INSTANTANEOUS FREQUENCY ESTIMATION ALGORITHM FOR POWER SYSTEMS

Received 15 July 2021; Revised 18 October 2021; Accepted 22 October 2021

Abstract
This paper deals with the problem of the Instantaneous Frequency (IF) estimation of power systems, to be adopted for example in Phasor Measurement Units (PMUs) used for monitoring, control, and protection. The main target is to provide an accurate estimation of power system frequency in real time with minimum delay. We introduce a novel algorithm based on Gabor transform for the estimation of the instantaneous frequency. Also, we review a number of frequency estimation algorithms dealt with in the literature, namely the Zero Crossing algorithm, The Three-level Discrete Fourier Transform algorithm and the Differential Evolution algorithm. Simulation tests are made to compare the relative performance of the 4 algorithms. These tests include stationary frequency, the tracking of a changing frequency, the case when both frequency and amplitude are time varying, and also when the signal contains harmonics, white noise or DC component. Simulation tests revealed that under these conditions Gabor Algorithm provided the lowest Root Mean Square Error (RMSE) almost in all cases. It takes Gabor Algorithm a frame of 4 cycles or less with a sampling rate of 200samples/s to estimate the instantaneous frequency precisely. By overlapping the frames an accurate estimation can be even deduced each cycle. Zero RMSE is achieved by Gabor algorithm for stationary frequency case, under the above conditions of sampling rate and number of cycles.

1. Introduction

The instantaneous frequency is one of the most important quantities in the operation of power systems. It is an important operational parameter about the power system safety, stability and efficiency to provide the monitoring, protection and control of power systems especially the power systems that need prerequisite frequency estimation for rapid-response applications like load shedding and generator protection. This paper proposes the use of the Gabor transform for the frequency estimation, and compares this algorithm with the known methods, namely the Zero Crossing (ZC), the Three Levels DFT (3-Level DFT) and the Differential Evaluation (DE).

A. C. F. Aziz\textsuperscript{1}  
H. Selim \textsuperscript{2}  
M. Nayel \textsuperscript{3}

Keywords
Instantaneous Frequency, Phasor Measurement Unit, Gabor Transform.

\textsuperscript{1,2,3} Electrical Engineering Department, Faculty of Engineering, Assiut University, Assiut, Egypt
The zero-crossing algorithm is one of the simplest ways to estimate the power system frequency by measuring the time interval between consecutive zero crossings. In addition, the three-level discrete Fourier transform (DFT) method for frequency estimation is to provide an accurate estimate of power system frequency in real time by three levels. In the first level, the signal will be decomposed into two orthogonal signals one is cosine and the other sine and then both are filtered. The second and third levels are used to determine the amplitude ratio of the cosine- and sine-filtered signals. The Differential Evaluation Algorithm is one of the most recent populations based stochastic evolutionary optimization techniques for minimizing non-linear functions.

The measured signals in real power systems typically contain harmonic distortion, which may introduce significant errors, thus one of the tests introduces the 3rd, 5th, 7th harmonics. The remainder of this paper is organized as follows: Section I reviews of the instantaneous frequency estimation algorithms. In Section II, the performance evaluation simulation tests are detailed. In section III the performance evaluation test results of the four algorithms are compared. The conclusion is drawn in Section IV.

2. Literature Review

Many researchers have dealt with the topic of the estimation of the instantaneous frequency of power systems. Together with the Zero Crossing algorithm \cite{2, 4, 6}, the Three Levels DFT algorithm \cite{3}, and the Differential Evaluation algorithm \cite{7, 8, 9} which will be dealt with extensively in this paper, many other algorithms appear in the literature. In \cite{12} Szafrane et al. suggested the application of orthogonal signal components obtained with the use of 2 orthogonal FIR filters. This algorithm ensures 1.5 mHz accuracy of estimation over typical (±2 Hz) range of measured frequency deviation but needs a delay of 80ms. A time-domain based power system frequency estimation algorithm is proposed in \cite{13}. It however needs signal de-noising and high frequency components removal. In \cite{14} a Viterbi algorithm has been applied to the cubic phase function and chirp-rate estimation. By a sampling rate of 1/128 sample/s it needs 256 samples i.e. 2 seconds for the estimation. In \cite{15} an algorithm is introduced based on the consideration of the relationship among the samples within every four consecutive sliding windows and the use of the Wiener filtering approach and an adaptive filter trained by the least mean square (LMS) algorithm. The accuracy of the scalar frequency estimator however depends strongly on the initial phase of the sine wave.

3. Instantaneous Frequency Estimation Algorithms

3.1. Gabor Algorithm

Gabor transform is a special case of the short-time Fourier transform which is used to determine the sinusoidal frequency of a signal. A power signal can be defined by its magnitude, phase and frequency (A, θ and ω):

\[ f(t) = A\cos(\omega t + \theta) \]

It can be represented as a complex number with magnitude and phase (A, and θ).

\[ F = A e^{j\theta} = A (\cos(\theta) + j\sin(\theta)) \]

Gabor transform is usually used in the non-stationary signals as in signal processing.
Use of the Gabor Transform allows the determination of the sinusoidal frequency of local sections of a signal. Some parameters are to be adjusted such as the number of cycles (window or frame width) and the sampling frequency to mitigate noise effects [1].

3.1.1. Fundamentals of Gabor
First, the signal to be transformed is multiplied by Blackman window function. The window function assures that the signal being analysed will have higher time weight as in fig (1)

![Fig 1. Blackman window](image)

The Blackman window exhibits an even lower maximum stop band ripple. Blackman windows are defined as:

$$\alpha[n] = \begin{cases} a_0 - a_1 \cos\left(\frac{2\pi n}{N}\right) + a_2 \cos\left(\frac{4\pi n}{N}\right) \\
0 \end{cases}$$

$$a_0 = \frac{1 - \alpha}{2}; \ a_1 = \frac{1}{2}; \ a_2 = \frac{\alpha}{2}$$

Second, from the resulting signal we can derive the time frequency analysis by Fourier transform. The Gabor transform of a signal $x(t)$:

$$G_x(\tau, w) = \int_{-\infty}^{\infty} x(t) e^{-\pi(t-\tau)^2} e^{-jw\tau} dt$$

A frame of the signal of a number of cycles is to be windowed with Blackman window, then the discrete Fourier transform is evaluated for the expected line frequency range namely from 49 to 51 Hz in steps of 0.01 Hz. The location of the peak of this transform will determine the signal sinusoidal frequency. Also, instead of evaluating the discrete Fourier transform at 200 values of the frequency, a rough resolution may be used at first, say in steps of 0.1 Hz to determine the approximate location of the peak, and then followed by the finer resolution of 0.01Hz around the peak to reach the instantaneous frequency of the signal.

3.2. Zero crossing
This method tracks the frequency or the period of a periodic signal by measuring the number of cycles of a reference signal in a certain time of the periodic signal. The mechanism of the zero crossing algorithms is by counting the number of crosses of the signal through zero in a certain period [4].

Zero crossing is the point of choice for measuring phase and frequency (time and phase) coordinates of the real and imaginary zero. The zero-crossing technique is one of the methods enabling to evaluate the delay time of propagating waves. The main idea of this technique is that using some threshold level, the half period of the signal exceeding this level is determined. This technique has a main feature that is no delay time because of its ability to reconstruct the segment of the phase velocity dispersion curves [6].
This reduces the errors caused by phase noise by making the perturbations in zero crossings small relative to the total period of the measurements. However, this results in slow measurement rates to obtain an accurate measurement. Zero crossing is the point of choice for measuring phase and frequency by selecting the closest one to zero samples, which have the best accuracy [2] [4].

3.3. Three level discrete Fourier transform

The Discrete Fourier Transform (DFT) is the equivalent of the continuous Fourier Transform for signals well known only at instants separated by sample times (i.e., a finite sequence of data). If a power system signal has a purely sinusoidal waveform with amplitude \( A \), power system frequency \( f \), and phase \( \theta \); it can be described in discrete time steps as:

\[
x(n) = A \cdot \cos \left( 2\pi \frac{f}{f_0} \frac{n}{N_0} + \theta \right)
\]

Where \( f_0 \) the nominal frequency and \( N_0 \) is i.e., the number of samples per cycle at \( f_0 \). The power system signal can be decomposed into two orthogonal signals via DFT, using cosine and sine filters. The coefficients of the cosine filter in the DFT are

\[
H_c(n) = \frac{2}{N_0} \cos \left( 2\pi \frac{n}{N_0} + \frac{n}{N_0} \right) \quad n = 0, ..., N_0 - 1
\]

The amplitude and phase response of the cosine filter can be found from [3]. First of all, the input signal preferred to pass through a sine filter instead of a cosine filter to suppress harmonics and inter-harmonics that might be included in the input signal of a power system. To determine this amplitude ratio, a second level DFT is applied to the output signals of the first level DFT, as shown in Figure (2) [3],[5],[11].

By the second and third levels of DFT, it provides four and six output signals respectively [3]. The resulting estimated frequency is implemented through these three levels. These frequencies levels are \( f_1 \), \( f_2 \) and \( f_3 \) steps known as ramp up, ramp down and a sinusoidal frequency variation to improve the estimated frequency to reach higher accuracy [3].

3.4. Differential Evaluation Algorithm

Differential evaluation (DE) is like genetic algorithm (GA) which was proposed by storm [7], it uses the crossover, mutation, and selection operators. In reference [8] the frequency is estimated accurately
by the method that shows its superiority on other methods such as fast Fourier transform [9]. Like in GA, first generation is initialized randomly, and further generations can be created through the application of evolutionary operator until a stopping criterion is reached. The optimization process has four basic operations namely: Initialization, Mutation, Crossover and Selection. DE starts with the number of populations of D-dimensional search variable vectors as:

\[ x^i = (x_{i1}, x_{i2}, ..., x_{iD}) \]

If the \( i^{th} \) parameter of the given problem has its lower and upper bound as \( x_{iL} \) and \( x_{iU} \), respectively, then we may initialize the \( i^{th} \) component of population members as:

\[ x_{i,j} = x_{iL} + \text{rand}(0,1) \cdot (x_{iU} - x_{iL}) \]

The following steps are iterative until stopping criteria. By our simulation, the differential evolution iteration took a long time so the stop criterion for all cases was chosen to be 500 iterations.

### 4. Tests used for the Performance Evaluation

The performance of these algorithms is evaluated by simulation tests under the following conditions:

#### 4.1. Stationary Signal by Off-nominal Frequencies

Stationary signal is a signal wave that is generated by keeping the time and spectral content value constant.

\[ V(t) = A \sin(2\pi ft + 0.3) \]  

Where \( A \) is a constant and the frequency is chosen to be: \( f = 49.25 \text{Hz} \).

#### 4.2. Tracking the frequency change

Here the frequency of the signal is time varying. This frequency change is chosen to be:

\[ f(t) = 45.6 + 3.6(1 + 0.4e^{-t} \cos(1.5 - 0.1) + 0.36e^{-0.7t} \cos(12t)) \]  

\[ t=0 \rightarrow 2 \text{sec} \]

#### 4.3. Both frequency and amplitude are time-varying:

\[ f(t) = 49.5 + \sin(2\pi t), \quad A(t) = 2 + 0.3 \cos(3\pi t) \]  

\[ t=0 \rightarrow 2 \text{ sec} \]

#### 4.4. Signal containing harmonics, noise, and dc component

##### 4.4.1. Signal containing 3rd, 5th, 7th harmonics

\[ v(t) = \sqrt{2} \sin(2\pi ft + 0.3) + 0.2\sqrt{2} \sin(6\pi ft) + 0.2\sin(10\pi ft) + 0.3\sqrt{2} \sin(14\pi ft) \]  

\[ f = 49.5 \text{ Hz}, \quad t=0 \rightarrow 0.3 \text{ sec} \]

##### 4.4.2. Signal containing exponential component

\[ v(t) = 0.5\frac{e^{-t}}{0.3} + 2 \sin(2\pi ft + \frac{\pi}{6}); \quad f=50\text{Hz}, \quad t=0 \rightarrow 1 \text{sec} \]
4.4.3. Signal containing white noise

\[ v(t) = A \sin (2\pi f t + 0.3) + \varepsilon \quad ; \quad f = 50 \text{ Hz}, \quad t = 0 \rightarrow 1 \text{ sec} \quad (12) \]

The parameter \( \varepsilon \) represents the noise that can be produced by a random function on MATLAB. The signal to noise ratio is \( \text{SNR} = 20 \log (1/0.01) = 40\text{dB} \).

5. Performance Evaluation using MATLAB

MATLAB was used to evaluate the performance of the four algorithms: Gabor, Zero-Crossing, Three-level DFT and Differential evolution. The results were compared with each other using root mean square error (RMSE).

**Fig. 3.** Simulation results by a stationary frequency \( f = 49.25\text{Hz} \)

(a) Gabor (b) Zero-crossing (c) Three Level DFT (d) Differential Evaluation (e) RMSE comparison of the 4 algorithms
Fig. 4. Simulation results for the frequency tracking test
(a) Gabor (b) Zero-crossing (c) Three Level DFT (d) Differential Evaluation
(e) RMSE comparison of the 4 algorithms
Fig. 5. Simulation results for time-varying frequency and amplitude test
(a) Gabor (b) Zero-crossing (c) Three Level DFT (d) Differential Evaluation
(e) RMSE comparison of the 4 algorithms
Fig. 6. Simulation results for Signal containing 3rd, 5th, 7th harmonics
(a) Gabor (b) Zero-crossing (c) Three Level DFT (d) Differential Evaluation
(e) RMSE comparison of the 4 algorithms
Fig. 7. Simulation results for Signal containing DC exponential component
(a) Gabor (b) Zero-crossing (c) Three Level DFT (d) Differential Evaluation
(e) RMSE comparison of the 4 algorithms
6. Results and Discussion
MATLAB was used to evaluate the performance of the four algorithms: Gabor, Zero-Crossing, Three-level DFT and Differential evolution. The results were compared with each other using root mean square error (RMSE).

5.1. Stationary Signal by Off-nominal Frequencies
The input signal is given by:

\[ V(t) = A \sin (2\pi ft + 0.3), \]
where \( A \) is a constant, at a stationary frequency \( f = 49.25 \text{Hz} \).

Fig. 3a depicts the result of using Gabor Algorithm to get the instantaneous frequency; thereby the number of cycles \( m=4 \) cycles (1 frame), and the sampling frequency is \( f_s=200 \text{ Hz} \). The estimated instantaneous frequency as found to be \( f=49.25 \text{ Hz} \), exactly equal to the input stationary frequency. The resulting RMSE is thus equal to zero. The Zero Crossing (fig.3b) shows a RMSE of 0.0401 by a frame length \( m=1.6 \) cycles, but with a sampling frequency \( f_s=25600\text{Hz} \).
By the same test signal the ‘Three Level DFT’ (fig.3c) scored a high RMSE of 0.4610 by m=1.6 cycles, and a sampling frequency fs=200 Hz. By the Differential Evaluation case (fig.3d), high-accuracy frequency estimate is obtained with nearly zero RMSE of around 3.1489e-06 depending on the random values taken at the start of the application of the algorithm. Hereby the frame length m=1.6 cycles, and the sampling frequency is fs=200 Hz. A comparison of the RMSE of the 4 algorithms is given in fig.3e.

It is however to be noted that all algorithms operate with a sampling frequency fs=200 sample/s, the Zero Crossing algorithm, however, needs a sampling of 25600 samples/s in order to be able to detect the zero-crossing instant with precision.

5.2. Tracking the frequency change:
The frequency is varying with time as in equation (8)

\[
f(t) = 45.6 + 3.6(1 + 0.4e^{-t}\cos(1.5t - 0.1) + 0.36e^{-0.7t}\cos(12t) \quad t=0 \rightarrow 2 \text{sec}
\]

By Gabor Algorithm (fig. 4a), the output frequency is seen to be almost the same as the input frequency. The calculated RMSE at a number of cycles m=3 and by a sampling frequency fs=200 Hz is equal to 0.0132. Fig. 4b depicts the behaviour of the Zero Crossing by this frequency tracking test. By m=1.6 and fs=25600 Hz, an unacceptable RMSE of 1.3020 is achieved.

By the ‘Three Level DFT’ (fig. 4c) the output frequency oscillated as the tracking input frequency giving an RMSE equal 0.6113 using a number of cycle m=1.6 and a sampling frequency fs of 200 Hz. Again, the Differential Evaluation (fig. 4d) resulted in an RMSE of 0.4710 when using a frame length of m=3 and a sampling frequency fs=200 Hz. A comparison of the RMSE of the 4 algorithms is given in fig.4e.

5.3. Both frequency and amplitude are time-varying:
In this case both frequency and amplitude are varying with time according to equation (9):

\[
f(t) = 49.5 + \sin(2\pi t) \quad \text{and} \quad A(t) = 2 + 0.3\cos(3\pi t) \quad t = 0 \rightarrow 2 \text{sec}
\]

An RMSE of 0.0016 results by the simulation of Gabor Algorithm for the case that both amplitude and frequency are varying (fig.5a). The number of cycles being m=2, and the sampling frequency = 200 sample/s. Again, the Zero Crossing the showed (fig.5b) an unacceptable RMSE of 1.3020 by m=1.6 and fs=25600 Hz.

By the ‘Three Level DFT’ (fig.5c) the estimated frequency average is 50.0652 and the RMSE=0.3737, which is not acceptable when compared with the other algorithms. It uses a frame length of m=2, and a sampling frequency of 200 Hz. The RMSE scored 0.4779 for the case of the Differential Evaluation (fig. 5d). Thereby the frame length m=3 and 200 samples/s were required. A comparison of the RMSE of the 4 algorithms is given in fig.5e.

5.4. Signal containing harmonics, noise, and dc component

5.4.1. Signal containing 3rd, 5th, 7th harmonics
According to equation (10):

\[
v(t) = \sqrt{2} \sin(2\pi ft + 0.3) + 0.2 \sqrt{2} \sin(6\pi ft) + 0.2 \sin(10\pi ft) + 0.3 \sqrt{2} \sin(14\pi ft)
\]

\[f = 49.5 \text{ Hz}, \quad t=0 \rightarrow 0.3 \text{ sec}\]
The Zero Crossing algorithm (fig. 6b) claims the lowest RMSE of 0.0382 but still needs a high sampling rate of 25600 samples/s to be able to detect the zero-crossing instant with precision. The second-best score of RMSE of 0.183 got the Gabor algorithm (fig. 6c). The RMSE for both the ‘Three Level DFT’ and the Differential Evaluation (fig.6 c, d) are unacceptably high. The comparison of the RMSE of the 4 algorithms is depicted in fig.6e.

5.4.2. Signal containing DC exponential component
As in equation (11):

\[ v(t) = 0.5 \frac{\sin t}{0.3} + 2 \sin(2\pi ft + \frac{\pi}{6}) \quad f = 50 \text{Hz}, \ t = 0 \rightarrow 1 \text{sec} \]

All 4 algorithms proved to be insensitive to this type of noise, and gave highly acceptable RMSE (fig. 7)

5.4.2.1. Signal containing white noise:
According to equation (12):

\[ v(t) = A \sin(2\pi ft + 0.3) + \varepsilon, \ f = 50 \text{Hz}, \ t = 0 \rightarrow 1 \text{sec} \]

the parameter \( \varepsilon \) represents the noise that can be produced by a random function on MATLAB.

The signal to noise ratio is \( \text{SNR} = 20 \log (1/0.01) = 40 \text{dB} \).

Except for the Differential Evaluation algorithm with a high RMSE of 1.927, all other algorithms have very acceptable error values (fig.8) However, when the number of cycles (m) and/or the sampling frequency (fs) are increased, we can get Zero RMSE by Gabor Algorithm, namely zero RMSE is reached by Gabor Algorithm by the following conditions:

1) By Signal containing harmonics: \( fs = 500, \ m = 8 \).
2) By Signal containing exponential component (DC): \( fs = 200, \ m = 8 \).
3) By Signal containing white noise: \( fs = 5000, \ m = 8 \).

Thereby it is to be noted that by overlapping the frames an accurate estimation can be even deduced each cycle.

7. Conclusions

As the results of simulation above show, Gabor Algorithm outperforms the other methods by Stationary Signal Frequencies, by Frequency Tracking, and when both frequency and amplitude are time-varying. It even results in zero RMSE by Stationary Frequencies. By Signal containing 3rd, 5th, 7th harmonics Zero Crossing is the best followed by the Gabor Algorithm. By the Signal containing exponential component the Three Level DFT was the best, but also Gabor Algorithm closely follows. By Signal containing white noise the Zero Crossing topped the race followed by the Gabor Algorithm.

Summing up, we can deduct from the simulation tests that the Gabor algorithm is reliable by all types of signal disturbances and by the frequency tracking, and it needs a frame m of 4 samples or less. Even by power signals containing harmonics, The RMSE reduces to zero if the sampling rate is raised to 500 sample/s. On the other hand, the other 3 algorithms exhibited unacceptable RMSE by one or more of the tests. Also, it is to be noted that the Zero crossing algorithm needs a sampling of 25600s amplexes/s in order to be able to detect the zero-crossing instant with precision.
Table 1. List of Abbreviation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-Level DFT</td>
<td>Three Level DFT</td>
</tr>
<tr>
<td>DE</td>
<td>Differential Evolution</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>GA</td>
<td>genetic algorithm</td>
</tr>
<tr>
<td>IF</td>
<td>Instantaneous Frequency</td>
</tr>
<tr>
<td>LMS</td>
<td>least mean square</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root mean square error</td>
</tr>
<tr>
<td>PMU</td>
<td>Phasor Measurement Unit</td>
</tr>
<tr>
<td>ZC</td>
<td>Zero Crossing</td>
</tr>
<tr>
<td>μPMU</td>
<td>Micro Phasor Measurement Unit</td>
</tr>
</tbody>
</table>

Declaration of Competing Interest

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[5] Sang-Hee Kang, Woo-Seok Seo * and Soon-Ryul Nam’A Frequency Estimation Method Based on a Revised 3-Level Discrete Fourier Transform with an Estimation Delay Reduction Technique,” 2020, J(3)(9), 2256


خوارزمية تقدير التردد الفوري لأنظمة الطاقة

الملخص بالعربية:

تتناول هذا البحث موضوع تقدير التردد اللحظي (IF) لنظام الطاقة الذي سيتم استخدامه على سبيل المثال في وحدات قياس الطور (PMUs) المستخدمة للمراقبة والتحكم والحماية.

الهدف الرئيسي هو عمل تقدير دقيق لتردد نظام الطاقة في الوقت الفعلي وبأقل تأخير. نقدم خوارزمية جديدة تعتمد على تحويل جابور Gabor Transform لتقييم التردد اللحظي. كما قمنا بمراجعة عدد من خوارزميات تقدير التردد التي تم تناولها في الأبحاث والمقترح تطبيقها في أنظمة الطاقة وهي خوارزمية ترتيبه المنفصلة ثلاثية المستوى DFT و خوارزمية تحويل فورييه المنفصلة Three Level DFT و خوارزمية تحويل دلتا المنفصلة Delta Evaluation.

تم عمل المحاكاة لمقارنة الأداء النسبي للخوارزميات الأربعة. تشمل هذه الاختبارات اختبار التردد الثابت، تتبع التردد المتغير، الاختبار عندما يتغير كل من التردد والسعة، وأيضًا عندما تحتوي الإشارة على التوافقيات الثالثة والسابعة أو الضوضاء البيضاء أو إشارة التيار المستمر.

كشفت اختبارات المحاكاة أنه في ظل العديد من الظروف، قدمت خوارزمية Gabor أدنى مستوى لـ RMSE تقريبًا في جميع الحالات أو على الأقل في المرتبة الثانية. تحتاج خوارزمية Gabor إطارات من 4 دورات أو أقل لتقدير التردد اللحظي بدقة. ويمكن من خلال تداخل الإطارات، يمكن حتى استنتاج تقدير دقيق بعد كل دورة. وجدنا أن خطأ مربع متوسط الجذر RMSE يكون من صفر بواسطة Gabor خوارزمية في حالة التردد الثابت، في ظل الظروف المذكورة أعلاه ل有所不同 الإشارة لأذن عند عدد الدورات. ومع ذلك، يمكن أيضًا الوصول إلى هذا الصفر لـ RMSE من خلال ظروف الإشارة الأخرى إذا تم رفع تردد عدد العينات أو عدد الدورات قليلاً.