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COORDINATES TRANSFORMATIONS OF THE PANORAMIC PHOTOGRAPHS ONTO A SPHERE

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Photography of the Earth onto tangent and tilted planes and its transformation to panoramic photographs from an external exposure station, if done mathematically, is useful for interpreting the photos taken by a camera mounted on a space vehicle located anywhere in space and pointed towards any part of the Earth. General formulas for vertical and oblique photographs have been established which will be used to find the panoramic coordinates for different points on the Earth. Inverse formulas of transformation are included. A numerical example is given.

***KEYWORDS**: Panoramic coordinates, panoramic photos, transformation.*

INTRODUCTION

A panoramic photograph is a picture of a strip of terrain taken transverse to the direction of flight. The exposure is made by a specially designed camera which scans laterally from one side of the flight path to the other. The lateral scan angle may be as great as 180^o, in which case the photograph contains a panorama of the terrain from horizon to horizon.

By observation of parts of the Earth from certain outer point, which is the camera lens and whose altitude h above the Earth, we can determine their geographic coordinates ϕ and λ, from knowing their coordinates on panoramic photos. Special case of polar panoramic photos is given also. A single numerical example is given for all the forward and inverse coordinate transformations.

The Coordinates onto the Tangent Plane

Figure (1) shows the geometry of the oblique panoramic photograph of the reference sphere with radius R, the camera lens L is at altitude h above the earth. The principal axis LO is perpendicular to the sphere and intersecting its surface at point P_o "principal point" at geodetic latitude ϕ_o and geodetic longitude λ_o. Through P_o a tangent plane T to the sphere is drawn. A central projection of a point P (ϕ ,λ) on the sphere onto T is formed by mapping the point with the intersection P_1 of the line LP with T.

At first the relative coordinates of P to P_o are to be calculated. They are:
The zenithal distance measured as the arc P_oP and represented by the central angle δ = angle (P_oOP), and the geographic azimuth α; that is the spherical angle NP_oP at P_o, measured from the meridian λ_o.

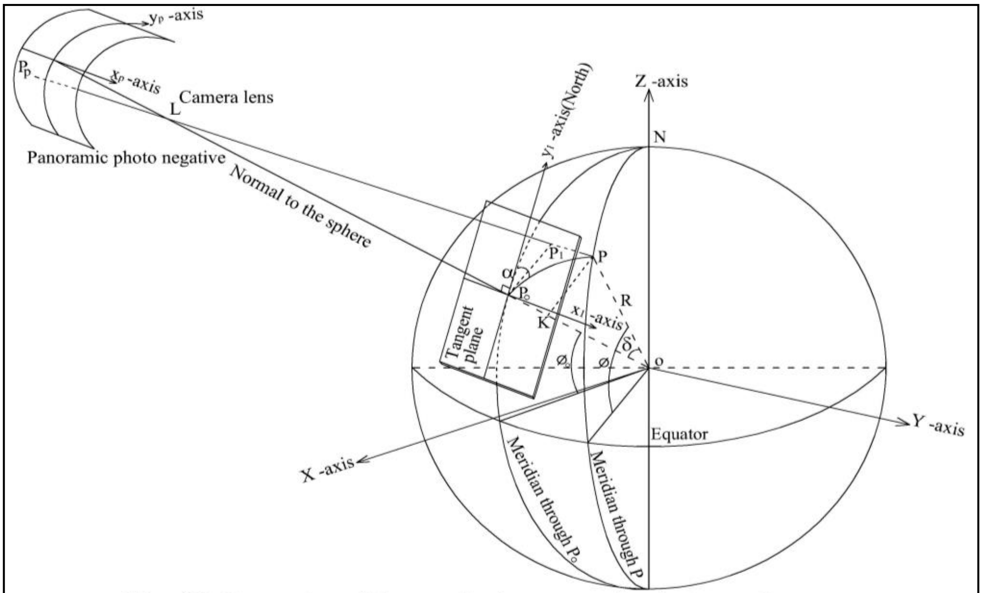

Fig. (1) Geometry of the vertical panoramic photograph

In the tangent plane *T*, let the assumed system of rectangular coordinates (x_1, y_1) be such that the y_1-axis coincides with the central meridian through P_o and directed toward the North, while the x_1-axis is perpendicular to the y_1 -axis, positive east.

Figure (2) shows the polar coordinates (r, α) of the point P_1 onto the plane *T*.

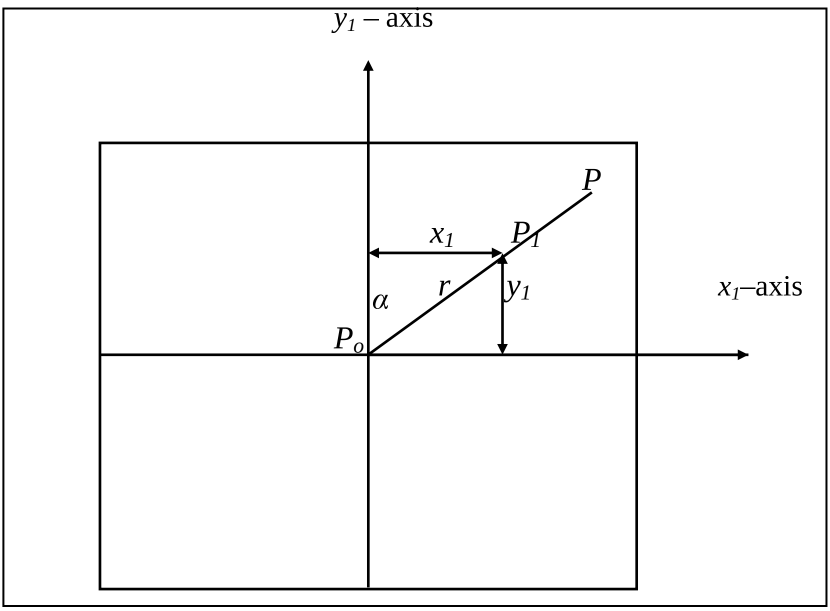

Fig. (2) Systems of rectangular and polar coordinates of the plane *T*.

Since α is the geographic azimuth, and r is the distance P_oP_1 (the projection of arc P_oP on the plane), the coordinates (x_1, y_1) are determined as follows:

In the plane of arc P_oP, defined by the rays LP_oO and LP_1P, and due to the similarity of the triangles LPK and LP_1P_o, we get:

$$\frac{r}{R\ \sin\delta} = \frac{h}{h + R\,(1 - \cos\delta)} \tag{1}$$

Since, $x_1 = r \sin\alpha$

and, $y_1 = r \cos\alpha$,

then substituting from equation (1), we obtain:

$$x_1 = \frac{h\ \sin\delta\ \sin\alpha}{(1 + \frac{h}{R}) - \cos\delta} \tag{2}$$

$$y_1 = \frac{h\ \sin\delta\ \cos\alpha}{(1 + \frac{h}{R}) - \cos\delta} \tag{3}$$

Applying the sine law to the spherical triangle NPP_o, shown in Figure (3), we get:

$$\frac{\sin d\lambda}{\sin\delta} = \frac{\sin\alpha}{\sin(\frac{\pi}{2} - \phi)} = \frac{\sin\alpha}{\cos\phi},$$

hence, $\sin\delta \sin\alpha = \cos\phi\ \sin d\lambda$. (a)

$\sin\delta\cos\alpha = \sin\phi\ \cos\phi_o - \cos\phi\ \sin\phi_o \cos d\lambda$ (b)

The cosine formula for the side δ gives:

$\cos\delta = \sin\phi\ \sin\phi_o + \cos\phi\ \cos\phi_o \cos d\lambda$ (c)

Substituting a, b & c into (2) and (3), we obtain; after some algebraic reductions:

$$x_1 = h\,\frac{C \sin d\lambda}{G - S \sin\phi_o - C \cos\phi_o \cos d\lambda} \tag{4}$$

$$y_1 = h\,\frac{S \cos\phi_o - C \sin\phi_o \cos d\lambda}{G - S \sin\phi_o - C \cos\phi_o \cos d\lambda} \tag{5}$$

Where, $C = \cos\phi$ (6)

$S = \sin\phi$ (7)

$G = (1 + \frac{h}{R})$ (8)

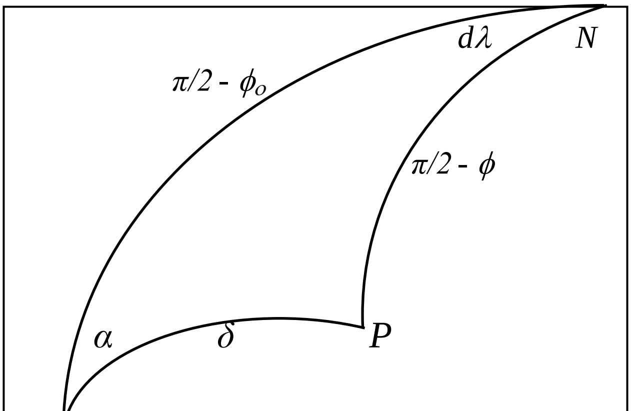

Fig. (3) The spherical triangle NPP_o

These equations represent the plane rectangular coordinates of the projection of any point on the sphere onto the tangent plane in terms of the geodetic coordinates (ϕ , λ). Replacing $d\lambda$ by its value ($\lambda - \lambda_o$), that is λ for P and λ_o for P_o , we obtain the general formulas of the coordinates for the image of a point on the sphere onto a tangent plane:

$$x_1 = h \frac{C \cos\lambda_o \sin\lambda - C \sin\lambda_o \cos\lambda}{G - S \sin\phi_o - C \cos\phi_o \cos\lambda_o \cos\lambda - C \cos\phi_o \sin\lambda_o \sin\lambda} \quad (9)$$

$$y_1 = h \frac{S \cos\phi_o - C \sin\phi_o \cos\lambda_o \cos\lambda - C \sin\phi_o \sin\lambda_o \sin\lambda}{G - S \sin\phi_o - C \cos\phi_o \cos\lambda_o \cos\lambda - C \cos\phi_o \sin\lambda_o \sin\lambda} \quad (10)$$

Rotation of the Coordinate Axes

We considered in the tangent plane that the y_1-axis is due North. This is not always the case, so we'll rotate the coordinate axes (x_1, y_1) in the tangent plane to the axes (x_2, y_2). The y_2–axis makes clockwise angle γ with the North direction. The new coordinates x_2 and y_2 of P_1 are:

$$x_2 = x_1 \cos\gamma - y_1 \sin\gamma \quad (11)$$

$$y_2 = x_1 \sin\gamma + y_1 \cos\gamma \quad (12)$$

Projection onto the Tilted Plane

Let P be projected through L onto a tilted plane I through x_2-axis. As shown in Figure (4), the tilted plane I makes an angle t with plane T. The coordinates (x_3 , y_3) of the projection P_2 in the tilted plane I, with the origin of axes at P_o (ϕ_o , λ_o) are to be found. Let the x_3-axis lie at the intersection of the two planes T and I (i.e. $x_3 = x_2$), then the y_3-axis lies in the principal plane LP_oy_2. The rectangular coordinates (x_3 , y_3) of P_2 on the plane I depend on the coordinates (x_2 , y_2). From the triangle $P_1P_2P_o$, we get [8]:

$$y_3 = h\ y_2 / (h \cos t + y_2 \sin t) \quad (13)$$

$$x_3 = h \cos t\ x_2 / (h \cos t + y_2 \sin t) \quad (14)$$

Translation and Rotation of the Axes onto the Tilted Plane

On the inclined space-photographs, P_o can't be precisely located, while center V_I along the optical axis, can be located if the picture has been adjusted for this aim. So the need arises to translate the origin of the axes from P_o to V_I along the ordinate axis $P_o y_3$, as shown in Figure (4).

Consequently, all ordinates from equation (13) become shorter by distance $(P_oV_I = h \sin t)$, while the abscissas remain unchanged. The point P_2 has now the translated coordinates x_4 & y_4 as follows:

$$x_4 = x_3 \quad (15)$$

$$y_4 = y_3 - h \sin t \quad (16)$$

If the x_4 , y_4 coordinates are rotated by an angle θ around V_I in the plane I, and if the x and y are the new axes, as shown in Figure (4), then the rectangular coordinates of P_2 related to this system, become:

$$x = x_4 \cos\theta - y_4 \sin\theta \quad (17)$$

$$y = x_4 \sin\theta + y_4 \cos\theta \quad (18)$$

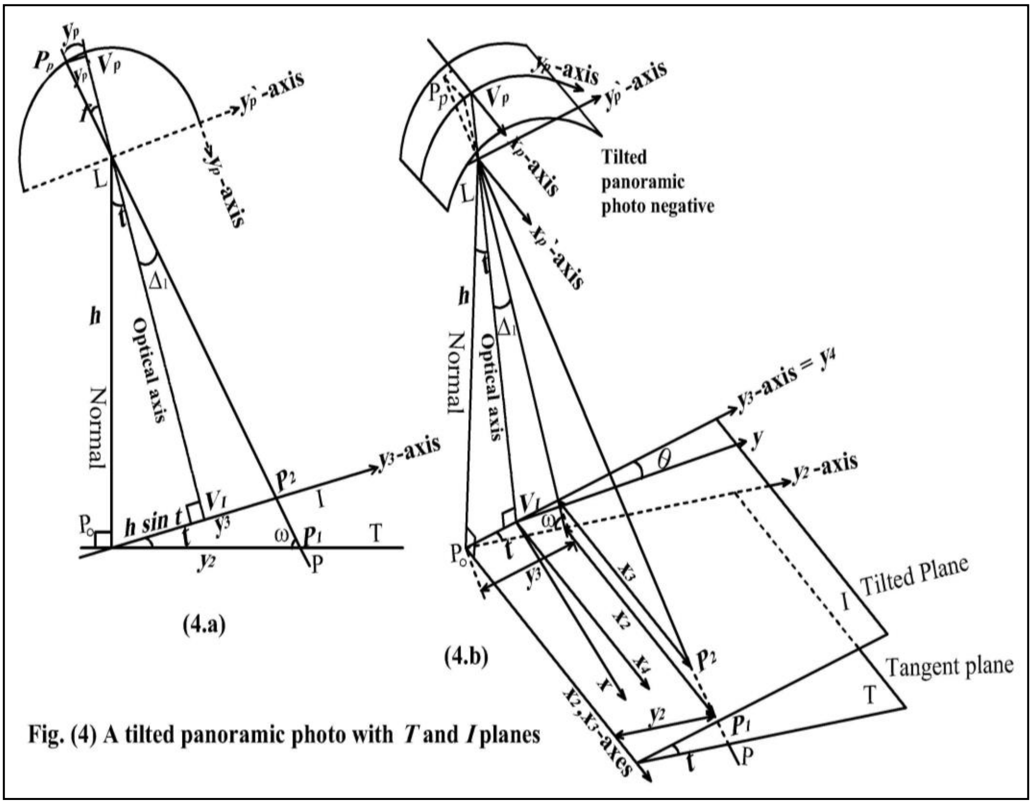

Fig. (4) A tilted panoramic photo with *T* and *I* planes

After we group the constants terms from those containing ϕ and λ, we reach to the synthetical formulas of all the previous changes of the whole system of photos of points on the sphere onto a plane indicated when we know the parameters:

$$x = \frac{c_1\ X + c_2\ Y + c_3\ Z + c_4}{c_5\ X + c_6\ Y + c_7\ Z + 1} \qquad (19)$$

$$y = \frac{c_8\ X + c_9\ Y + c_{10}\ Z + c_{11}}{c_5\ X + c_6\ Y + c_7\ Z + 1} \qquad (20)$$

Where, $X = C \cos \lambda$

$Y = C \sin \lambda$

$Z = S$, where C, S are $\cos\phi$ and $\sin\phi$ respectively.

The expressions of the eleven parameters required to calculate the rectangular coordinates on the space-photographs or its mathematical model, x and y, when the geographical coordinates ϕ & λ are given, are as follows:

$c_1 = [\ (K_2 \cos\theta - K_6`\sin\theta)\ h \cos t\ ] / K_{12}$

$c_2 = [\ (K_1 \cos\theta - K_5`\sin\theta)\ h \cos t\ ] / K_{12}$

$c_3 = [\ (K_3 \cos\theta - K_7`\sin\theta)\ h \cos t\ ] / K_{12}$

$c_4 = -K_8\, h\ \sin\theta\ \cos t\ / K_{12}$

$c_5 = K_{10} / K_{12}$

$c_6 = K_9 / K_{12}$

$c_7 = K_{11} / K_{12}$

$c_8 = [\ (K_2 \sin\theta + K_6`\cos\theta)\ h \cos t\ ] / K_{12}$

$c_9 = [\,(K_1 \sin\theta + K_5\grave{}\cos\theta)\, h \cos t\,] / K_{12}$

$c_{10} = [\,(K_3 \sin\theta + K_7\grave{}\cos\theta)\, h \cos t\,] / K_{12}$

$c_{11} = K_8\ h\ \cos\theta\ \cos t\ / K_{12}$

where, $K_1 = \cos\lambda_o \cos\gamma + \sin\phi_o \sin\lambda_o \sin\gamma$

$K_2 = -\sin\lambda_o \cos\gamma + \sin\phi_o \cos\lambda_o \sin\gamma$

$K_3 = -\cos\phi_o \sin\gamma$

$K_5 = \cos\lambda_o \sin\gamma - \sin\phi_o \sin\lambda_o \cos\gamma$

$K_6 = -\sin\lambda_o \sin\gamma - \sin\phi_o\ \cos\lambda_o \cos\gamma$

$K_7 = \cos\phi_o \cos\gamma$

$K_8 = -G \sin t$

$K_9 = K_5 \sin t - \cos\phi_o \sin\lambda_o \cos t$

$K_{10} = K_6 \sin t - \cos\phi_o \cos\lambda_o\ \cos t$

$K_{11} = K_7 \sin t - \sin\phi_o\ \cos t$

$K_{12} = G \cos t$

$K_5\grave{} = K_5 \cos t + \cos\phi_o \sin\lambda_o \sin t$

$K_6\grave{} = K_6 \cos t + \cos\phi_o \cos\lambda_o\ \sin t$

$K_7\grave{} = K_7 \cos t + \sin\phi_o\ \sin t$

In equations (19) and (20), the constants in the denominators are dimensionless, but those in the numerators have the same units as *h* and *R*. By these formulas as well as with their numberless variants, we can transform the geographical coordinates of points onto the Earth or any planet in rectangular coordinates of the corresponding point on the space-photographs or pseudo-space-photographs.

FINAL COORDINATES TRANSFORMATION TO THE TILTED PANORAMIC PHOTOGRAPH

In the tilted panoramic photo taken from the exposure station *L* , the $y_p\grave{}$-axis is taken through *L* parallel to the *y* –axis in the tilted plane *I*, and the $x_p\grave{}$-axis is perpendicular to $y_p\grave{}$-axis in the direction of flight, as shown in Figure (4), the coordinates $(x_p\grave{}, y_p\grave{})$ are determined as follows:

$$y_p\grave{} = f\ \sin\left(\tan^{-1}\left[\frac{y}{h \cos t} \right] \right) \qquad (21)$$

and by similarity, we find:

$$x_p\grave{} = \frac{x}{y}\ f\ \sin\left(\tan^{-1}\left[\frac{y}{h \cos t} \right] \right) \qquad (22)$$

where, f = the camera focal length.

Equations (21) and (22) permit the computation of the coordinates $(x_p\grave{}, y_p\grave{})$ of the point P_p in terms of the coordinates (x, y) of the point P_2 onto the tilted plane *I*.

If we develop the cylindrical surface of the panoramic film, then any point P_p on the panoramic photo will have the coordinates (x_p, y_p), as shown in Figure (4), the x_p-axis is taken parallel to the $x_p\grave{}$-axis in the direction of flight passing through the position of zero scan angle, and the y_p-axis is taken along the trace of the intersection of the camera lens optical axis with the film during scanning, so we have:

$$x_p = x_p\grave{} \qquad (23)$$

$$y_p = f\ (\pi / 180^o)\,\Delta_1 = f\ (\pi / 180^o)\,\sin^{-1}(y_p\grave{} / f) \qquad (24)$$

Δ_1 is in degree units.

These equations expressing the panoramic coordinates (x_p, y_p) in terms of the coordinates $(x_p`, y_p`)$.

By substituting from equations (21), (22) into (24), (23), we get:

$$x_p = \frac{x}{y} f \sin(\tan^{-1}[\frac{y}{h \cos t}]) \tag{25}$$

$$y_p = \frac{f \pi}{180^o} \tan^{-1}[\frac{y}{h \cos t}] \tag{26}$$

Equations (25), (26) permit the computation of the panoramic coordinates (x_p, y_p) of the point P_p in terms of the coordinates (x, y) of the point P_2. It is to be noticed that, a parallel displacement of the photo in the direction of the optical axis change only the scale of the photograph.

Inverse Transformations

To calculate the coordinates (x, y) of the tilted plane I in terms of the coordinates (x_p, y_p) of the point P_p in the panoramic photo, from equation (26) by solving it for y, we obtain:

$$y = h \cos t \ \tan[\frac{180^o y_p}{f \pi}] \tag{27}$$

and from equation (25) by solving it for x, we get:

$$x = \frac{y x_p}{f \sin(\tan^{-1}[\frac{y}{h \cos t}])} \tag{28}$$

Substituting from equations (11) through (18), we obtain:

$$x_1 = h \frac{D_1 y + D_2 x + D_3}{h \cos^2 t + (x \sin \theta - y \cos \theta) \sin t} \tag{29}$$

$$y_1 = h \frac{D_4 y + D_5 x + D_3}{h \cos^2 t + (x \sin \theta - y \cos \theta) \sin t} \tag{30}$$

Where,
$D_1 = \sin \gamma \cos t \cos \theta + \cos \gamma \sin \theta$
$D_2 = \cos \gamma \cos \theta - \sin \gamma \cos t \sin \theta$
$D_3 = h \sin \gamma \cos t \sin t$
$D_4 = \cos \gamma \cos t \cos \theta - \sin \gamma \sin \theta$
$D_5 = -(\cos \gamma \cos t \sin \theta + \sin \gamma \cos \theta)$
$D_6 = h \cos \gamma \cos t \sin t$

Knowing x_1 and y_1, we can obtain the geodetic latitude ϕ and longitude λ, as follows:

I. Geodetic latitude ϕ

From the spherical triangle NPP_o in Figure (3), by cosine law, we obtain:

$$\phi = \sin^{-1}[\sin\phi_o \cos \delta + (y_1 / r) \cos\phi_o \sin \delta] \tag{31}$$

II. Geodetic longitude λ

From the spherical triangle NPP_o, using the sine and cosine laws, and with some algebraic reductions, we obtain:

$$\lambda = \lambda_o + \tan^{-1}\left(\frac{x_1}{r\cos\phi_o \cot\delta - y_1 \sin\phi_o}\right) \tag{32}$$

Where, $$\sin\delta = \frac{G + \sqrt{1 - \frac{r^2}{hR}(G+1)}}{\frac{r}{h} + \frac{h}{r}} \quad , \quad r = \sqrt{x_1^2 + y_1^2}$$

Special case

Polar panoramic photos (origin at *N*) with γ = 0 of the sphere

The polar panoramic photographs are simple form of the general case in which the origin P_o at the pole N, where $(\phi_o = \pi / 2)$, as shown in Figure (5).

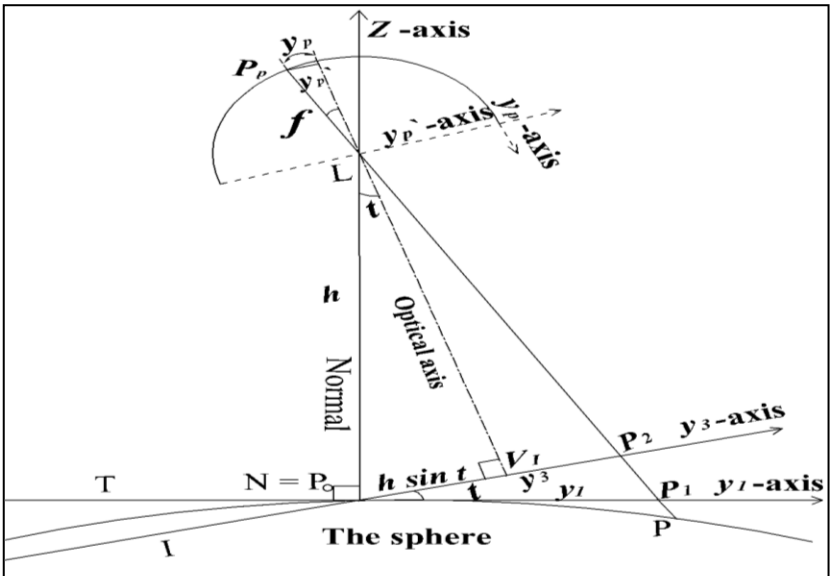

Fig. (5) Polar panoramic photo with $\gamma = 0$

The Coordinates onto the Tangent Plane T

The coordinates (x_1, y_1) in terms of (ϕ, λ) in equations (4) and (5) become:

$$x_1 = h\,\frac{C\,\sin(\lambda - \lambda_o)}{G - S} \tag{33}$$

$$y_1 = -h\,\frac{C\,\cos(\lambda - \lambda_o)}{G - S} \tag{34}$$

The Coordinates onto the Tilted Plane I

The coordinates (x_3, y_3) in terms of (ϕ, λ) become:

$$x_3 = h\cos t\,\frac{C\sin(\lambda - \lambda_o)}{(G - S)\cos t - C\sin t\,\cos(\lambda - \lambda_o)} \tag{35}$$

$$y_3 = -h \frac{C \cos(\lambda - \lambda_o)}{(G - S)\cos t - C \sin t \cos(\lambda - \lambda_o)} \quad (36)$$

Coordinates Transformation to a Tilted Polar Panoramic Photo

We can obtain the coordinates $(x_p`, y_p`)$ and (x_p , y_p) on the panoramic photo in terms of the tilted coordinates (x_3 , y_3) of the point P_2 , as shown in Figure (5):

$$y_p` = f \sin(\tan^{-1} [\frac{y_3 - h \sin t}{h \cos t}]) \quad (37)$$

$$x_p` = \frac{x_3}{y_3 - h \sin t} f \sin(\tan^{-1} [\frac{y_3 - h \sin t}{h \cos t}]) \quad (38)$$

and by substituting into (23), (24), we obtain:

$$x_p = \frac{x_3}{y_3 - h \sin t} f \sin(\tan^{-1} [\frac{y_3 - h \sin t}{h \cos t}]) \quad (39)$$

$$y_p = \frac{f \pi}{180^o} \tan^{-1} [\frac{y_3 - h \sin t}{h \cos t}] \quad (40)$$

Numerical Example

An example is given to calculate all the forward and inverse coordinates with the help of the equations. Given:

R = 6371000 m ,h = 10000 m ,$\phi_o = 25^o$ N. ,$\lambda_o = 0$,$\phi = 30^o$ N. ,$\lambda = 30^o$ E.

$\gamma = 40^o$ clockwise from the North, $t = 20^o$,$\theta = 20^o$ clockwise from the positive y_4–axis. To find x_1 and y_1 from the given ϕ and λ , equations (4) to (8) are used and we get:

$C = \sqrt{0.75}$, $S = 0.5$, $G = 1.001569612$

$x_1 = 39176.16101$ m , $y_1 = 12321.59951$ m

By rotating the axes an angle $\gamma = 40^o$ clockwise from the North, the coordinates x_2 and y_2 from equations (11) and (12) are:

$x_2 = 22090.50895$ m , $y_2 = 34620.84373$ m

To find x_3 and y_3 , using equations (13) and (14), we get:

$x_3 = 9774.147768$ m , $y_3 = 16301.40397$ m

To find x_4 and y_4 from (x_3 , y_3), using equations (15) and (16) we get:

$x_4 = 9774.147768$ m , $y_4 = 13881.20253$ m

To find x and y from (x_4 , y_4) for the given ϕ & λ , using equations (17) to (20) we have:

$x = 4779.063795$ m , $y = 15447.32639$ m

To calculate the panoramic coordinates $(x_p` , y_p`)$ and (x_p , y_p) from x and y , $f = 0.15$ m, equations (21) to (26), yield:

$x_p` = 0.039647146$ $m$ , $y_p` = 0.128151127$ m

$x_p = 0.03964714$ m , $y_p = 0.15364218$ m

To calculate the inverse coordinates (x , y) from the coordinates (x_p , y_p), using equations (27) and (28), we get:

$y = 15447.326\ m$, $x = 4779.0638\ m$

and from equations (29) and (30), we get:

$x_1 = 39176.161\ m$, $y_1 = 12321.6\ m$

To find the inverse coordinates ϕ and λ from x_1 and y_1, equations (31) and (32) yield:

$\phi = 30^o$, $\lambda = 30^o$

with, $r = 41068.15562\ m$, $\sin\delta = 0.45392485037$, $\delta = 26^o\ 59`\ 44.8``$

CONCLUSIONS

In this paper, photography of the celestial bodies and transformations that to panoramic photographs are discussed. This resembles the actual photographs taken by space vehicles. We considered the Earth as a sphere.

The resulting formulas for coordinates transformations x_1 and y_1 were deduced in terms of the geodetic coordinates. General formulas are established which are used to find the panoramic coordinates for different points on the Earth. The axes must be rotated clockwise angle γ with the North direction to allow any azimuth.

The Inverse transformations permit the calculations of ϕ and λ for any selected point on the photo. Special case of the polar panoramic photographs of the sphere, are derived. Finally we checked the accuracy of all the forward and inverse formulas for coordinates transformations by a numerical example.

REFERENCES

1. Wolf P. R., "Elements of photogrammetry", McGraw-Hill, Inc., 1974.
2. Maling D. H., "Coordinate Systems and Map Projections", Second Edition, Pergoman Press, Oxford, New York, 1992.
3. Maling D. H., "Measurements from Maps ", Pergoman Press, Oxford, New York, 1989.
4. Francis H. M., Edward M. M., "Photogrammetry", Third Edition, Harper and Row Inc., New York, 1980.
5. Borkowski K. M., "Transformation of Geocentric to Geodetic Coordinates without Approximations", D. Reidel Publishing Company, Astrophysics and Space Science 139, pp.1-3, 1987.
6. Schmid E., "The Earth as Viewed From Satellite", United States Government Printing Office, Washington, Technical Bulletin No.20, pp.1-21, 1962.
7. Tomas S., Larry D., "Coordinate System Used in Geodesy: Basic Definitions and Concepts", ASCE, Journal of Surveying Engineering, Vol.114, No.2, May, 1988.
8. Youssef A. , "Triple projection of a topographic surface from an external perspective center", Ph.D Thesis, Civil Engineering Department, Faculty of Engineering, Assiut University , Assiut , Egypt ,1993.

" تحويل الإحداثيات للصور البانورامية على كرة "

الصورة البانورامية هي صورة لشريحة من سطح الأرض مأخوذة عرضياً لاتجاه الطيران، حيث يتم التصوير بواسطة كاميرا مصممة بطريقة خاصة للتصوير العرضي من أحد جانبي ممر الطيران إلى الجانب الآخر. حيث أن زاوية التصوير العرضي قد تصل إلى 180^{o}, لذلك فهي تغطي مساحة كبيرة من سطح الأرض مقارنة بالصورة العادية.

و في الصور البانورامية نستخدم فقط الجزء المركزي من عدسة الكاميرا, لذلك فإن درجة الوضوح للصور البانورامية أكبر من مثيلتها للصور العادية.

و تمثيل سطح الأرض على مستويات أفقية و مائلة (مستويات الصورة) و تحويل ذلك إلى صورة بانورامية في الحالة العامة، إذا تم صياغته حسابيا سيكون نافعاً في تفسير و تحليل الصور المأخوذة من كاميرات مثبتة في المركبات الفضائية أو الأقمار الصناعية المتحركة في الفضاء و الموجهة إلى أي مكان على سطح الأرض، و معرفة الإحداثيات الجيوديسية لهذا المكان بمعلومية الإحداثيات البانورامية من الصورة.

و يشتمل البحث على المعادلات العكسية للتحويل بين الإحداثيات. و يشتمل أيضاً على حالة خاصة للصور البانورامية القطبية للأرض. وأخيراً التعويض بقيم عددية في المعادلات العادية و العكسية للتحقق من صحتها.