR = 7.5 dB. However, at SNR = 9dB, the sensing range covers up to 10m. Figure (1) depicts the sensing ranges for different detection metrics  $(\alpha, \beta)$ .

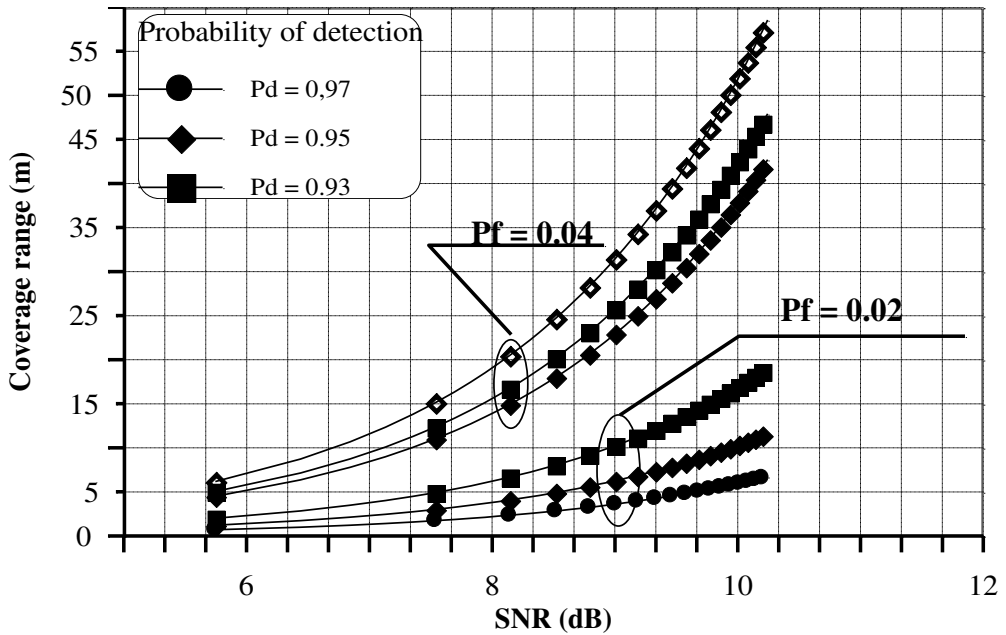


Fig.(1), Coverage range estimation

Next, in Figure (2), we show an illustrative example on the optimum transmission power for the sensor-to-fusion communication links. In this illustrative example, the fading coefficient  $\mathcal{H}$ , is set to unity.

This is done in order to highlight the effects of different communication metric values  $(\alpha, \beta)$  on the power assignment process. As expected, the higher is  $(\alpha, \beta)$ , the higher is the transmission power necessary to compensate for effects of the path losses over the communication range (i.e., twice the sensing range) as given by Equ. (15).

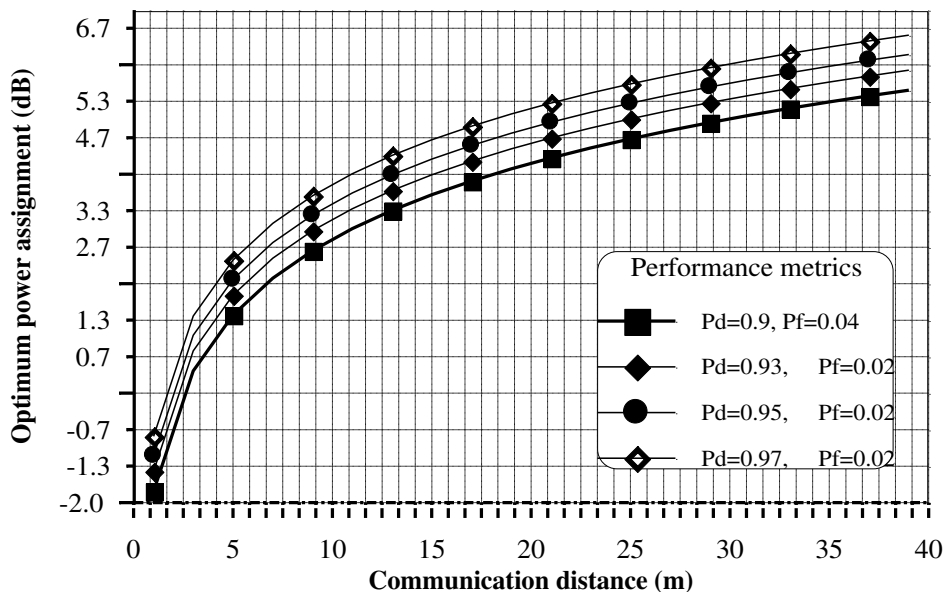


Fig.(2), Optimum power assignment

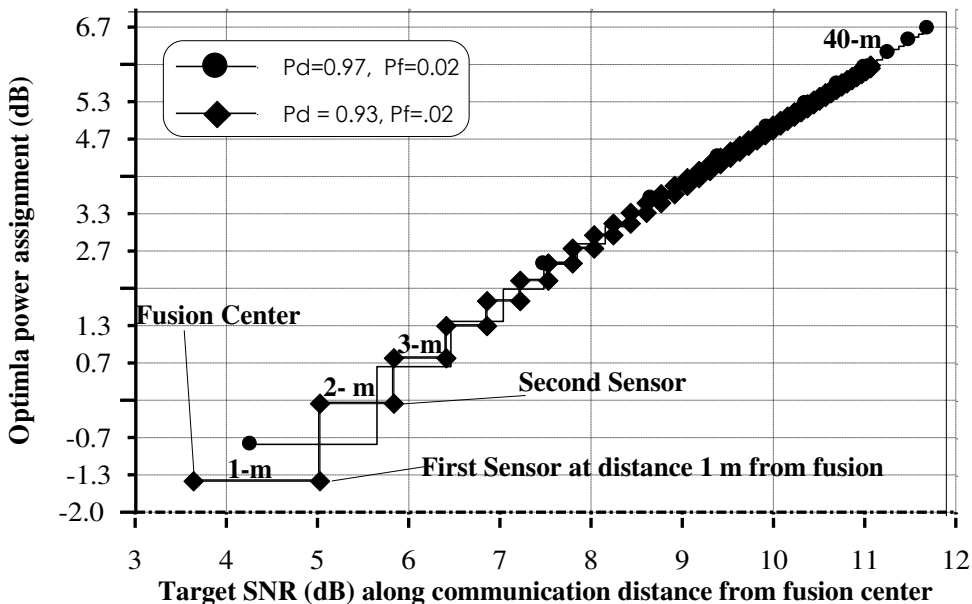


Fig.(3), Optimum power adaptation

Figure (3), presents a practical implementation for the proposed optimal power assignment strategy. Assume that the sensor knows its (discrete) location. Assume further that each sensor sends a pilot signal to the fusion center at the target SNR as shown in Figure (3). Upon receiving the pilot signal from the sensor and based on the



actual channel characteristics, the fusion center performs an estimation of the optimum transmission power, (Equ. (15)), necessary to achieve the target  $(\alpha, \beta)$  metrics and sends it as an update to the sensor. This way, our power assignment strategy would guarantee that the network operational point lies in the optimal SNR region.

## 5. SUMMARY AND CONCLUSIONS

We have proposed a probabilistic sensing coverage estimation scheme to evaluate area coverage in a randomly deployed wireless sensor network. The proposed algorithm takes into account the effect of path loss variations in sensing behavior.

Next, in the presence of fading, an optimal transmission power necessary to compensate for path loss variations is obtained by constraining the error probabilities over the communication channel. An optimal transmission power adaptation strategy is, then, presented. Analytic results reflect the effectiveness of the proposed algorithms in predicting the optimal transmission power as well as the sensing range over which a predefined detection and false alarm probabilities are satisfied.

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### تقدير مدى الحس والقدرة الأمثل للتراسل في شبكات الحساسات اللاسلكية

يقدم هذا البحث نموذج لتقدير مدى التغطية للحساسات في شبكات الحساسات اللاسلكية. يعتمد هذا النموذج على معاملات قنوات التراسل وكذلك مستوى الضوضاء الناتج بها. ثم يقدم البحث نموذج لتقدير المستوى الأمثل لقدرة التراسل بناء على التغيرات التي تحدث في معاملات قنوات التراسل مثل الخفوت ومستويات التداخل وكذلك بناء على بعد الحساسات عن مركز التراسل. أظهرت النتائج التحليلية قدرة النماذج المقترحة على تقدير مدى التغطية وكذلك المستوى الأمثل لقدرة الإرسال مما ينبئ بالقدرة على توفير شبكة حساسات تعمل بأقل نسبة أخطاء عند أقل مستوى قدرة