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SECOND – ORDER ANALYSIS IN BRACED SLENDER COLUMNS PART I: APPROXIMATE EQUATION FOR COMPUTING THE ADDITIONAL MOMENTS OF SLENDER COLUMNS

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ABSTRACT

Second- order analysis in braced slender columns was investigated in this study. The present study is concerned with two main points. The first point is focusing on how to compute the additional moments in slender columns according to accredited equations in different codes as well as the basics and the assumptions of these equations. In the second point, approximate equation was suggested to compute the additional moments in slender columns. This equation was proved in elastic analysis and for the cases of single curvature in slender columns. The equation was proved by considering the column supported on two pin supports with rotational springs. The rotational springs represent the connected beams with the columns. The suggested equation gave matching values of the induced additional moments of slender columns compared with finite element results in elastic analysis.

Keywords: *Second order; Finite element; Additional moment; Slender column.*

1. Introduction

A slender column is defined as a column that is subjected to additional moments due to lateral deflections. These moments cause a pronounced reduction in axial-load capacity of the column. In first-order analysis, the effect of the deformations on the internal forces in the members is neglected. In second-order analysis, the deformed shape of the structure is considered in the equations of equilibrium. However, because many engineering calculations and computer programs are based on first-order analyses, methods have been derived to modify the results of first-order analysis to approximate the second-order effects. Second order analysis in slender columns is recommended in many codes by using approximate equations to compute the additional moments. The additional moments in slender columns according to accredited equations in different codes, as well as the basics and the assumptions of these equations will be discussed in next section.

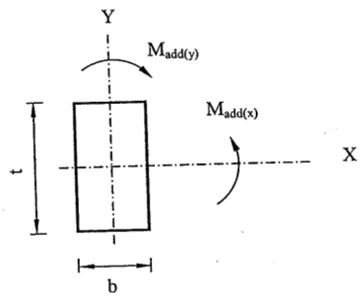
2. Calculation of the additional bending moments (M_{add}) in slender columns according to different codes

The Egyptian Code[1] takes into consideration increasing the applied moments in slender columns by adding an additional moment to the original moment. The secondary moments are assumed to be induced due to interaction of the axial load with the lateral deformation of the column.

According to ECP, ($M_{add.}$) is induced by the deflection (δ) is given by:-

$$M_{add.} = P.\delta \tag{1}$$

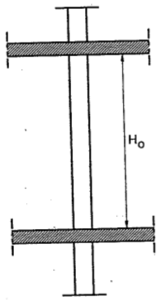
If the column is slender in t direction,

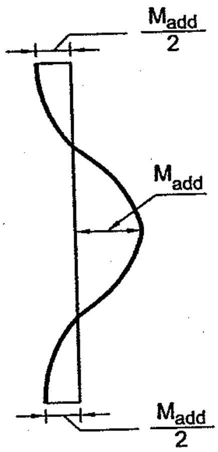


$$\delta_t = \frac{\lambda_t^{\,2} t}{2000} \tag{2}$$

$$M_{add.} = P.\delta_t \tag{3}$$

However, if the column is slender in b direction,





$$\delta_b = \frac{\lambda_b^{\,2} b}{2000} \tag{4}$$

$$M_{add.} = P.\delta_b \tag{5}$$

$$\lambda_b = \frac{H_e}{b}$$

$$H_e = k.H_0 \tag{6}$$

where H_e is the effective height of the column,

H_0 is clear height of the column,

k is length factor which depends on the conditions of the end column and the bracing conditions.

For braced columns, k is the smaller of Eqs. (7) and (8)

$$k = (0.7 + 0.05(\alpha_1 + \alpha_2)) \le 1.0 \tag{7}$$

$$k = (0.85 + 0.05(\alpha_{\min})) \le 1.0 \tag{8}$$

where α_1, α_2 are ratio of the columns stiffnesses sum to the beams stiffnesses sum at the column lower and upper ends, respectively.

$$\alpha = \frac{\sum (E_c I_c / H_o)}{\sum (E_b I_b / L_b)} \tag{9}$$

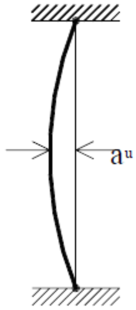
In fact, ECP doesn't mention the basis of equation (5), or the presuppositions which the equation was based on.

The British Code [2] uses the same equation of ECP for the computation of M_{add} in slender column. ***Prab Bhatt et al [3]*** illustrated the basis and the assumptions of the British code as follows:

Additional moment is a function of the columns lateral displacement. The code aims to predict the deflection at mid-height at the moment of concrete failure.

The shape of the curvature is assumed, and the central deflection

a_u is assumed to be given by,



$$a_u = \frac{P.a_u}{EI}\beta l_e^2 \tag{10}$$

$$\frac{1}{r} = \frac{P.a_u}{EI} \tag{11}$$

where $\frac{1}{r}$ is the curvature. The curvature will vary typically along the column and the code assumes a sinusoidal value of $\frac{1}{\pi^2}$. Thus the central lateral deflection a_u is assumed to be:

$$a_u = \left(\frac{1}{\pi^2}\right) l_e^2 \left(\frac{1}{r}\right) \tag{12}$$

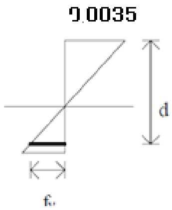


Fig. (1-a)

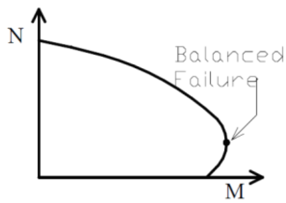


Fig. (1-b)

Fig. (1-a): Strain diagram in ultimate stage

Fig. (1-b): The interaction diagram between the bending and the normal force

The column curvature $\left(\frac{1}{r}\right)$ is calculated by considering the M-N curve. At the balanced failure, where the compressive concrete strain at its maximum and the steel tensile strain at its yield, the corresponding local curvature to this distribution of strain is given by the following equation

$$\frac{1}{r_b} = \frac{(0.0035 + 0.002)}{d} \tag{13}$$

The maximum deflection for the case set out above is given in the code by the following expression:

$$a_u = \frac{0.0005 l_e^2}{h} \tag{14}$$

$$a_u = \frac{h}{2000}.(\frac{l_e}{h})^2 = \frac{\lambda^2}{2000}.h \tag{15}$$

Where: l_e and h are effective buckling height of column and column thickness, respectively.

Some important notes can be observed from basics of the used equations in the British and Egyptian codes,

- In these equations, the deflection shape is assumed as sine curve, even if the columns are subjected to end moments.
- These equations are valid only in the ultimate stage. This means that these equations are not valid in working stress design or at any stage before the ultimate.
- The equations take into account the connected beams rigidity in the calculation of the column effective length without considering the effect of these beams in the reversal moments, which can be induced at the connection between them and the columns.

In American Code[4], the moment magnifier method is used in the analysis of secondary moments. In the moment-magnifier analysis, unequal end moments are applied on the column shown in Fig. 2-a. The column is replaced with a similar column subjected to equal moments at both ends, which is shown in Fig. 2-b. The bending moments are chosen where the maximum magnified moment is the same in both columns. The expression for the factor C_m was originally derived for use in the steel beam-columns design and was adopted without change for concrete design.

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \tag{16}$$

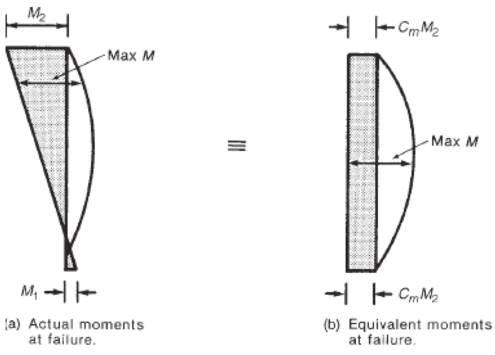


Fig. 2. Equivalent moment factor C_m

M_2, M_1 are the larger and smaller end moments, respectively, calculated from a conventional first-order elastic analysis. If a single curvature bending without a point of contra flexure between the ends is occurred by the moments M_1 and M_2, $\frac{M_1}{M_2}$ is positive. However, if the moments cause double curvature with a point of zero moment between the two ends, that $\frac{M_1}{M_2}$ is negative. The moment magnifier equation in the cases of no sway according to ACI is,

$$M_c = \delta_{ns}.M_2 \tag{17}$$

The subscript **ns** refers to no sway. The moment M_2 is defined as the greater end moment acting on the column. ACI Code goes on to define δ_{ns} as follows:

$$\delta_{ns} = \frac{C_m}{1 - 0.75\frac{P}{P_c}} \tag{18}$$

The 0.75 factor in Eq. (18) is the stiffness reduction factor **ϕ*K***, which is based on the probability of under strength of a single isolated slender column.

The nomograph given in Fig. 3 is used to compute k. To use these nomographs, ψ is calculated at both ends of the column, from Eq. (19), and the appropriate value of k is found as the intersection of the line labeled k and a line joining the values of at the column ends.

$$\psi = \frac{\sum(EI_c / L_c)}{\sum(EI_b / L_b)} \tag{19}$$

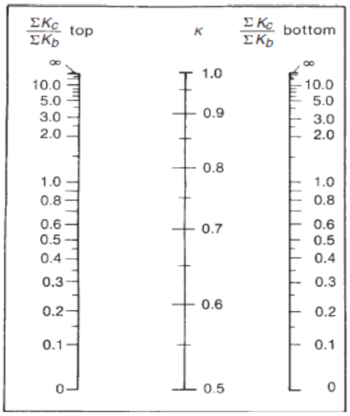


Fig. 3. Nomograph for effective length factors

where L_c, L_b are the lengths of columns and beams and are measured center to center of the joints, and I_c, I_b are the moments of inertia of the columns and the beams, respectively.

The effective length factor for a compression member, such as a column, wall, or brace, considering braced behavior, ranges from 0.5 to 1.0. It is recommended that a ***k*** value of 1.0 be used. If lower values are used, the calculation of ***k*** should be based on analysis of the frame using ***I*** values given in table 1.

Table (1-a).
I and A permitted for elastic analysis at factored load level

Member and condition		Moment of Inertia	Cross-sectional area
Columns		$0.70I_g$	$1.0A_g$
Walls	Uncracked	$0.70I_g$	
	Cracked	$0.35I_g$	
Beams		$0.35I_g$	
Flat plates and flat slabs		$0.25I_g$	

Table (1-b).
Alternative I for elastic analysis at factored load

Member	Alternative value of *I* for elastic analysis		
	Minimum	*I*	Maximum
Columns and walls	$0.35I_g$	$\left(0.80+25\frac{A_{st}}{A_g}\right)\left(1-\frac{M_u}{P_u h}-0.5\frac{P_u}{P_o}\right)$	$0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g$	$(0.10+25\rho)\left(1.2-0.2\frac{b_w}{d}\right)I_g$	$0.5I_g$

The critical load ***(Pc)*** shall be calculated as

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kL_u)^2} \tag{20}$$

The effects of cracking and creep are considered by using a reduction factor for stiffness ***EI***. In calculating the critical axial buckling load, the primary concern is the choice of a stiffness **(*EI*)*eff*** that reasonably approximates the variations in stiffness due to cracking and the concrete nonlinearity.

For non-composite columns, (EI)eff shall be calculated in accordance with equations 21- (a), (b), or (c): Where

$$\text{(a)}\ (EI)_{eff} = \frac{0.4E_c I_g}{1+\beta_{dns}} \tag{21-a}$$

$$\text{(b)}\ (EI)_{eff} = \frac{(0.2E_c I_g + E_s I_{se})}{1+\beta_{dns}} \tag{21-b}$$

$$\text{(c)}\ (EI)_{eff} = \frac{E_c I}{1+\beta_{dns}} \tag{21-c}$$

$$\beta_{dns} = \frac{Max.\ factored\ sustained\ axial\ load}{Max.\ factored\ axial\ load\ associated\ with\ the\ same\ load\ combination}$$

3. Suggested approximate equation for computing of additional moment (M_{add}) in slender columns

In this section, approximate equation for computation of the M_{add} in slender columns will be proved. The derivation of suggested equation in the elastic analysis and for the cases of single curvature of the columns. The closed frame in Fig.(4) as an example will be used in this derivation. By using any method in the structural analysis, the first order analysis can be done.

3.1. Modeling the slender column

After determination the moments and the deformations induced in the columns, the columns can be modeled as shown in Fig. (5).

Firstly, the left column "1" will be the studied column considering that no M_{add} induced in the right column. The column is modeled as a pin supported member with rotational spring supports. The upper and lower beams are as spring supports for the column with rotational stiffness.

Where, K_{b1} is the rotational stiffness of upper beam and K_{b2} is the rotational stiffness of lower beam.

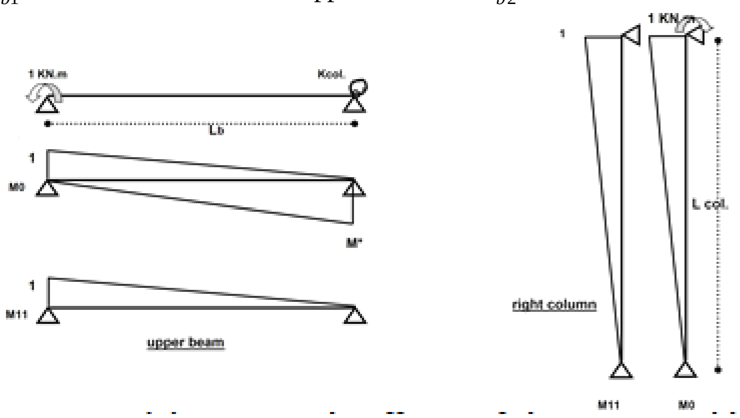


Fig. 4. Modeling of the studied column

3.2. Calculation of the rotational stiffness of the connected beam

The rotational stiffness (K_{b1}) can be determined approximately by applying unit force at the end of the beam which is connected with the studied column. The other end of the beam is considered as rotation restriction by the other column which is considered as a spring support ($K_{col.}$). By using virtual work method, the rotation of the free end beam is computed, and then the rotational rigidity is computed as $K_{b1} = 1/\theta_b$. Also, $K_{col.}$ can be found approximately in the same manner but considering the other end of the column as pin supported and neglecting the rotational restriction at this end.

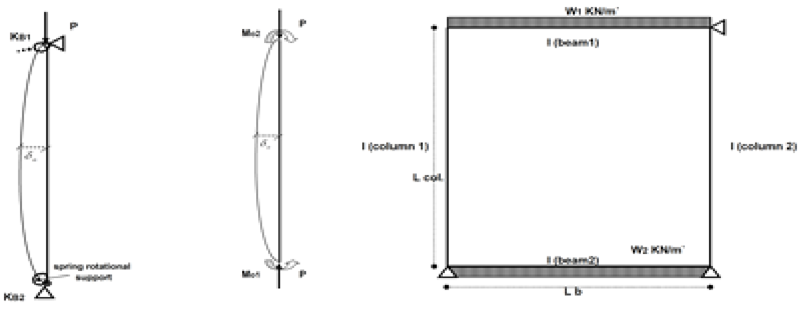


Fig. 5. Rotational stiffness of the connected beam

The rotation at the loaded end right column

$$\theta_{R.col.} = \frac{1}{EI_{R.col}} \int_0^L M_o . M_{11} = \frac{L_{R.col.}}{3EI_{R.col}} \tag{22}$$

where $L_{R.col}$ and $I_{R.col}$ are The length and moment of inertia of the right column**.**

The rotational stiffness of the right column to the upper beam is:

$$K_{R.col} = \frac{3EI_{R.col}}{L_{R.col}} \tag{23}$$

M* is the reaction moment at the right of the shown beam. This moment can be determined by the force method, as follows:

$$M^* = \left(\frac{L_{b1}/6}{\frac{EI_{b1}}{K_{R.col}} + \frac{L_{b1}}{3}} \right) \tag{24}$$

By using the virtual work, the rotation at the other end of the upper beam can be determined as follows:

$$\theta_{b1} = \frac{1}{EI_{b1}} \left[\frac{1}{2} . \frac{L_b}{3} .2 - \frac{L_b^2}{12} . \frac{1}{3} \frac{1}{\frac{EI_b}{K_{col}} + \frac{L_b}{3}} + \frac{L_b^2}{36} . \frac{1}{3} \left[\frac{1}{\frac{EI_b}{K_{col}} + \frac{L_b}{3}} \right]^2 \right] \tag{25}$$

The rotational rigidity is computed as $K_{b1} = 1/\theta_b$.

$$K_{b1} = \frac{EI_{b1}}{L_{b1} \left(0.33 - \frac{L_b}{36} . \frac{1}{\frac{EI_b}{K_{col}} + \frac{L_b}{3}} + \frac{L_b^2}{108} \left(\frac{1}{\frac{EI_b}{K_{col}} + \frac{L_b}{3}} \right)^2 \right)} \tag{26}$$

The rotational stiffness of the lower beam **(** K_{b2}) is determined in the same manner.

Generally, Eq. (26) for calculation of K_b is considered as the rotational rigidity of any element end which is directly connected to the studied column. Also Eq. (23) for calculation of K_{col} is the rotational rigidity of any element end which is indirectly connected to the studied column. The shown example in Fig.(6) illustrates generally how to find the rotational stiffness at the column ends.

In this example, the aim is finding the rotational rigidity of the column AB. The rotational rigidity at node B to the column AB can be expressed as $\sum K_{BD} + K_{BE} + K_{BC}$,

where $K_{BD} = \dfrac{3EI_4}{L_4}$

$$K_{BE} = \frac{EI_5}{L_5\left(0.33 - \frac{L_5}{36}\cdot\frac{1}{\frac{EI_5}{K_{EF}} + \frac{L_5}{3}} + \frac{L_5^2}{108}\left(\frac{1}{\frac{EI_5}{K_{EF}} + \frac{L_5}{3}}\right)^2\right)}$$

…….. where $K_{EF} = \dfrac{3EI_7}{L_7}$

$$K_{BC} = \frac{EI_3}{L_3\left(0.33 - \frac{L_3}{36}\cdot\frac{1}{\frac{EI_3}{K_{CG} + K_{CJ}} + \frac{L_3}{3}} + \frac{L_3^2}{108}\left(\frac{1}{\frac{EI_3}{K_{CG} + K_{CJ}} + \frac{L_3}{3}}\right)^2\right)}$$

……. where $K_{CG} = \dfrac{3EI_1}{L_1}$,, $K_{CJ} = \dfrac{3EI_2}{L_2}$

Then, $K_B = K_{BE} + K_{BC} + K_{BD}$

Also, the rotational stiffness at the other end of column AB is as follows:

$$K_{AB} = \frac{EI_8}{L_8\left(0.33 - \frac{L_8}{36}\cdot\frac{1}{\frac{EI_8}{K_{FA}} + \frac{L_3}{3}} + \frac{L_8^2}{108}\left(\frac{1}{\frac{EI_8}{K_{FA}} + \frac{L_8}{3}}\right)^2\right)}$$

$$K_{CG} = \frac{3EI_7}{L_{FE}}$$

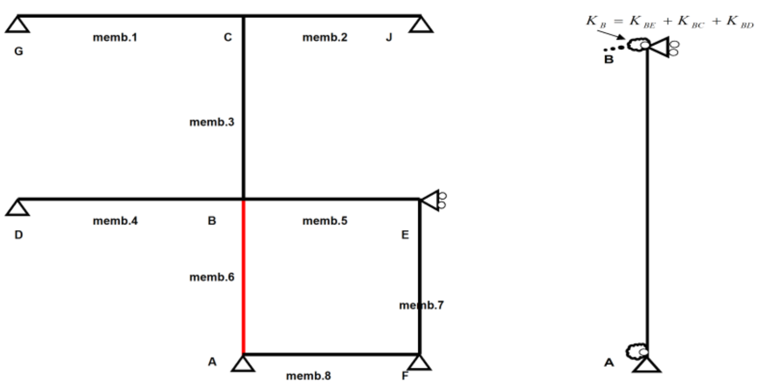


Fig. 6. Rotational stiffness at the column ends**.**

From many trials, it was found that, for any element that is connected to a column whether it was another column or a beam, and has a cross section greater than or equal to the studied column cross section, the rotational rigidity in Eq. (26) can be simplified to Eq.(23)

$$K = \frac{3EI_{element}}{L_{element}}$$

3.3. Additional moment in pin slender column subjected to external end moments

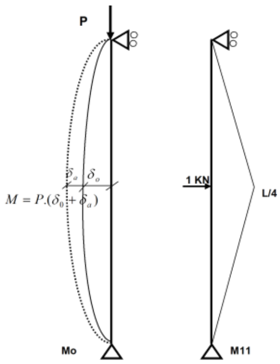


Fig. 7. M_{add} in pin slender column under equally end moments

The column in Fig.(7) is deformed under the action of the equal end moments by an amount δ_o.This will be referred to as the first-order deflection. When the axial load P is applied, the deflection increases by the amount δ_a. The final deflection at mid span is $(\delta_o + \delta_a)$. This total deflection will be referred to as the second-order deflection. It will be assumed that the final deflected shape is 2nd degree parabolic curve`. Because the deflected shape is assumed to be 2nd degree parabolic curve, the moment diagram is also 2nd degree parabolic curve. The Max.deflection at mid span due to equal end moments is $\delta_o = \frac{M_0 L^2}{8EI}$.By using virtual work method, the additional deflection can be found as follows:

$$\delta_a = \int_0^L M_0^{`} M_{11} dx \qquad M_0^{`} = P.(\delta_0 + \delta_a)$$

$$\delta_a = \frac{1}{EI}(2 * \frac{2}{3} P(\delta_0 + \delta_a).\frac{L}{2}.\frac{5}{8}.\frac{L}{4})$$

$$\delta_a = \frac{\delta_0 5PL^2 / 48EI}{1 - \frac{5PL^2}{48EI}} \quad ……. \qquad \text{And, we can put } \frac{1}{P_e} = \frac{5L^2}{48EI}$$

Where P_e is Euler load

$$\delta_a = \frac{\delta_0}{1-\frac{P}{P_e}} \quad \ldots\ldots \text{ with putting } \quad \eta = \frac{1}{1-\frac{P}{P_e}}$$

Then, $M_{add} = P\delta_o\eta \ldots$ 2nd parabolic curve (27)

Similarly, when the column is deformed under unequal end moments, the moments are divided into two parts $M = M_0 + \Delta M$ .The deflection curve of ΔM is 3rd degree parabolic curve, and the maximum deflection occurs when $\frac{d\delta}{dx} = 0$. The maximum deflection in this case is at 0.54L from the smaller moment. Thus Max. deflection due to ΔM is given by $\delta_0^{`} = \frac{\Delta ML^2}{15.56EI}$.

Thus, the additional deflection is $\delta_a^{`} = \frac{\delta_0^{`}}{1-\frac{P}{0.94P_e}}$ (28)

By rounding the term $\frac{1}{1-\frac{P}{0.94P_e}}$ to η, it is considered that the total additional deflection in the case of unequal end moments is at distance equal to 0.54L from the smaller moment, and is expressed as:

$$\delta_{at} = (\delta_o + \delta_o^{`})\eta \tag{29}$$

So, when the column is deformed under unequal end moments, the maximum M_{add} through the middle of the span can be considered as:

$$M_{add.} = P\eta\left(\frac{M_0L^2}{8EI} + \frac{\Delta ML^2}{15.56EI}\right) \tag{30}$$

3.4. The additional moments in restricted slender column.

After obtaining the formula which is expressed on the rotational stiffness at the column ends. Now the modeled column in Fig.(4) is analyzed to find M_{add} at the ends and mid span of the column. Solving this model can be analyzed in five cases as shown in Fig.(8).

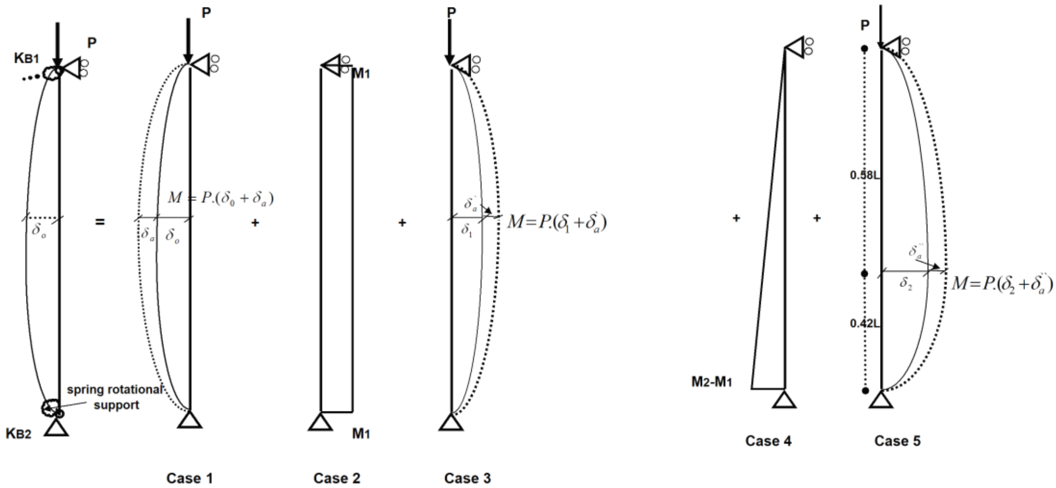


Fig. 8. M_{add} in restricted slender column

where:-

case 1:- $M_{add.} = P\eta(\frac{M_0 L^2}{8EI} + \frac{\Delta M L^2}{15.56EI})$:- M_{add} at mid span due to the axial force and the deflections from moments.

Case 2:- M_1 is the induced moments at the spring support "1"

Case 3:- $P(\delta_1 + \delta_a^{`})$ is M_{add} at mid span due to the axial force and the deflections from M_1.

Case 4:- $\Delta M = M_2 - M_1$ is the difference between the larger moments induced at spring support "1" and the smaller moments induced at spring support "2".

Case 5:- $P(\delta_2 + \delta_a^{``})$ is M_{add} at mid span due to the axial force, the deflections from ΔM and its additional deflection.

Using the moment area method to solve these cases as follows:-

$$\delta_1 - \delta_2 = -\theta_a . x_{ab} + \frac{A_m}{EI_{col.}} . x^{`} \tag{31}$$

where A_m is the area of moments between the nodes1,2.

$$\frac{M_1 . EI_{col.}}{K_{b1}} . L_{col.} = \frac{P\eta L_{col.}^2}{EI_{col.}} \begin{bmatrix} \frac{2}{3} . \frac{M_0 L_{col.}^2}{2*8EI_{col.}} + 0.65*0.46*\frac{\Delta M_0 L_{col}^2}{15.56EI_{col}} - \frac{2}{3} . \frac{1}{2} . \frac{M_1 L_{col.}^2}{8EI} - \\ 0.65*0.46*\frac{\Delta M_1 L_{col.}^2}{15.56EI} - \frac{M_1 L_{col.}^2}{2} - \frac{1}{2} . \Delta M_1 . \frac{L_{col.}^2}{3} \end{bmatrix} \tag{32}$$

$$\frac{M_1.}{K_{b1}}.L_{col.}=\frac{1}{EI_{col.}}\left[\begin{array}{l}P.\eta.L^2(\frac{2}{3}.\frac{M_0L^2_{col.}}{2*8EI_{col.}}+0.65*0.46*\frac{\Delta M_0L^2_{col}}{15.56EI_{col}}-\frac{2}{3}.\frac{1}{2}.\frac{M_1L^2_{col.}}{8EI}-\\0.65*0.46*\frac{\Delta M_1L^2_{col.}}{15.56EI})-\frac{M_1L^2_{col.}}{2}-\frac{1}{2}.\Delta M_1.\frac{L^2_{col.}}{3}\end{array}\right] \tag{33}$$

$$\frac{M_1L_{col}}{K_{b1}}+\frac{L^2_{col}}{2}M_1+\frac{M_1L^2_{col}}{24EI_{col}}.P\eta L^2_{col}=P\eta L^2_{col}Q-\frac{\Delta ML^2_{col}}{6}-P\eta L^2_{col}\frac{0.31\Delta ML^2_{col}}{EI_{col}} \tag{34}$$

By putting $$Q=\frac{M_0L^2_{col}}{24EI_{col}}+0.315\frac{\Delta M_0L^2_{col}}{15.56EI_{col}} \tag{35}$$

$$M_1=\left(\frac{1}{\frac{L_{col}EI_{col}}{K_{b1}}+\frac{L^2_{col.}}{2}+\frac{P\eta L^4_{col}}{24EI}}\right).\left[\left[P\eta L^2_{col}Q-\Delta M(\frac{L^2_{col}}{6}+\frac{0.315P\eta L^4_{col}}{15.56EI_{col}}\right]\right] \tag{36}$$

$$putting....Z_1=(\frac{L_{col.}EI_{col.}}{K_{b1}}+\frac{L^2_{col}}{2}+\frac{L^2_{col.}}{24EI_{col}}) \tag{37}$$

$$A=(-\frac{L^2_{col}}{6}-1.94\frac{P\eta L^4_{col}}{100EI}).\frac{1}{Z_1} \tag{38}$$

$$M_1=\left(\frac{1}{Z_1}\right).\left(P\eta L^2_{col}Q-\Delta MA\right) \tag{39}$$

$$\theta_1-\theta_2=\frac{A_m}{EI_{col.}}$$

$$\frac{\Delta M.EI_{col.}}{K_{b2}}+M_1(\frac{EI_{col.}}{K_{b2}}+\frac{EI_{col.}}{K_{b1}})=\left[\begin{array}{l}-M_1L_{col.}-\frac{2}{3}.\frac{P\eta M_1L^3_{col.}}{8EI_{col}}-\frac{\Delta ML_{col.}}{2}-0.65.\frac{P\eta\Delta ML^3_{col}}{15.56EI_{col.}}\\+P\eta L_{col}(\frac{2}{3}\frac{M_0L^2}{8EI_{col}}+0.65\frac{\Delta M_0L^2}{15.56EI})\end{array}\right] \tag{40}$$

$$M_1(\frac{EI}{K_{b2}}+\frac{EI}{K_{b1}}+L_{col}+\frac{P\eta L^3_{col}}{12EI_{col}})=P\eta L_{col}Q^*-\Delta M(\frac{L_{col}}{2}+\frac{0.65P\eta L^3_{col}}{15.56EI_{col}}+\frac{EI_{col}}{K_{b2}}) \tag{41}$$

Where $$Q^*=\frac{2}{3}.\frac{M_0L^2_{col}}{8EI_{col}}+0.656\frac{\Delta M_0L^2_{col}}{15.56EI_{col}} \tag{42}$$

And putting

$$EI_{col}(\frac{1}{K_{b2}}+\frac{1}{K_{b1}})+L_{col}+\frac{P\eta L_{col}^{3}}{12EI_{col}}=Z_{2} \tag{43}$$

$$M_{1}\left(\frac{EI_{col.}}{K_{b2}}+\frac{EI_{col.}}{K_{b1}}+L_{col}+\frac{2}{3}.P\eta\frac{L_{col}^{3}}{8EI_{col}}\right)=2QL_{col}P\eta+\Delta M(-0.5L_{col}-\frac{0.67*P\eta L_{col}^{3}}{15.56EI_{col}}-\frac{EI_{col}}{K_{b2}}) \tag{44}$$

$$M_{1}=\frac{1}{Z_{2}}\left[2QL_{col}P\eta+\Delta M(-0.5L_{col}-\frac{0.67*P\eta L_{col}^{3}}{15.56EI_{col}}-\frac{EI_{col}}{K_{b2}})\right] \tag{45}$$

By solving equations 36 and 45

$$\Delta M=\left(\frac{Q^{*}P\eta L_{col}}{Z_{2}}-\frac{QP\eta L^{2}}{Z_{1}}\right).\left[\begin{array}{l}\frac{1}{Z_{2}}\left(\frac{L_{col}}{2}+\frac{41.67P\eta L_{col}^{3}}{1000EI_{col}}+\frac{EI_{col}}{K_{b2}}\right)\\+\frac{1}{Z_{1}}\left(-\frac{20P\eta L_{col}^{2}}{100EI_{col}}-\frac{L_{col}^{2}}{6}\right)\end{array}\right] \tag{46}$$

And putting

$$B=.\left[\frac{1}{Z_{2}}\left(\frac{L_{col}}{2}+\frac{41.67P\eta L_{col}^{3}}{1000EI_{col}}+\frac{EI_{col}}{K_{b2}}\right)\right] \tag{47}$$

The additional moment can be obtained in the final formulas as follows:

$$M_{1}=\frac{Q^{*}P\eta L_{col.}}{Z_{2}}-\Delta MB \tag{48}$$

$$\Delta M=P\eta L_{col.}\left(\frac{Q^{*}}{Z_{2}}-\frac{QL_{col.}^{2}}{Z_{1}}\right).\frac{1}{[A+B]} \tag{49}$$

$$M_{2}=M_{1}+\Delta M \tag{50}$$

Where M_1 at the beam which has the smaller rigidity and M_2 at the beam this has higher rigidity. And the terms of equation Q, Z_1, A, Q^*, Z_2 and B are shown in equations (35, 37, 38, 42, 43 and 47), respectively.

$$M_{mid.}=-P.(\delta_{o}^{*}-\delta_{m}^{*})+M_{1}+0.58\Delta M \tag{51}$$

Where

M_{mid} is the additional moments through the middle of column length.

$$\delta_{0}^{*}=(\frac{M_{0}L_{col}^{2}}{8EI_{col.}}+\frac{\Delta M_{0}L_{col}^{2}}{15.56EI_{col.}})*\eta \tag{52}$$

$$\delta_m^* = \left(\frac{M_1 L_{col}^2}{8EI_{col.}} + \frac{\Delta M L_{col}^2}{15.56EI_{col.}}\right) * \eta \qquad (53)$$

3.5 Effect of additional moment in the adjacent columns on the studied column

From the proved previous equations, additional moments at the ends and at mid span of the column can be computed. These equations are proved hence there are no M_{add} considered in the adjacent columns.

To take into consideration the effect of M_{add} of other columns on the studied column, assuming in the example in Fig.(3) that the M_{add} induced in the right column are determined according to Eqs. (48 to 51). Also, as shown in Fig.(9), part of these moments will transfer to the studied left column through the beams. By considering one of the ends of these beams is subjected to M_{add} which are coming from the ends of the right column, and the other end of the beams is rotationally restrained by the studied left column as shown in Fig.(9). The force method can be used to find the transferred moments to the studied column.

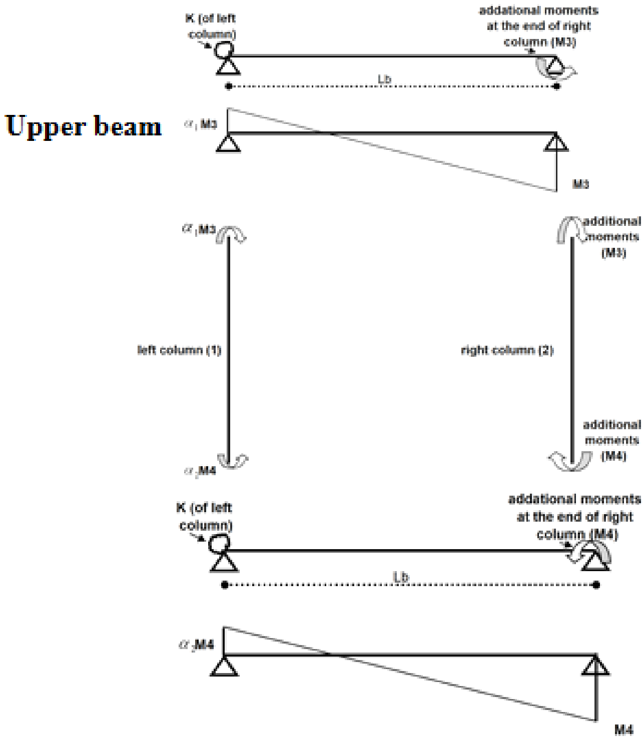


Fig. 9. The effect of M_{add} in the adjacent columns

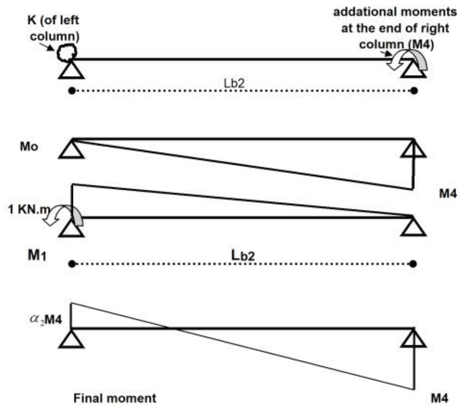


Fig. 10. The transferred moments to the lower end of column 1

For example to find the transferred moments to the bottom end of column 1,

$$\theta_{10} + M^*\theta_{11} = \theta = \frac{M^*}{K_{col}} \tag{54}$$

where M^* is The moments at lower of col. 1 due to M_4

$$\frac{1}{EI_b}(-0.5M_4L_b * \frac{1}{3} + (0.5L_b * \frac{2}{3})M^*) = \frac{M^*}{K_{col.}} \tag{55}$$

$$M^* = \left(\frac{L_{b2}/6}{\frac{EI_{b2}}{K_{col.1}} + \frac{L_{b2}}{3}} \right) M_4 = \alpha_2 M_4 \tag{56}$$

$$\alpha_2. = \left(\frac{L_{b2}/6}{\frac{EI_{b2}}{K_{col1}} - \frac{L_{b2}}{3}} \right) \tag{57}$$

Similarly for the upper beam

$$\alpha_1.M_3 = \left(\frac{L_{b1}/6}{\frac{EI_{b1}}{K_{col1}} - \frac{L_{b1}}{3}} \right) M_3, \tag{58}$$

where $$\alpha_1. = \left(\frac{L_{b1}/6}{\frac{EI_{b1}}{K_{col1}} - \frac{L_{b1}}{3}} \right) \tag{59}$$

Thus the total additional moments induced in the upper and lower ends of the studied column respectively are as follows:

$$M_{t1} = M_1 + \alpha_1 M_3 \tag{60}$$

$$M_{t2} = M_2 + \alpha_2 M_4 \tag{61}$$

where:- M_1 is M_{add} at the weakest column end without considering the effect of transferred moments from other columns.

M_2 is M_{add} at the strongest column end without considering the effect of transferred moments from other columns.

M_3 and M_4 are M_{add} at adjacent column ends without considering the effect of transferred moments from other columns.

α_1 and α_2 are the ratios of transferred moments from the other columns.

It is found from many proceeded trials that as the inertia moment of beams is greater than the column inertia moment, the factors α_1, α_2 will be small values, and in this case the effect of additional moments from other columns can be neglected . Also the additional moments can be simplified to Eqs. (48 to 51).

The suggested equation in this paper can be applied manually or easily by any computational program.

3.6. Summary for the suggested equation by solving example

The multistory frame shown in Fig. (11) is analyzed in the first order to calculate the moments and the normal forces in column AB.

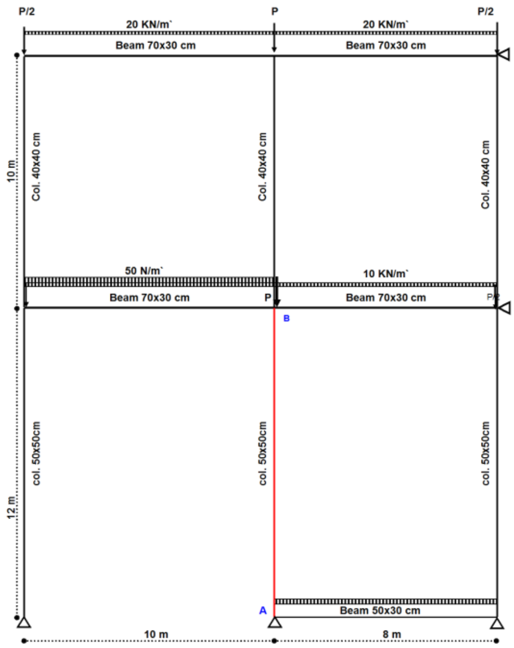


Fig. 11. Multi-story frame

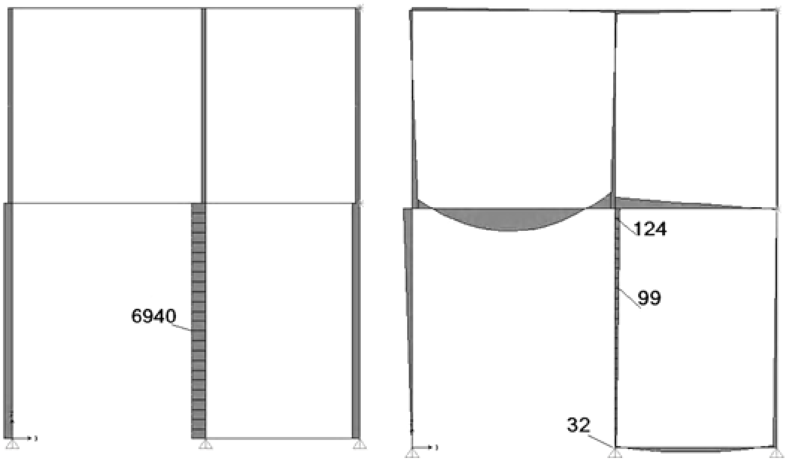


Fig. 12. First order analysis by finite element method

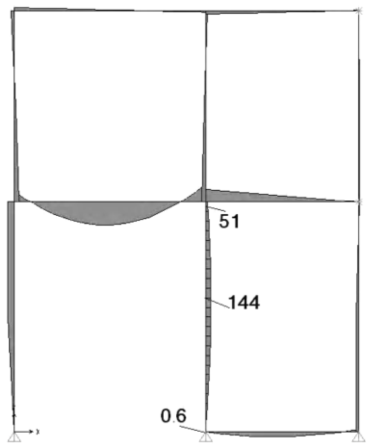


Fig. 13. Second order analysis by finite element method

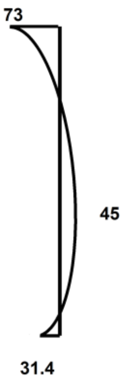


Fig. 14. M_{add} In column AB by finite element method

The first order analysis of the frame, the second order analysis by the finite element method and the induced M_{add} in column AB are given in Figs (12,13 and14) respectively.

Calculating M_{add} in column AB by the suggested equation is as follows:

N.F = 6940 KN. $E_c = 248.211x10^5 KN/m^2$ $I_c = 0.0052m^4$

EI**col**. = 124106 KN.m ^{2}

$M_0 = 32KN.m$ $\Delta M_o = 129 - 32 = 90KN.m$

The moment of inertia of the beams which are connected to the column is greater than the moment of inertia of the column. Thus the rotational stiffness at the column ends can be determined according to simplified equation (23). Also, M_{add} can be determined by using equations (48-51) and neglecting the effect of the additional moments in the other columns.

$$K = \frac{3EI}{L}$$

$$K_2 = \frac{3E(0.7)^3 * 0.3/12}{10} + \frac{3E(0.7)^3 * 0.3/12}{8} + \frac{3E*(0.4)^3 * 0.4/12}{10} = 160000KN.m/rad$$

$$K_1 = \frac{3E(0.5)^3 * 0.3/12}{8} = 29087KN.m/rad \qquad P_e = \frac{\pi^2 EI_{col}}{(L_{col})^2} = 8851KN \cdot \qquad \eta = \frac{1}{1 - \frac{P}{P_e}} = 4.63$$

The terms of equation are found as follows:

$$.Z_1 = (\frac{L_{col.} EI_{col.}}{K_{b1}} + \frac{L_{col}^2}{2} + \frac{L_{col.}^2}{24EI_{col}}) = 340.11$$

,,

$$A = (-\frac{L_{col}^2}{6} - 1.94\frac{P\eta L_{col}^4}{100EI}).\frac{1}{Z_1} = -0.36459$$

$$Q^* = \frac{2}{3}.\frac{M_0 L_{col}^2}{8EI_{col}} + 0.656\frac{\Delta M_0 L_{col}^2}{15.56EI_{col}} = 0.0074,, \quad Q = \frac{M_0 L_{col}^2}{24EI_{col}} + 0.315\frac{\Delta M_0 L_{col}^2}{15.56EI_{col}} = 0.0036,$$

$$Z_2 = EI_{col}(\frac{1}{K_2} + \frac{1}{K_1}) + L_{col} + \frac{P\eta L_{col}^3}{12EI_{col}} = 53$$

$$B = .\left[\frac{1}{Z_2}\left(\frac{L_{col}}{2} + \frac{41.67P\eta L_{col}^3}{1000EI_{col}} + \frac{EI_{col}}{K_2}\right)\right] = .4661$$

$$\Delta M = P\eta L_{col.}\left(\frac{Q^*}{Z_2} - \frac{QL_{col.}^2}{Z_1}\right).\frac{1}{[A+B]} = 44.92KN.m$$

$$M_1 = \frac{Q^* P\eta L_{col.}}{Z_2} - \Delta MB = 31.1KN.m$$

$$M_2 = M_1 + \Delta M = 76.02KN.m$$

$$M_{mid.} = -P.(\delta_o^* - \delta_m^*) + M_1 + 0.58\Delta M = -48.55KN.m$$

Where

M_{mid} : - M_{add} at the mid span of column.

$$\delta_0^* = \left(\frac{M_0 L_{col}^2}{8EI_{col.}} + \frac{\Delta M_0 L_{col}^2}{15.56 EI_{col.}}\right) * \eta = 0.05171m$$

$$\delta_m^* = \left(\frac{M_1 L_{col}^2}{8EI_{col.}} + \frac{\Delta M L_{col}^2}{15.56 EI_{col.}}\right) * \eta = 0.03679m$$

Table 3.
Comparison of M_{add} results between F.E.M and suggested equation

method	M_1	M_2	M_{mid}
F.E.M	31.4	73	-45
Sugg. Equ.	31.1	76	-48.55

3.7. Checking the efficiency of the suggested equation

Sixteen closed frames were analyzed by the suggested equation and finite element method by using SAP 2000 program [5]to verify the efficiency of the suggested equation. These frames were analyzed with changing many of factors such as cross section of top and lower beams, the ratio of the axial force to Euler load, slenderness ratio and the rigidity of the upper beam to lower beam. The left column is the target column in this analysis. The main model of the analyzed frame is shown in Fig.(15) and the results of the analyzed frames are shown in figures (16 to 19).

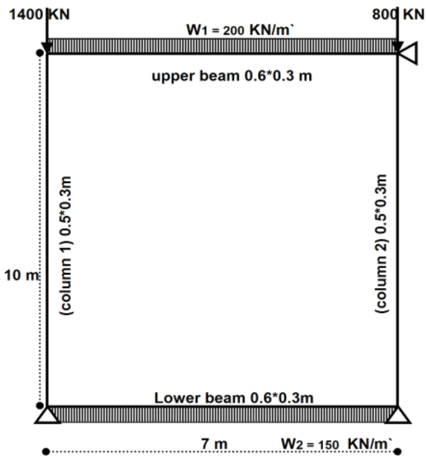


Fig. 15. Closed frame "main model".

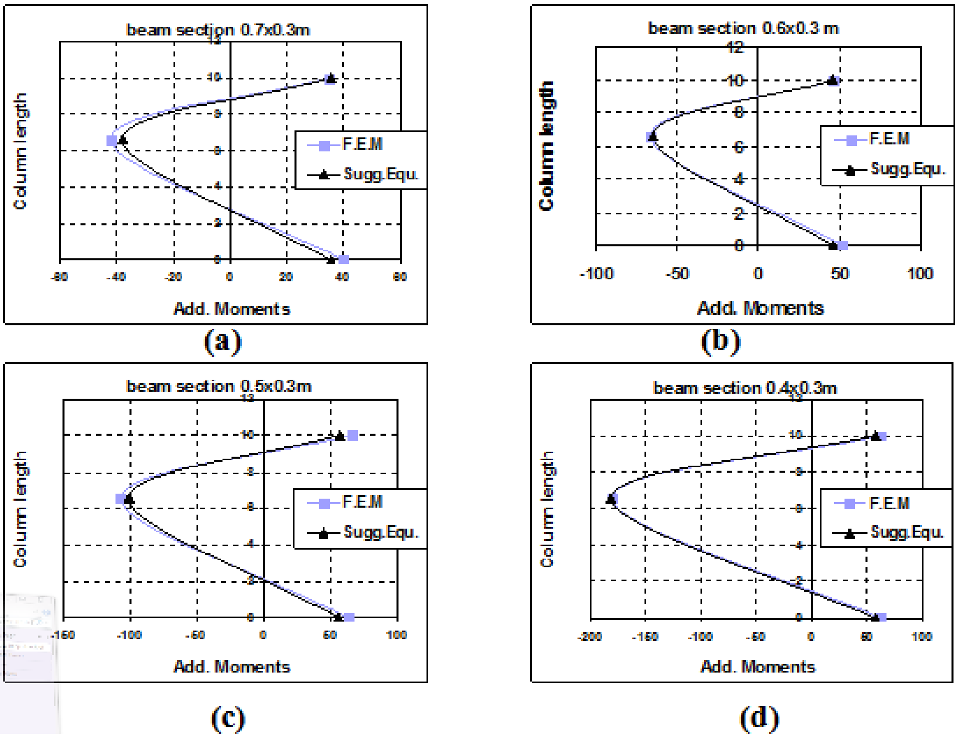


Fig. (16 a-d). The change of the rigidity of beams.

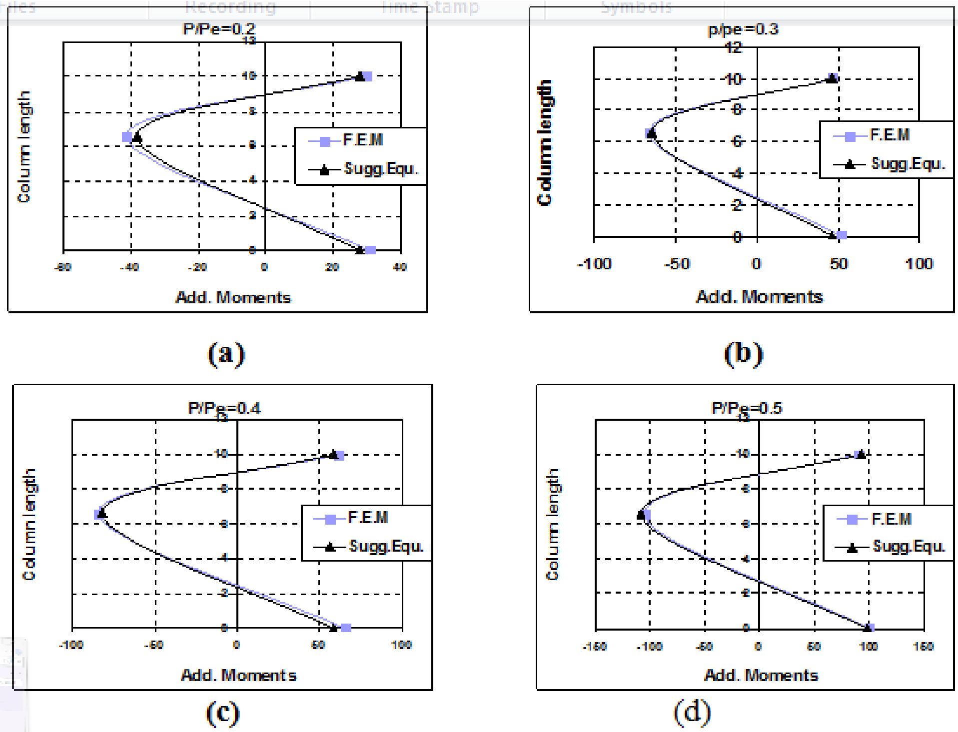


Fig. (17 a-d) The change in the ratio of axial force to Euler load "for left column".

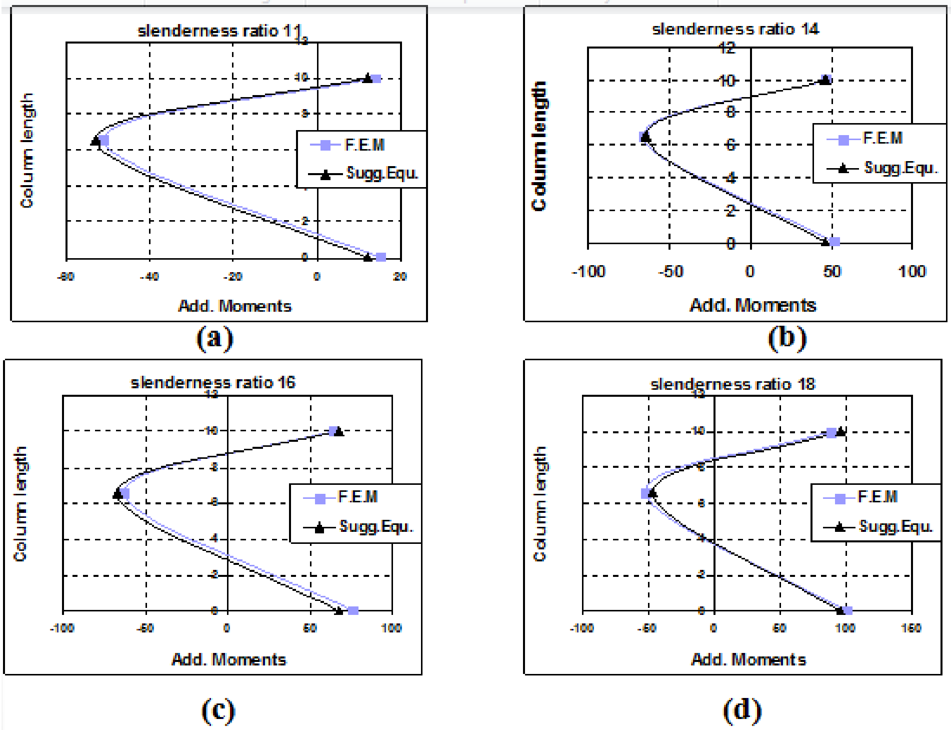


Fig. (18 a-d). The change of the slenderness ratio "for left column".

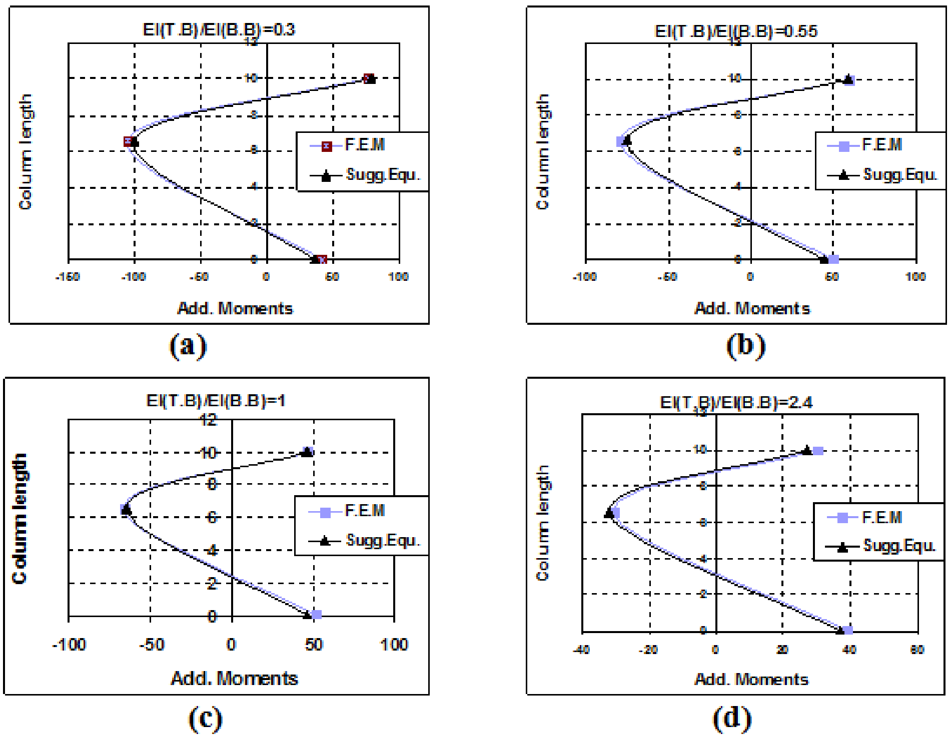


Fig. (19 a-d). The change in the rigidity of the upper beam to the lower beam

From the results in Figs (16 to 19), it was observed that the suggested equation gives matching results compared with that of finite element method. By the suggested equation, we can find the additional moments at the mid length of slender columns and at the connection between them and the connected beams. Comfortably, this equation can be modified in future to consider stiffness reduction due to the cracks and the yielding.

4. Conclusions

The following conclusions have been drawn out of the presented study:

- Approximate equation for computing the additional moments in braced slender columns was suggested in this paper. This equation was proved in elastic analysis and for the cases of single curvature in the slender column.
- The additional moments which are induced in the slender column were considered, not only in the mid span of the columns, but also the rigidity of the connected beams and the induced additional moments between them and the columns were also considered.
- The suggested equation gave matching values of the induced additional moments of the slender columns compared with the results of finite element in elastic analysis.
- The suggested equation in this paper is as a prelude for the modified equation in near future. The stiffness reduction in the column and the rotational springs due to the cracking and yielding will be taken into account in the modified equation. Then, the results will be compared with accredited equations in different codes.

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تحليل العزوم الإضافية فى الأعمدة النحيفة المقيدة جانبيا
الجزء الأول : معادلة مقترحة لحساب العزوم الإضافية في الأعمدة النحيفة

الملخص العربي

هذه الدراسة اختصت بتحليل العزوم الإضافية في الأعمدة النحيفة والناتجة من فعل التشكلات الحادثة مع القوة الداخلية العمودية. هذا البحث يختص بنقطتين رئيسيتين. النقطة الأولي وهى إلقاء الضوء على المعادلات الموصى بها في الكودات المختلفة والتي من خلالها يتم الحصول على العزوم الإضافية المتولدة في الأعمدة النحيفة وكيفية التعامل مع هذه المعادلات والفروض والأسس التي بنيت عليها هذه المعادلات. أما النقطة الثانية في هذا البحث هي اقتراح معادلة تقريبية يتم من خلالها حساب العزوم الإضافية المتولدة في الأعمدة النحيفة. هذه المعادلة تأخذ في الاعتبار الجساءة الدورانية للكمرات بالنسبة للأعمدة والعزوم الإضافية المتولدة عند اتصال الأعمدة بهذه الكمرات. هذه المعادلة المقترحة قد تم إثباتها في هذا البحث على أساس التحليل المرن وفي الحالات ذات الانحناء الأحادي للأعمدة. وقد تم التحقق من كفاءة هذه المعادلة بحل العديد من الأمثلة والحصول على نتائج متوافقة مع التحليل بطريقة العناصر المحدودة.