EQUIVALENT STRUT WIDTH FOR MODELING R.C. INFILLED FRAMES

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ABSTRACT

The macro-models method is one of the main categories for modeling infills based on the equivalent strut method. The basic parameter of these struts is their equivalent width, which affects the stiffness and strength. This paper presents a general review of several expressions proposed by researchers to calculate this equivalent width. The comparative study of different expressions shows that the Paulay and Priestley equation is the most suitable choice for calculating the diagonal equivalent strut width, due to its simplicity and because it gives an approximate average value among those studied in this paper. Consequently, the model will be used in our further study for analysis of RC infilled frames.

1. Introduction

Infilled RC frames have been used in many parts of the world over a long time. It is a structural composite system which consists of a reinforced concrete frame with masonry or concrete panels filling the planar rectangular voids between lower and upper beams and side columns. In these structures, the infill walls are typically considered as nonstructural elements and are often overlooked in the structural analysis and design[1]. However, they can interact with the bounding frames under seismic loads and alter the load resisting mechanism and failure pattern of the RC frame.

For modelling infills, several methods have been developed. They are grouped in two main categories: macro-models, and micro-models. The first one is based on the equivalent strut method (Figure 1) and the second is based on the finite element method. The main advantages of macro-modelling are computational simplicity and the use of structural mechanical properties obtained from masonry tests, since the masonry is a very heterogeneous material and the distribution of material properties of its constituent elements is difficult to predict [2].

The single strut model is most widely used as it is simple and evidently most suitable for large structures [3]. Thus, R.C. frames with masonry infilled walls can be modeled as equivalent braced frames with infill walls replaced by equivalent diagonal strut which can be used in rigorous nonlinear pushover analysis. The basic parameter of these struts is their equivalent width, which affects their stiffness and strength.

Polyakov [4], [5] conducted one of the first analytical studies based on elastic theory. From his study, complemented with tests on masonry walls diagonally loaded in compression, he suggested that the effect of the masonry panels in infilled frames subjected to lateral loads could be equivalent to a diagonal strut (see Figure 1). This suggestion was subsequently adopted by Holmes [6] who replaced the infill with an equivalent pin-jointed diagonal strut made of the same material and having the same thickness as the infill panel and he arbitrarily assumed that its width was the one-third part

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of the diagonal between the two compressed corners. Since then, many studies have been performed in order to give a proper determination of the equivalent strut's width [6–18].
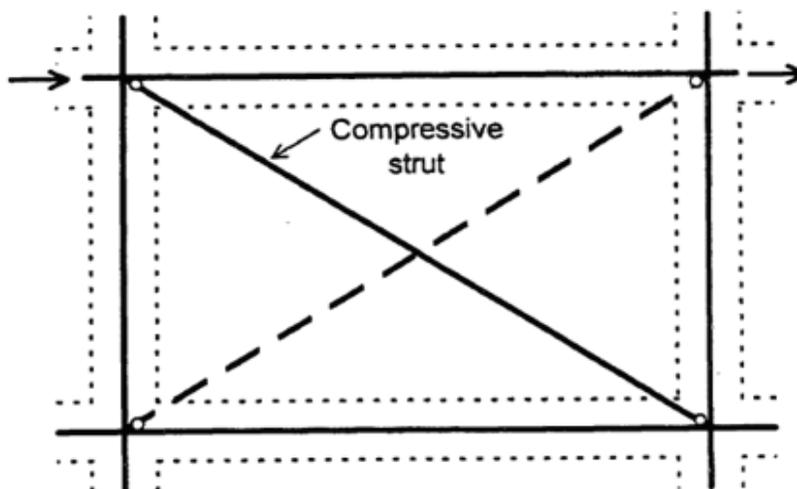

Fig. 1. Diagonal strut model for infilled frames

This paper is a preliminary study for a study being prepared by the authors on some parameters affecting the deformations of reinforced concrete multistory frame buildings subjected to Earthquakes. And the presence of infills in the frame is one of these parameters; one of the problems they faced was how to model the infills using the equivalent strut method and how to choose the appropriate expression to calculate the equivalent strut width. Therefore, this study will focus on giving a general review of several expressions proposed by researchers to calculate this equivalent width and applying the different expressions to one-bay one-story frame by using ABAQUS program for analysis and the results compared so as to arrive at a rational modelling scheme for masonry infilled

2. General description Of ABAQUS software

ABAQUS, which was used as the basic program for this study, is a powerful engineering simulation program, based on the finite element method, and can solve problems ranging from relatively simple linear analyses to the more complex nonlinear simulations [19]. ABAQUS is used throughout the world for stress, heat transfer, and other types of analysis in mechanical, structural, civil, biomedical, and related engineering applications.

2.1. Software validation

To validate the software program, an experimental test by Mehrabi[20] has been modeling; a selective sample is chosen (test No. 1) from Mehrabi collection test[20]. Material properties and geometric specifications and its designed details of reinforced concrete frames are same as laboratory testing according to Figure 2 for numerical modeling. Analysis results are plotted together with the test data as in Figure 3. The graphs indicate that the models predict the behavior with acceptable accuracy.
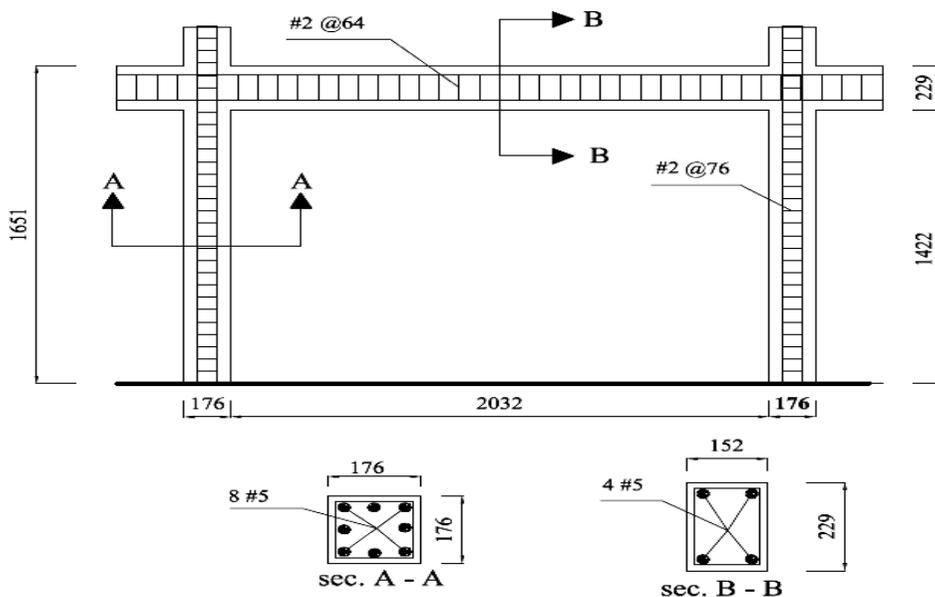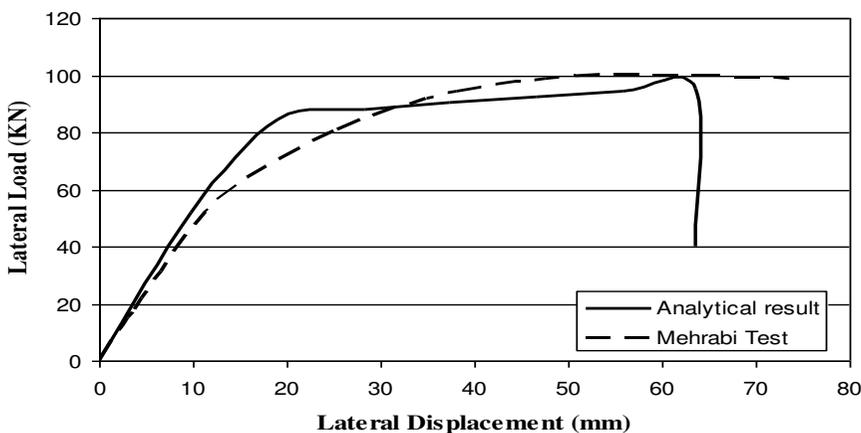

Fig. 2. Frame number 1 from Mehrabi collection tests[20]

Fig. 3. Comparison between analytical results and experimental results obtained by Mehrabi [20].

2.2. Determination of the equivalent strut width

The width of the equivalent diagonal strut (w) can be found out by using a number of expressions given by different researchers. Holmes (1961) [6] states that the width of equivalent strut to be one third of the diagonal length of infill, which resulted in the infill strength being independent of frame stiffness

$$w = \frac{1}{3} d_{\text{inf}} \tag{1}$$

Where d_{inf} is the diagonal length of infill

Later Stafford Smith and Carter (1969) [7] proposed a theoretical relation for the width of the diagonal strut based on the relative stiffness of infill and frame.

$$w = 0.58 \left(\frac{1}{H} \right)^{-0.445} \cdot \left(\lambda_h \cdot H_{\text{inf}} \right)^{0.335 d_{\text{inf}} \left(\frac{1}{H} \right)^{0.064}} \tag{2}$$

$$\lambda_h = \sqrt[4]{\frac{E_{\text{inf}} t \sin 2\theta}{4 E_c I_c H_{\text{inf}}}}$$

Where: t, H_{inf}, and E_{inf} are the thickness, the height and the modulus of the infill respectively, θ is the angle between diagonal of the infill and the horizontal, E_c is the modulus of elasticity of the column, I_c is the moment of inertia of the columns, H is the total frame height, and λ_h is a dimensionless parameter (which takes into account the effect of relative stiffness of the masonry panel to the frame).

Mainstone (1971) [8] gave equivalent diagonal strut concept by performing tests on model frames with brick infills. His approach estimates the infill contribution both to the stiffness of the frame and to its ultimate strength.

$$w = 0.16 d_{\text{inf}} \left(\lambda_h H_{\text{inf}} \right)^{-0.3} \tag{3}$$

Mainstone and Weeks [10] and Mainstone [9] (1974), also based on experimental and analytical data, proposed an empirical equation for the calculation of the equivalent strut width:

$$w = 0.175 d_{\text{inf}} \left(\lambda_h H_{\text{inf}} \right)^{-0.4} \tag{4}$$

Bazan and Meli (1980) [11], on the basis of parametric finite-element studies for one-bay, one-story, infilled frames, produced an empirical expression to calculate the equivalent width w for infilled frame:

$$w = (0.35 + 0.22\beta)h$$

$$\beta = \frac{E_c A_c}{G_{\text{inf}} A_{\text{inf}}} \tag{5}$$

Where: β is a dimensionless parameter, A_c is the gross area of the column, $A_{inf} = (L_{inf}\, t)$ is the area of the infill panel in the horizontal plane and G_{inf} is the shear modulus of the infill. Figure 4 illustrates the ratio $w/d_{inf.}$, according to Eq. 5. It is also important to note that it is difficult to compare these results with previous expressions because they are related to two different parameters ($\theta = 50^o\, and \quad \theta = 25^o$).

Liauw and Kwan (1984) [12] proposed the following equations based on experimental and analytical data:

$$w = \frac{0.95 H_{\text{inf}} \cos\theta}{\sqrt{\lambda_h H_{\text{inf}}}} \tag{6}$$
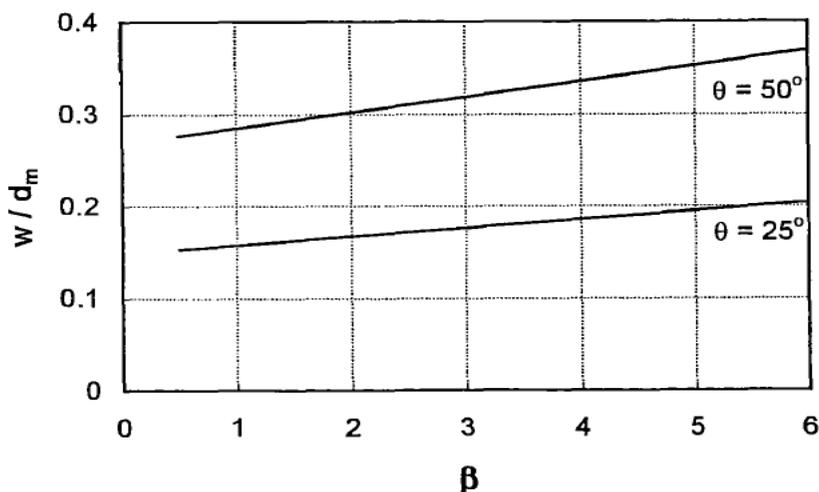

Fig. 4. Ratio w/d; for framed masonry structures according to Bazan and Meli [11]

Paulay and Preistley (1992) [13] pointed out that a high value of *w* will result in a stiffer structure, and therefore potentially higher seismic response. They suggested a conservative value useful for design proposal, given by:

$$w = 0.25 d_{\text{inf}} \tag{7}$$

Durrani and Luo (1994) [14] analyzed the lateral load response of reinforced concrete infilled frames based on Mainstone's equations. They proposed an equation for effective width of the diagonal strut, w, as

$$w = \gamma \sqrt{L^2 + H^2} \sin 2\theta \tag{8}$$

where :

$$\gamma = 0.32\sqrt{\sin 2\theta}\left[\frac{H^4 E_{inf} t}{m E_c I_c H_{inf}}\right]^{-0.1} ; \; m = 6\left[1 + \frac{6 E_c I_b H}{\pi E_c I_c L}\right]$$

L is the Length of frame c/c

FEMA (1998) [15] proposed that the equivalent strut is represented by the actual infill thickness that is in contact with the frame (t_{inf}) and the diagonal length (d_{inf}) and an equivalent width, W, is given by:

$$w = 0.175 d_{inf}\left(\lambda_h H_{inf}\right)^{-0.4} \tag{9}$$

Hendry (1998) [16] has also presented equivalent strut width that would represent the masonry that actually contributes in resisting the lateral force in the composite structure:

$$w = 0.5\sqrt{\alpha_h^2 + \alpha_L^2}$$

$$\alpha_h = \frac{\pi}{2}\left[\frac{4 E_c I_c H_{inf}}{E_{inf} t \sin 2\theta}\right]^{1/4} \; and \; \alpha_L = \pi\left[\frac{4 E_c I_b L_{inf}}{E_{inf} t \sin 2\theta}\right]^{1/4} \tag{10}$$

Where α_h , α_L are contact length between wall and column and beam respectively at the time of initial failure of wall, I_b is the moment of inertia of the beam, and L_{inf} is the length of the infill (clear distance between columns).

Al-Chaar 2002 [17] proposed that the equivalent masonry strut is to be connected to the frame members as depicted in Figure 5. The infill forces are assumed to be mainly resisted by the columns, and the struts are placed accordingly. The strut should be pin-connected to the column at a distance l_{column} from the face of the beam. This distance is defined by the following equations

$$l_{column} = \frac{w}{\cos \theta_{column}} \tag{11}$$

$$\tan \theta_{column} = \frac{H_{inf} - \dfrac{w}{cas \theta_{column}}}{L_{inf}}$$

Where the strut width (*w*) is calculated by using Mainstone and Weeks Equation without any reduction factors:

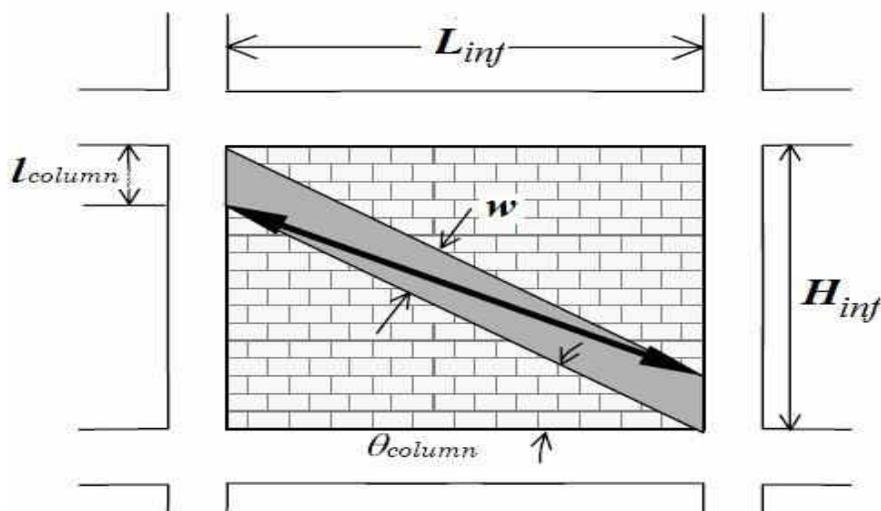$$w = 0.175 d_{inf}\left(\lambda_h H_{inf}\right)^{-0.4} \tag{12}$$

Fig. 5. Placement of strut [17]

Papia *et al.* 2008 [18] developed an empirical equation for the effective width of the diagonal strut as

$$w = \frac{c}{z}\frac{1}{\lambda^*}d_{\inf} \tag{13}$$

Where:

$$c = 0.249 - 0.0116v_{\inf} + 0.567v_{\inf}^2$$

$$\beta = 0.146 + 0.0073v_{\inf} + 0.126v_{\inf}^2$$

$$\lambda^* = \frac{E_{\inf}t.H_{\inf}}{E_c A_c}\left(\frac{H_{\inf}^2}{L_{\inf}^2} + \frac{A_c L_{\inf}}{4A_b H_{\inf}}\right)$$

$$z = 1 \quad if \quad \frac{L_{\inf}}{H_{\inf}} = 1 \quad \& \quad z = 1.125 \quad if \quad \frac{L_{\inf}}{H_{\inf}} \geq 1.5$$

Where z is an empirical constant, λ^* is the stiffness parameter, $v_{\inf}$ is the poison ratio for the infill, E_c was the Young's modulus of the frame, A_c was the cross sectional area of the column and A_b was the cross sectional area of the beam.

Applying these expressions to a one-bay one-story frame example with geometric specifications and its designed details of reinforced concrete frames are same as laboratory

testing according to Figure 2, the study proposes a comparison of the results and indicates the most suitable relation to be used in practical design. The geometrical parameters of the frame members and properties of the materials are indicated in Table 1.

Table 1.
Geometrical parameter and material properties of frame members

Parameter	Data	Units
Beam width	152	mm
Beam depth	229	mm
Moment of inertia of beam I_b	$1.52*10^8$	mm^4
Column width	176	mm
Column depth	176	mm
Moment of inertia of column I_c	$1.42*10^8$	mm^4
Height of frame c/c H	1536	mm
Length of frame c/c L	2338	mm
Concrete strength f'_c	20.7	MPa
Young's modulus of concrete E_c	21.52	GPa
Poisson ratio of concrete v_c	0.2	-
Infill thickness t	100	mm
Height of infill H_{inf}	1422	mm
Length of infill L_{inf}	2032	mm
Young's modulus of infill E_{inf}	12	GPa
Poisson ratio of infill v_m	0.15	-
Diagonal length of infill d_{inf}	2795	mm
Angle made by strut with horz. θ	33.32	degrees

3. Analytical modeling

The frame was assumed to be fixed at the bottom. The columns and beams of the frame are modeled in using C3D8 element. The reinforcement is modelled as rods embedded in the concrete surfaces. This means that the end nodes of the steel rods are considered to be slave nodes to the concrete master nodes, and thus, that the steel nodes follow the deformations of the concrete nodes[19]. Masonry infill walls were modelled as one equivalent diagonal struts using two nodded beam elements and finite elements using shell elements. The transfer of bending moments from frame to masonry wall was prevented by specifying the moment releases at both ends of the struts. The Geometrical parameter are those presented in Table 1.

Three different modelling possibilities were considered as follows:

Model 1 - bare frame model, in which the strength and stiffness of masonry infills were not considered; **Model 2** – full infill frame, with masonry modelled using finite element method; **Model 3** – masonry modelled as a single strut with using the different widths of the strut calculated with different methods.
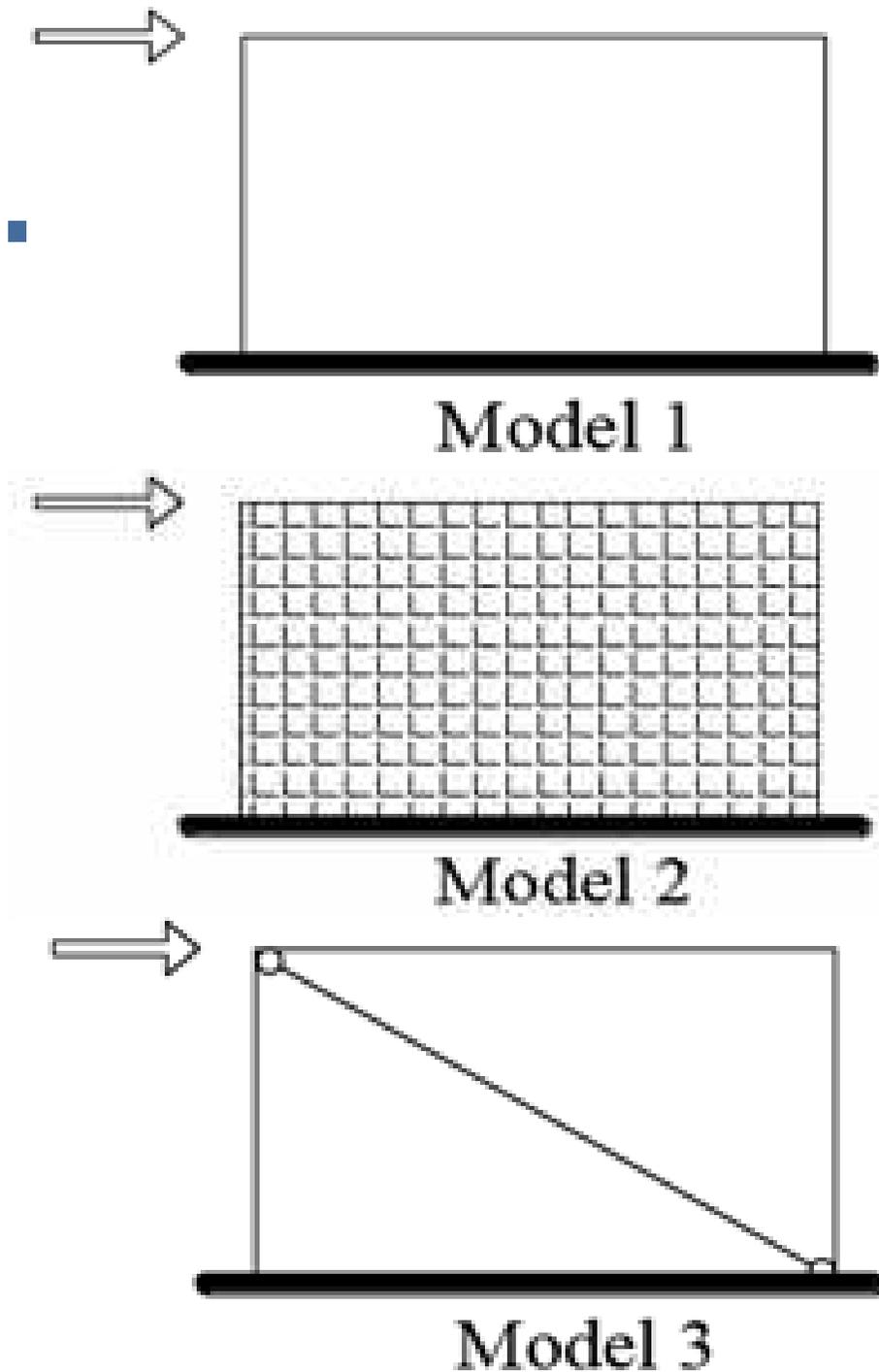
Fig. 6. Different analytical models

4. Material properties

The concrete material is modeled as Concrete Damaged Plasticity Model (CDP). This model takes into consideration the degradation of the elastic stiffness induced by plastic straining both in tension and compression. It also accounts for stiffness recovery effects under cyclic loading. The compressive and tension stress-strain relation can be seen in Figure 7. The compressive behavior is elastic until initial yield and then is characterized by stress hardening followed by strain softening after the ultimate point. After the onset of microcracking (failure stress) the response is softened, inducing strain localizations in the concrete structure. In tension behavior the stress-strain relation is assumed to be linear until the failure stress, which corresponds to the onset of macrocracking, is reached. This is most often followed by softening which induces strain localization. The parameters used of Concrete Damaged Plasticity Model are listed in Table 2.

The steel reinforcing bars were considered as elastic perfectly plastic materials in both tension and compression. The assumed uniaxial stress-strain curve of the steel bars is shown in Figure 8. The main parameters used of steel materials are listed in Table 3.

5. Results

The values of equivalent strut width defined by different methods are shown in Table 2 and Figure 6. It shows that the ratio of the estimated equivalent strut width to the diagonal length of infill (w/d_{inf}) are ranging between about 0.1 to 0.33 except the result calculated by using Stafford Smith and Carter method equation which generate large value for the equivalent strut width. It shows that Holmes's expression (*Eq.1*) gives the highest value ($w/d_{inf} = 0.33$) and Mainstone and Al-Chaar expressions (*Eq.4*) gives the lowest ($w/d_{inf} = 0.09$). Whereas Paulay and Priestley (*Eq.7*) gives an average value of the equivalent strut width.
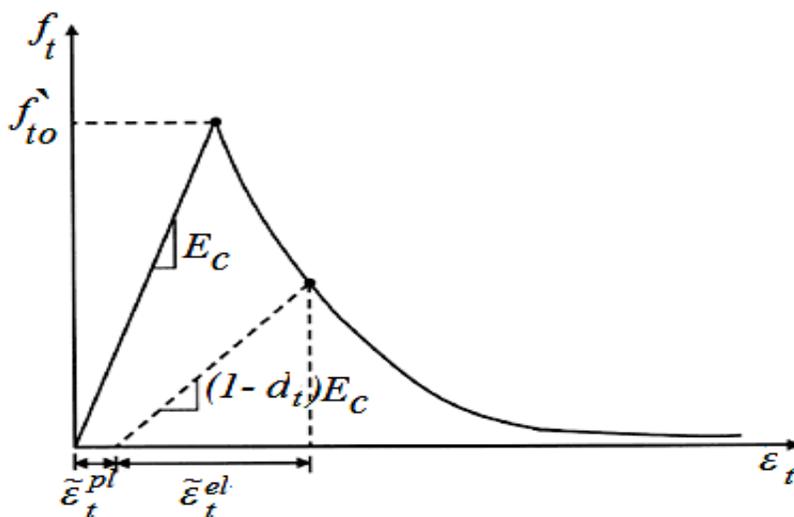
Table 2.

Parameters used for concrete damaged plasticity model

Young modulus E (GPa)	Poisson ratio υ	Density ρ (Kg/m3)	Delatation angle ψ	Eccen-tricity	f_{co} MMPa	f_{cu} MMPa	f_t MPa	f_{bo}/f_{co} *	Invariant stress ratio K_c
21.5	0.2	2400	33.32	0.1	115.4	220.7	11.85	11.16	0.667

$^{*}f_{bo}/f_{co}$ is the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress.
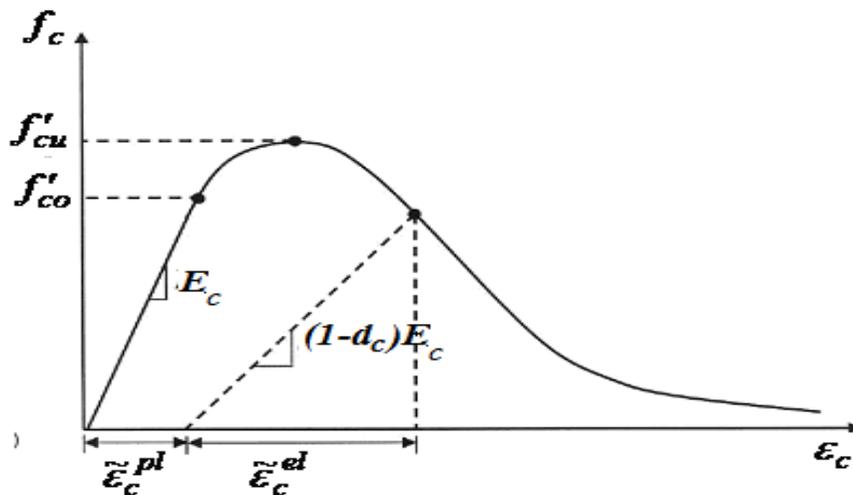
(a)

(b)

Fig. 7. Damage Plasticity: (a) uniaxial concrete compressive behavior, (b) tension response of concrete [19]
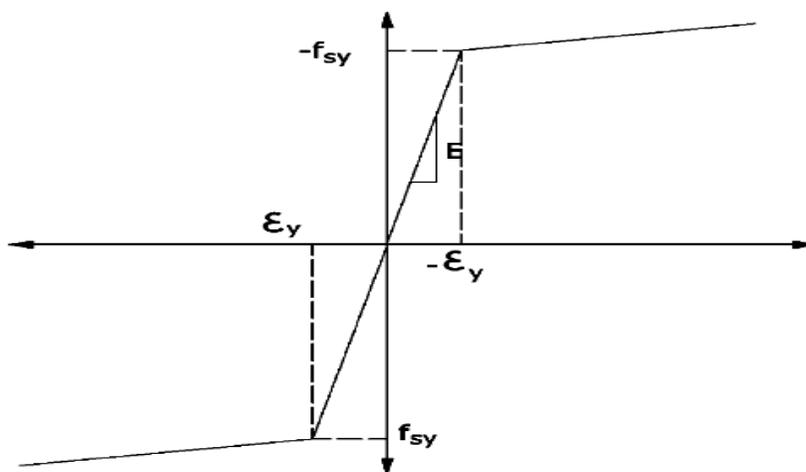
Fig. 8. Stress-strain curve for steel [19]

Table 3.
Parameters used for steel model

Young modulus E (GPa)	Poisson ratio υ	Density ρ Kg/m^3	Ff_y MMPa
210	0.3	7800	4420

Table 4.
Strut width and coefficient by various researchers

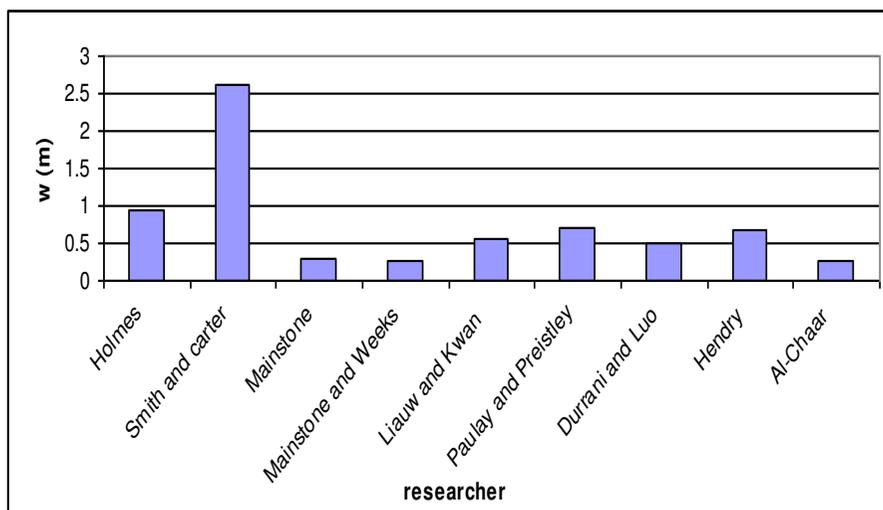| Researcher	*Eq. No.*	Strut width (m)	Coefficient (w / d_{inf})
Holmes [6]	1	0.93	0.333
Stafford Smith and Carter [7]	2	2.61	0.935
Mainstone[8]	3	0.29	0.103
Mainstone and Weeks [10]	4	0.27	0.097
Liauw and Kwan[12]	6	0.56	0.201
Paulay and Preistley [13]	7	0.7	0.250
Durrani and Luo [14]	8	0.49	0.176
Hendry [16]	10	0.68	0.244
Al-Chaar [17]	12	0.27	0.097
Papia *et al.*[18]	13	0.44	0.158

Fig. 9. Equivalent Strut Width

Figure 10 illustrates the variation of the ratio (w/dinf) as a function of (H/L) according to the previous expressions. Holmes's proposition (Eq.1) gives an upper-bound value for the strut width, and Mainstone's proposition (Eq.4) a lower-bound one. On the other hand, the constant value suggested by Paulay and Pristley [(Eq.7) gives a value that is more or less an average value of the two extremes.
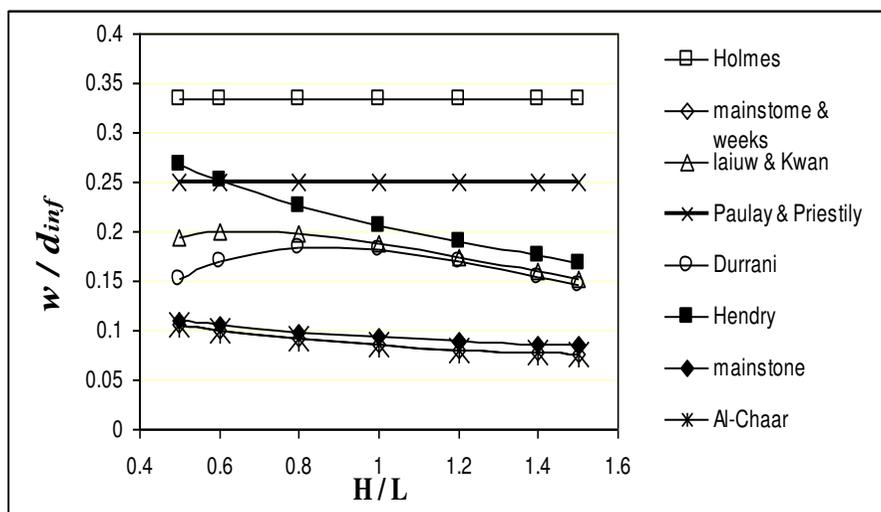

Fig. 10. Variation of the ratio *w/d$_{inf}$* for infilled frame as a function of (H/L)

Figure 11 illustrates the comparison of lateral force – lateral displacement relations according to the previous expressions. It was observed as shown in that all the different expressions methods used here to estimate equivalent strut width are relatively close. The Paulay and Priestley relation is recommended to be used because it gives an average value of the equivalent strut width and because of its simplicity.
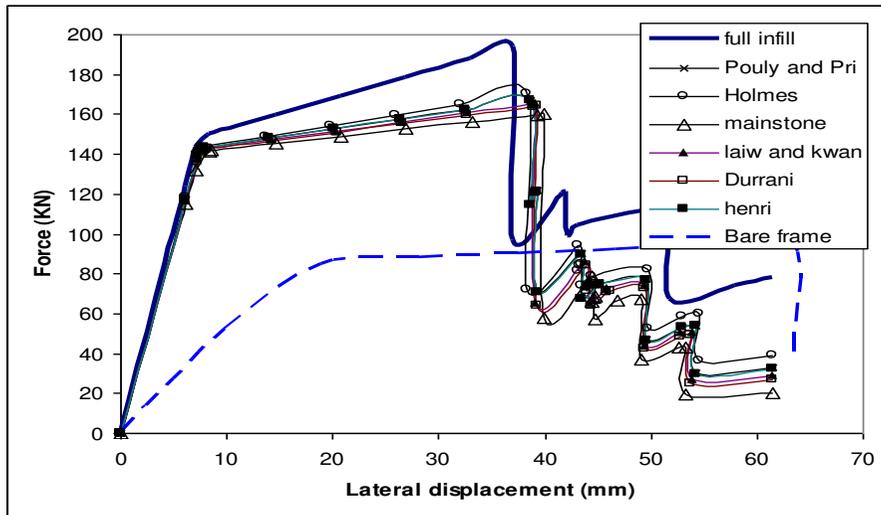

Fig. 11. Comparison Between Different Models

6. Conclusions

The comparative study of different expressions shows that the Paulay and Priestley equation is the most suitable choice for calculating the diagonal equivalent strut width, due to its simplicity and because it gives an approximate average value among those studied in this paper. Consequently, the model will be used in our further study for analysis RC infilled frames.

In the analysis involving analytical models for infilled frams in a single-storey, single-bay reinforced concrete frame, the single-strut model was found to be predicting the global behaviour of the system with reasonable accuracy.

In conclusion, the single-strut model is better to be used in analysis regarding the general behaviour of infilled frames, because it can be accepted as correct and due to its simplicity.

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العرض المكافئ للدعامة القطرية في نمذجة القواطيع المائلة للإطارات الخرسانية المسلحة

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ملخص:

تعد الهياكل الخرسانية المسلحة المملوءة بالقواطيع من أهم الانظمة الانشائية المستخدمة للمباني المتعددة الطوابق. ولدراسة وتحليل هذه الانظمة عادة ماتستخدم طريقة النموذج الكلي (Macro-models method) وفيها يتم تمثيل القواطيع بدعامة مكافئة قطرية (Equivalent Strut). ويعتبر عرض هذه الدعامات اهم عامل لتأثيره على المتانة والقوة. وهناك العديد من الدراسات التي قدمت مقترحات لحساب عرض الدعامات المكافئة تم استعراض عدد منها في هذا البحث وعمل مقارنة بينها للوصول الى المقترح المناسب، وتوصلت الدراسة الى ان المقترح المقدم من قبل بولي وبرستلي (Paulay and Priestley) هي أنسب المقترحات لسهولة تطبيقها ودقتها المقبولة. وسيتم استخدام هذا المقترح في بحث يقوم الباحثون على اعداده لدارسة وتحليل الهياكل الخرسانية المتعددة الطوابق.