

## THEORETICAL STUDY ON STATIC SHEAR BEHAVIOR OF HIGH STRENGTH CONCRETE BEAMS COMPARED WITH NORMAL STRENGTH CONCRETE BEAMS

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### ABSTRACT

The main aim of the present study is to present a comparison between the static shear behavior of high strength concrete beams and normal strength concrete beams. The concrete compressive strength of the beams ranged from 250 to 700 kg/cm<sup>2</sup>. Sixty reinforced concrete beams were analyzed under two point static loads. The variables were the compressive strength of concrete, shear span-to-depth ratio and shear reinforcement ratio (stirrup spacing S). The effect of each variable on the behavior of the beams is studied separately. The details of the beam specimens, material properties, and instrumentation are described in this paper. The results are presented and discussed and the influence of each design parameter is investigated. Analysis of the results is also compared with different existing approaches.

*Keywords:* Normal and High-strength concrete Beams, Shear reinforcement, Spacing of stirrup, Shear span to depth ratio, Cracking and Ultimate shear strength.

### 1. Introduction

In recent times, high strength concrete has been used widely in construction. For example Beams, Columns, Precast elements and Structures where durability is an important design parameter. This has resulted in the design of beams of smaller depths, which may undergo greater deflections. To give a simplified explanation, HSC is obtained by improving the compactness of the concrete mix, which increases the strength of both the paste and the interface between the paste and the coarse aggregate. However, an increase in the strength of the concrete produces an increase in its brittleness and smoother shear failure surfaces leading to some concerns about the application of high-strength concrete.

Since most of the current shear procedures are based on tests carried out on beams with a concrete compressive strength lower than 401.1 kg/cm<sup>2</sup> and one of the shear transfer mechanisms is shear-friction across the cracks, the failure shear strength needs to be re-evaluated. Moreover, shear failure in a beam without web reinforcement is sudden and brittle. Therefore, it is necessary to provide an amount of shear reinforcement, which must prevent sudden shear failure on the formation of first diagonal tension cracking and, in addition, must adequately control the diagonal tension cracks at service load levels.

Due to the higher tensile strength of high strength concrete, a higher cracking shear is expected and hence, would require a large amount of shear reinforcement. In some codes of practice, the shear strength of a reinforced concrete beam is taken as the sum of the

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shear force that is carried by the concrete ( $V_c$ ) and the web reinforcement ( $V_s$ ). The term ( $V_c$ ) in a diagonally cracked beam with web reinforcement represents the sum of three separate components. These components are:

- (a) Dowel action resistance of the longitudinal reinforcement,
- (b) Aggregate interlock resistance along the diagonal crack,
- (c) The shear resistance carried by the uncracked concrete compressive zone.

The term ( $V_s$ ) represents the vertical component of the shear force carried by the vertical (shear) reinforcement (strut mechanism).

## 2. Research Significance

This research was carried out for the following purposes:

- 1- To study the effect of compressive strength, a/d ratio, shear reinforcement and Ultimate shear strength of reinforced high strength concrete beams of rectangular cross section.
- 2- To compare the obtained analysis results and proposed equations results with the ACI Code.

## 3. Previous Works

Experimental tests were carried out to get a better understanding of the behavior of shear strength of high strength concrete beams and to estimate the amount of the shear reinforcement required to high strength concrete beams in order to prevent sudden failure of the beam and to improve its ductility. In ref. (1), the authors carried out a study to make a comparative analysis on shear behavior of high-strength concrete beams using various international design approaches like ACI [1], Canadian [2], AASHTO [3], European Code [4] and the method proposed by Zararis [5]. Reinforced concrete beams without web reinforcement were tested under three point loading. Based on the analysis of total of 122 similar beams, they observed that:

- 1- When shear span-to-effective depth ratio increases from 2 to 3, relative flexural strength decreases, however, this decrease is dependent upon the tensile steel ratio as the greater the steel ratio, the lower is the difference. On further increase of a/d ratio from 3 to 6, the relative flexural strength increases and a valley of diagonal shear failure was observed in the vicinity of shear span-to-depth ratio equal to 3.
- 2- For a constant value of a/d ratio, the relative flexural strength decreases and failure load increases with an increase in longitudinal reinforcement ratio.
- 3- Comparison of test results with various approaches reveals that the experimental shear strength is more in conformity with ACI 318-02 than other design approaches for beams having tensile steel ratio higher than 1 %. Current ACI 318 shear strength equation could be not conservative for lightly reinforced high-strength concrete beams having tensile steel ratio < 0.58%.
- 4- The current shear design approaches in various codes underestimate the shear carrying capacity of high-strength concrete beams up to shear span-to-depth ratio 2.5 and overestimate for slender beams having a/d ranging from 2.5 to 6.

5- Analysis of the research results revealed that shear strength and failure mode depends on shear span and longitudinal reinforcement ratio.

In ref. (2) An experimental investigation was carried out to study the shear behavior of (HSC) beams with constant width by varying shear span to depth ratio, the longitudinal reinforcement ratio and the minimum web reinforcement ratio. They compared the results with the different code equations and concluded that the longitudinal reinforcement ratio, strength of the concrete, shear span to depth ratio, value and depth of the beam are the most influencing parameters in the deformational and shear behavior of the HSC slender beams with web reinforcement. The result indicates that the reserve strength, increases with increase in the percentage of longitudinal reinforcement ratio. And as the longitudinal reinforcement ratio increases, the ultimate shear stress increases. He also found that as the shear span to depth ratio ( $a/d$ ) increases,

In ref. (3), the authors carried out a study on shear resistance of high strength concrete beams. The shear span-to-effective depth ratio is taken as main variable keeping all other parameters constant.

Most of the equations are under estimating the shear capacity at lower  $a/d$  ratios.

When the  $a/d$  ratio is less than (2.0), strut action prevails and the shear resistance is very high. For  $a/d$  ratios up to (2), the experimental values showed remarkable increase in shear strength compared to various design approaches.

In ref. (4) The authors carried out a study to predict the shear strength of high strength concrete beams (70 Mpa) with different shear span to depth ratios without web reinforcement. The reinforced concrete beams were tested under shear loading and were modeled in 'ANSYS', which is Finite Element Analysis software. The test results were compared with the 'ANSYS' model results

They indicate that the increase in  $a/d$  ratio has shown reduction in shear capacity of the beam. At lower  $a/d$  ratios the ultimate load was observed to be more than twice at diagonal cracking. The deflections increased with  $a/d$  ratio. The ANSYS model closely predicted the diagonal tension failure and shear compression failure of high strength concrete beams without shear reinforcement as observed in experiment.

## 4. Theoretical Program

### 4.1. Analyzed beams

We analyzed sixty reinforced concrete beams divided into five groups A, B, C, D and E using (ABAQUS) program using 3D model. All analyzed beams have over all depth 60 cm and 30 cm width. All beams were analyzed under two points static loading up to failure. Steel reinforcement of all beams have three bars 12 mm diameter as compression reinforcement, two bars 12 mm diameter and two bars 16 mm or four bars 16 mm as main reinforcement, and The stirrups were 8 mm diameter having a variable spacing. The concrete strength of analyzed beams 250, 400 and 700 kg/ $cm^2$  and  $a/d$  ratio 1, 2, 3, 4 and 6.

Details of the analyzed beams are given in Figs. (1. a), (1. b) and (1. c). The analysis program is given in table (1). This program consists of five groups A, B, C, D and E.

**Table 1.**  
analyzed specimens

| Secondary Group | Beam NO. | Length of beam | a (cm) | a/d ratio | (kg/) | S (cm) | Main steel ratio |
|-----------------|----------|----------------|--------|-----------|-------|--------|------------------|
| A 1             | A11      | 160            | 57.5   | 1         | 250   | 5      | 0.268%           |
|                 | A12      | 160            | 57.5   | 1         | 250   | 10     | 0.268%           |
|                 | A13      | 160            | 57.5   | 1         | 250   | 15     | 0.268%           |
|                 | A14      | 160            | 57.5   | 1         | 250   | 20     | 0.268%           |
| A 2             | A21      | 160            | 57.5   | 1         | 400   | 5      | 0.268%           |
|                 | A22      | 160            | 57.5   | 1         | 400   | 10     | 0.268%           |
|                 | A23      | 160            | 57.5   | 1         | 400   | 15     | 0.268%           |
|                 | A24      | 160            | 57.5   | 1         | 400   | 20     | 0.268%           |
| A 3             | A31      | 160            | 57.5   | 1         | 700   | 5      | 0.268%           |
|                 | A32      | 160            | 57.5   | 1         | 700   | 10     | 0.268%           |
|                 | A33      | 160            | 57.5   | 1         | 700   | 15     | 0.268%           |
|                 | A34      | 160            | 57.5   | 1         | 700   | 20     | 0.268%           |
| B 1             | B11      | 320            | 114    | 2         | 250   | 5      | 0.268%           |
|                 | B12      | 320            | 114    | 2         | 250   | 10     | 0.268%           |
|                 | B13      | 320            | 114    | 2         | 250   | 15     | 0.268%           |
|                 | B14      | 320            | 114    | 2         | 250   | 20     | 0.268%           |
| B 2             | B21      | 320            | 114    | 2         | 400   | 5      | 0.268%           |
|                 | B22      | 320            | 114    | 2         | 400   | 10     | 0.268%           |
|                 | B23      | 320            | 114    | 2         | 400   | 15     | 0.268%           |
|                 | B24      | 320            | 114    | 2         | 400   | 20     | 0.268%           |
| B 3             | B31      | 320            | 114    | 2         | 700   | 5      | 0.268%           |
|                 | B32      | 320            | 114    | 2         | 700   | 10     | 0.268%           |
|                 | B33      | 320            | 114    | 2         | 700   | 15     | 0.268%           |
|                 | B34      | 320            | 114    | 2         | 700   | 20     | 0.268%           |
| C 1             | C11      | 480            | 171    | 3         | 250   | 5      | 0.402%           |
|                 | C12      | 480            | 171    | 3         | 250   | 10     | 0.402%           |
|                 | C13      | 480            | 171    | 3         | 250   | 15     | 0.402%           |
|                 | C14      | 480            | 171    | 3         | 250   | 20     | 0.402%           |
| C 2             | C21      | 480            | 171    | 3         | 400   | 5      | 0.402%           |
|                 | C22      | 480            | 171    | 3         | 400   | 10     | 0.402%           |
|                 | C23      | 480            | 171    | 3         | 400   | 15     | 0.402%           |
|                 | C24      | 480            | 171    | 3         | 400   | 20     | 0.402%           |
| C 3             | C31      | 480            | 171    | 3         | 700   | 5      | 0.402%           |
|                 | C32      | 480            | 171    | 3         | 700   | 10     | 0.402%           |
|                 | C33      | 480            | 171    | 3         | 700   | 15     | 0.402%           |
|                 | C34      | 480            | 171    | 3         | 700   | 20     | 0.402%           |
| D 1             | D11      | 640            | 228    | 4         | 250   | 5      | 0.402%           |

| Secondary Group | Beam NO. | Length of beam | a (cm) | a/d ratio | (kg/) | S (cm) | Main steel ratio |
|-----------------|----------|----------------|--------|-----------|-------|--------|------------------|
|                 | D12      | 640            | 228    | 4         | 250   | 10     | 0.402%           |
|                 | D13      | 640            | 228    | 4         | 250   | 15     | 0.402%           |
|                 | D14      | 640            | 228    | 4         | 250   | 20     | 0.402%           |
| D 2             | D21      | 640            | 228    | 4         | 400   | 5      | 0.402%           |
|                 | D22      | 640            | 228    | 4         | 400   | 10     | 0.402%           |
|                 | D23      | 640            | 228    | 4         | 400   | 15     | 0.402%           |
|                 | D24      | 640            | 228    | 4         | 400   | 20     | 0.402%           |
| D 3             | D31      | 640            | 228    | 4         | 700   | 5      | 0.402%           |
|                 | D32      | 640            | 228    | 4         | 700   | 10     | 0.402%           |
|                 | D33      | 640            | 228    | 4         | 700   | 15     | 0.402%           |
|                 | D34      | 640            | 228    | 4         | 700   | 20     | 0.402%           |
| E 1             | E11      | 960            | 342    | 6         | 250   | 5      | 0.536%           |
|                 | E12      | 960            | 342    | 6         | 250   | 10     | 0.536%           |
|                 | E13      | 960            | 342    | 6         | 250   | 15     | 0.536%           |
|                 | E14      | 960            | 342    | 6         | 250   | 20     | 0.536%           |
| E 2             | E21      | 960            | 342    | 6         | 400   | 5      | 0.536%           |
|                 | E22      | 960            | 342    | 6         | 400   | 10     | 0.536%           |
|                 | E23      | 960            | 342    | 6         | 400   | 15     | 0.536%           |
|                 | E24      | 960            | 342    | 6         | 400   | 20     | 0.536%           |
| E 3             | E31      | 960            | 342    | 6         | 700   | 5      | 0.536%           |
|                 | E32      | 960            | 342    | 6         | 700   | 10     | 0.536%           |
|                 | E33      | 960            | 342    | 6         | 700   | 15     | 0.536%           |
|                 | E34      | 960            | 342    | 6         | 700   | 20     | 0.536%           |

Where  $f_c$  is the concrete compressive strength ( $kg/cm^2$ ), S is stirrup spacing (cm).

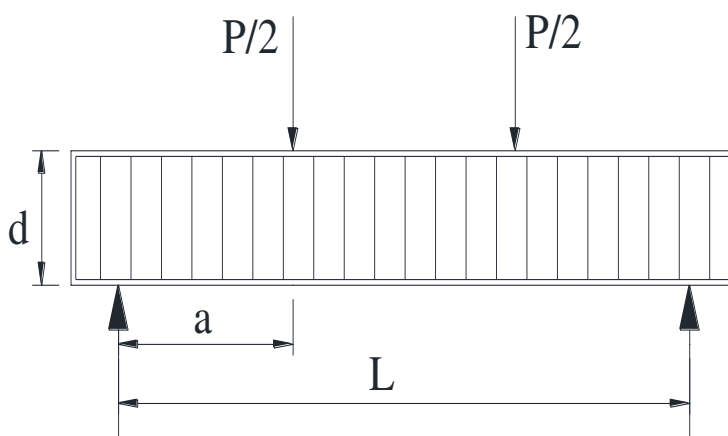
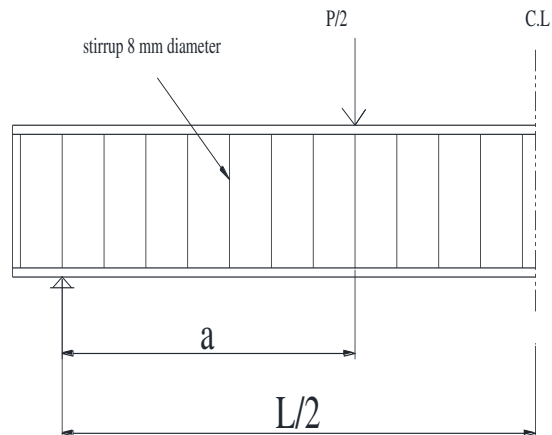
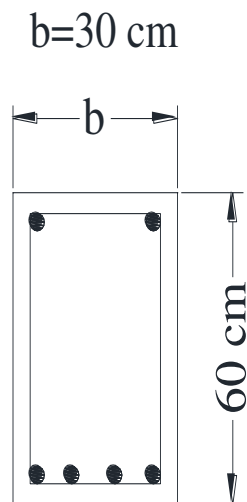


Fig. 1. a. details of beams having vertical stirrups



**Fig. 1. b.** details of beams



**Fig. 1. c.** cross section of analyzed specimens

## 5. Analysis Results and Discussions

### 5. 1. Load – deflection diagrams

At the position of mid-span, the deflection measured values have been plotted against the corresponding applied loads from starting of loading up to failure as shown in Figs (2) to (4).

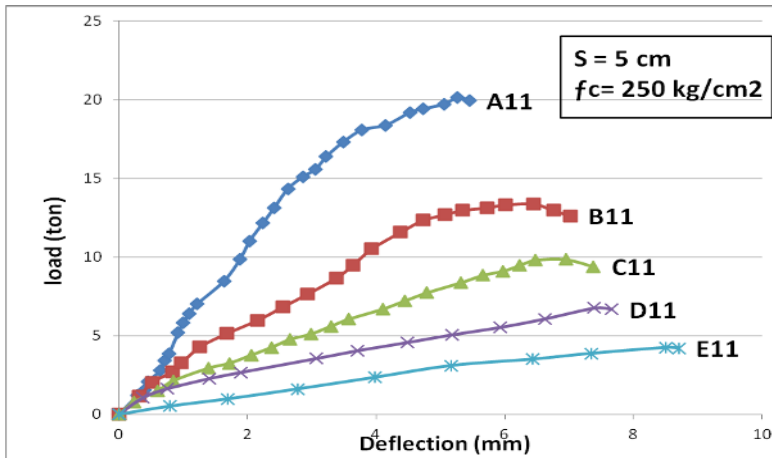


Fig. 2. Load-Deflection relationship for beams with different a/d ratios

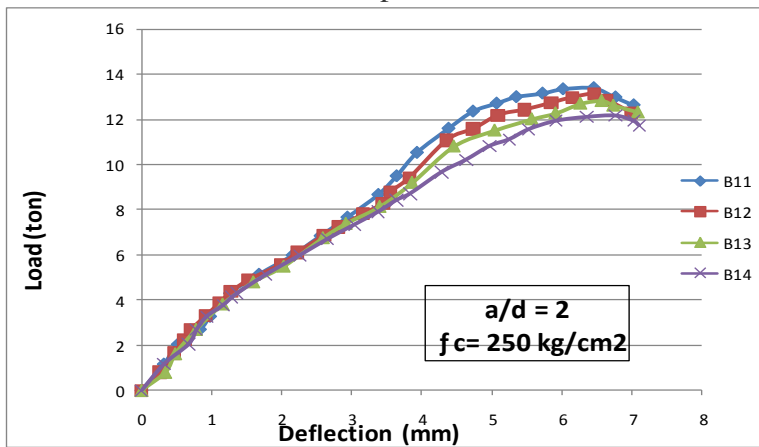


Fig. 3. Load-Deflection relationship for beams with different spacing of stirrups

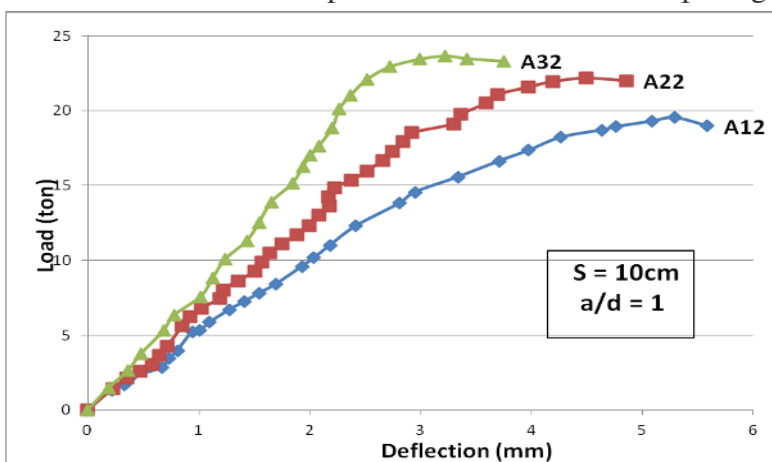


Fig. 4. Load-Deflection relationship for beams with different concrete strength

### 5. 2. Effect of compressive strength

The increasing of concrete compressive strength is led to the increase of the cracking load and ultimate load for concrete grades C250, C400 and C700. The concrete strength affects the deflection as it can be noticed that the measured deflection at a certain load decreases as the compressive strength increase.

### 5. 3. Effect of shear reinforcement

It is obvious that the cracking and ultimate load values decreases with the increase of spacing of stirrups. Increasing of the spacing of stirrups leads to a decrease of the maximum measured deflection for beams of either normal or high strength concrete. This means that as the stirrups increased in the beam, the ductility of that beam increases. Also, a decrease of the spacing of stirrups (increasing shear reinforcement) leads to a decrease of the maximum measured stirrup strain for the beams of grade C250, C400 and C700. As the number of stirrups in the beam increases, the shear load of one stirrup decreases. And this small shear load causes the stirrup not to be highly stressed.

## 6. Proposed Equations for Cracking and Ultimate Shear Strength

From results and the parametric analysis it appears that the cracking shear strength for R.C. beams depends mainly on the concrete strength ( $f_c$ ), the  $a/d$  ratio, and the amount of web reinforcement (spacing and  $A_s$ ). Based on best fit method and using a statistical program, the equation (1) is proposed to calculate the cracking shear strength for R.C. beams

$$v_{cr} = 0.559 \left( \sqrt{f_c} \times \frac{a}{d} \right) + 0.359 \left( \frac{\rho_w \cdot f_{yw}}{\sin \alpha} \right) - 1.58 \quad \text{kg/cm}^2 \quad (1)$$

Also from results and the parametric analysis it appears that the ultimate shear strength for R.C. beams depends mainly on the concrete strength ( $f_c$ ), the  $a/d$  ratio, and the amount of web reinforcement (spacing and  $A_s$ ). Based on best fit method and using a statistical program, the equation (2) is proposed to calculate the ultimate shear strength for R.C. beams.

$$v_u = 2.754 \left( \sqrt{f_c} \times \frac{d}{a} \right) + 0.205 \left( \frac{\rho_w \cdot f_{yw}}{\sin \alpha} \right) - 3.71 \quad \text{kg/cm}^2 \quad (2)$$

Where

$f_c$  the concrete compressive strength in  $\text{kg/cm}^2$ ,

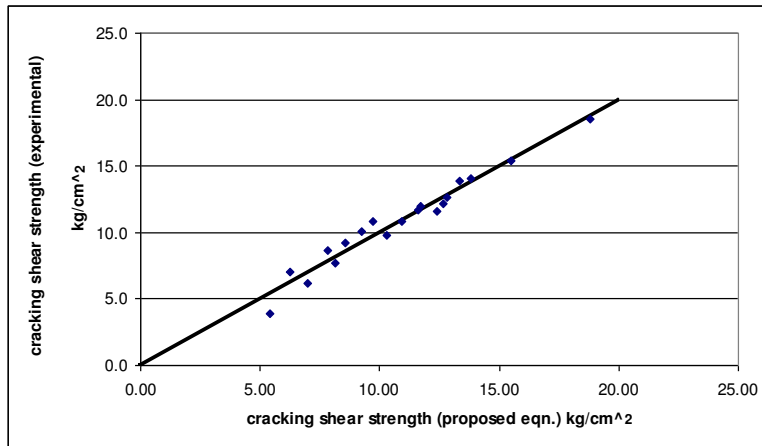
$d/a$  the depth to shear span ratio,

$\rho_w \cdot f_{yw}$  the shear reinforcement index ( $\text{kg/cm}^2$ ),

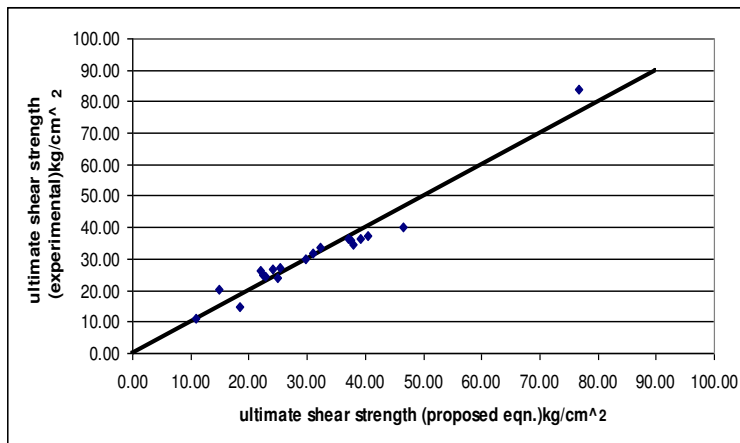
$\alpha$  the inclination angle of stirrups.



Comparison between the predicted cracking and ultimate shear strength from proposed equations (1) and (2) with the analyzed cracking and ultimate shear strengths is given in Fig. (5) and (6).



**Fig. 5.** Cracking shear strength versus calculated cracking shear strength



**Fig. 6.** Ultimate shear strength versus calculated ultimate shear strength

## 7. Ultimate Shear Capacity Adopted By the ACI Code of Practice

The present ACI code of practice (3) assumes that in beams with web reinforcement, the amount of shear stress resisted by the concrete at ultimate is equal to the amount of shear stress that would cause diagonal tension cracking. The amount of shear strength of the concrete was based and determined from analysis results on beams without web

reinforcement and with concrete compressive strengths up to  $401.1 \text{ kg/cm}^2$ . The shear strength of concrete without shear reinforcement is given by

$$v_{cr} = 1.9 \sqrt{f'_c} + 2500 \cdot \rho_w \cdot \frac{V_u \cdot d}{M_u} \quad (3)$$

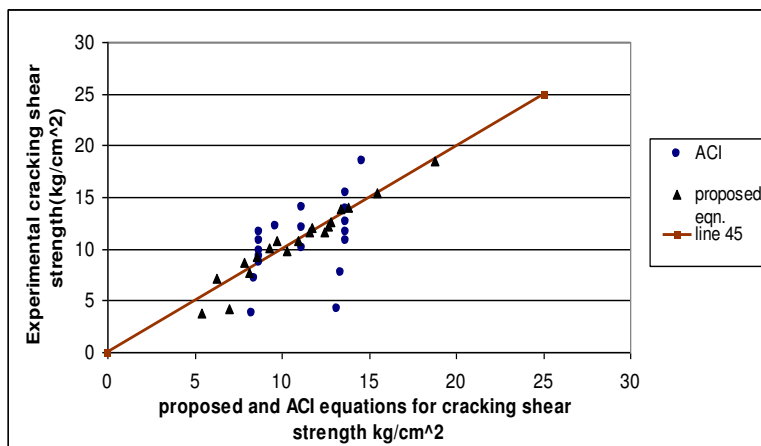
And not greater than  $3.5 \sqrt{f'_c}$

$$v_u = v_{cr} + \frac{A_{st} \cdot f_y}{b \cdot s} \quad (4)$$

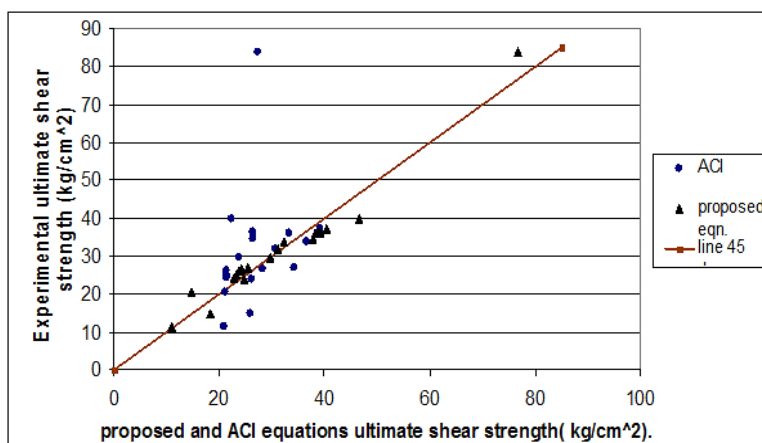
Where  $f'_c$  cylinder compressive strength (Psi),  $\rho_w$  main reinforcement ratio,  $A_{st}$  area of stirrups in spacing S,  $v_{cr}$  and  $v_u$  are in Psi units.

## 8. Comparison between the ACI Code and Analysis Cracking and Ultimate Shear Strength

From Figs. (7) and (8), it can be noticed that the ACI code equations for predicting cracking shear strength of R.C beams gives values smaller than the analytical values for beams of  $a/d = 1.0$  and greater than that of beams of  $a/d$  equal to 2, 3, 4 and 6 for normal and high strength concrete beams. Also the ACI code equation underestimate the effect of the concrete strength in high strength concrete.



**Fig. 7.** Comparisons between analytical and predicted cracking shear strengths based on proposed equation (1) and ACI codes



**Fig. 8.** Comparisons between analytical and predicted ultimate shear strengths based on proposed equation (2) and ACI codes

## 9. Conclusions and Remarks

On the basis of the results obtained in this study, the following conclusions have been reached:

- 1- Increasing the compressive strength and the amount of shear reinforcement leads to an increase of the cracking and ultimate shear strengths.
- 2- Increasing the amount of shear reinforcement leads to an increase in the beam ductility, so the minimum amount of shear reinforcement must be increased for high strength concrete.

## 10. References

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## دراسة نظرية عن سلوك القص الاستاتيكي في الكمرات عالية المقاومة ومقارنته بالكمرات ذات المقاومة العادية

### الملخص العربي

في هذه الدراسة تم تحليل ستون كمرة خرسانية ذات رتب خرسانية مختلفة باستخدام برنامج تحليل يسمى Abaqus. المتغيرات التي تمت دراستها هي:

1- مقاومة الخرسانة وهي تتراوح بين (250 , 400 و700 كجم/سم<sup>2</sup>)

2- النسبة بين - بحر القص والعمق الفعال وتتراوح بين (1,2,3,4,6)

3- نسبة حديد تسليح القص (عدد الكانات/متر)

تم تقسيم الكمرات الى خمس مجموعات وكل مجموعة مقسمة الى اثني عشرة كمرة ثانوية. تم تحليل النتائج واستنتاج تأثير هذه المتغيرات على سلوك الكمرات وتوقع العلاقة بين الحمل والتشكل ومنها تم استنتاج معادلات لحساب قيم اجهادات التشريح والانهيال للقص في الكمرات ذات المقاومة العالية والعادية وكذلك تمت مقارنة هذه النتائج مع القيم المتوقعة من الكود الأمريكي (ACI). ومن الدراسة أمكن استخلاص النتائج الآتية:

1- تزداد قيمة كلاً من اجهاد القص عند التشريح وعند الانهيال بزيادة كلاً من مقاومة الخرسانة وكمية حديد تسليح القص في الكمرات ذات الخرسانة العادية والعالية المقاومة.

2- زيادة نسبة حديد القص يؤدي إلى زيادة ممطولية الكمرات ذات الخرسانة العادية والعالية المقاومة.

3- الانفعال الأقصى المتولد في الكانات في الكمرات ذات الخرسانة العالية المقاومة أكبر منه في الكانات في الكمرات ذات الخرسانة العادية المقاومة ولذا يجب زيادة حديد تسليح القص (الكانات)

بزيادة رتبة الخرسانة.