

Predictive Model on Architectural Design Optimization for Minimizing Construction Cost with Design Constraints

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Abstract

In many different development projects, we need to minimize construction costs and usable area according to the design constraints assigned to each project. The research aims to contrive a mathematical model for calculating the dimensions and sizes of rooms in a residential plan to get the least cost and manage the resources of construction. This paper presents a mathematical model (nonlinear programming) to achieve the above-mentioned goals according to the various design constraints and conditions, depending on the nature of the project and the goal of its establishment. To check the efficiency and accuracy of results, the nonlinear objective function and nonlinear constraints are solved using Mathematica and GAMS programs in two different cases. Tables and graphical representations of the results are very useful to the problem stated in this study. The study was applied to one of the New Urban Communities Authority projects in Egypt.

1. Introduction

Design is the act of applying intellect and creativity to create a new and unique solution. Function, form, structure, sustainability, interior design, costing, adaptability, and the site and its surroundings, are all aspects of design. Good design requires ingenuity and should result in cost savings and simplicity. It has the potential to boost production while also improving service quality. It can also provide the facility with a competitive edge in terms of recruiting both consumers and employees. Good design adds value in many ways: functionality cost savings throughout the life cycle of the project, service enhancement, architectural excellence, and broader social and environmental advantages [1]. Most designers are inspired by the art of architecture. Most architects need to show the principles

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of aesthetics of their customers, users, and environment in their designs. All clients and users have genuine goals and want in terms of the physical look of the

buildings they live in. All Clients are interested in knowing the initial building cost and the fees of the architect. They want both of them to be as low as possible. Before the design stage for many types of projects (commercial, industrial, and housing) we have to prepare a project financial feasibility analysis. It depends on market analysis and project finance planning [2]. The main factors affecting the cost accuracy include multiple internal and external aspects such as construction team experience in building type, client's financial situation, and budget, completeness and reliability of cost information and details, stability of the market as well as the economic situation of the country [3]. The most significant factors affecting cost estimates are clear and detailed drawings, specifications and project documentation, experience and skill level of cost estimator, completeness of cost information (accuracy, quality, and details), materials (prices, availability, quality, and imports) and experience on similar projects [4]. In Egypt Actual cost of construction projects generally differs from the planned cost, where which could reach a value between 21% and 55% [5]. The factors affecting the cost accuracy of construction projects in Egypt were investigated [6].

A building whose cost exceeds budget can be devastating to both the client and the architect. If discovered this problem through the design process, it can be resolved by reducing the structure's size or increasing the goodness of resources and systems. On the other side, if not discovered early in the design phase, this might need to make redesigned by the architect.

In general, architectural design employs a mix of creative and scientific knowledge to make equilibrium between the building's functions, aesthetics, and economics. It is common knowledge that Building plan design is one of the sectors of architectural design that interfere with all types of knowledge [7-8]. It refers to the lowest construction cost by considering the economical aspect of dimensions and room sizes in a building plan's design. The unit cost approach, which is the value of total direct costs for a production divided by the production output, is a building estimating method. In the construction industry, the cost of construction per usable area is defined as unit cost. In general, the cost of a building unit is comprised of three components: materials, labor, and machinery. Currently, unit cost estimation is often used to estimate building costs generally before beginning work. The unit cost may be broken down into specific rooms in a building, such as the living room, bathroom, walkway, roof desk, and so on. The differences in unit costs for each room are determined by the rooms' functions and building expenses [7-8].

The purpose of this paper is to arrive at decision-making in many problems related to many aspects of life. In the present case study (one of the projects of Egypt's New Urban Communities Authority) and by using nonlinear programming after converting the problem to a nonlinear mathematical model (Objective function- Constraint functions) compatible with all the factors surrounding the present case study. The building plan had several rooms with various costs of construction. The overall construction cost of the project is influenced by the variances in construction costs for each component of the building. The present case study can be solved using a mathematical approach called nonlinear programming. The model's outputs:

 \checkmark Identify the dimensions and room sizes with the lowest building costs.

✓ Determine the minimum construction cost and the acceptable useable area in the building taking into consideration design restrictions and parameters.

1.1. Scope of the study:

- i. Room of the Living.
- ii. Kitchen.
- iii. Toilet Room.
- iv. Hall.
- v. Bedroom1
- vi. Bedroom2
- vii. Bedroom 3

The construction expense was divided into two sections. In the first section, the building cost of floors is calculated by dividing the unit cost of construction by the useable area of each room (L.E / m^2) and grouping purposes of each room. In the second section, the building cost of walls is calculated by dividing the unit cost of construction per wall area (L.E/ m^2) by identifying the categories of exterior and interior walls. In Fig.1 a schematic with bubbles expresses the two different cases of the relationships among areas that are prepared by an architect according to user behaviors and holder needs.



Fig. 1 Explain the relationships in cases (1&2) among areas of the building plan.

The area linkages were generated from the bubble diagram (1) to create the preliminary construction plan. Fig. 2 depicted the building's preliminary plan prior to the study.

2. Architectural Design Process

Building plan design is a subset of architectural design that uses a combination of creative and scientific knowledge to create a balance between the building's functions, aesthetics, and economics. The design process characterizes a well-organized sequence of phases or processes. Designers should not consider through this process have a point of beginning and end point [7-8]. Not all ventures necessitate a designer going through all the processes, but

most utility from putting up with the process. Each step necessitates critical thinking, problem-solving, and decision-making to accomplish the project. The information learned in academic degrees serves as a basis for these design talents. The design process involves a variety of actions and tasks that must be completed to provide the answer. Some require collaborating with clients and other project participants. Some missions need the equitation of graphic documents and perspectives. Other responsibilities contain gathering data, such as the relevant codes. Tasks need deep thinking, solving the issue, and taking a decision. As for the design process itself, it has been identified as having five stages (design programming; schematic design; design development; contract documentation; and contract administration), with several duties in every phase [7-8].



Fig.2 The residential building's preliminary plan.



Fig. 3. "Phases of the design process"

The data information is viewed as the plan programming stage. The creator gathers to acquire all suitable data as conceivable taking into thought the commitments in the extent of administrations. Relying on the venture, information will be gained from the client, laborers, the undertaking designer, civil workplaces (regarding codes), and others as needs are. A schematic plan is a stage at which essential preparation and early independent direction happen. Normal visual records are bubble outlines, charts of contiguousness, and ideas portrayed. The fashioner likewise makes plan idea and select starting item. Plan improvement is the stage during which the fashioner settles different venture-related choices with the client. The creator should likewise guarantee that the furnishings, decorations, and gear on the arrangement are possible and coordinate with the need of the client [7-8].

Contract reports are the development and establishment drawings, particulars, and other documentation expected to construct and introduce the venture. A portion of these resources incorporate dimensioned floor plans, lighting/reflected roof plans and finishing timetables. Drawings are expected to be composed by the task designer and workers for hire, who will likewise be capable of the venture's development establishment. Contract organization involves cutthroat offering or setting orders for work and components to be fabricated. The cutthroat offering is happening for business improvements like places of business. The fashioner might be responsible for obtaining the merchandise, materials, and administrations for more modest undertakings and numerous private activities [9].

The schematic plan stage incorporates bubble charts where each circle distinguishes a space. The air pocket addresses the space and how different spaces relate to it. The connections between spaces are communicated by straightforward lines. Bubble outlines help the creator in the investigation of data with the goal can accomplish satisfactory arrangements. The unit technique utilizes verifiable and list and figure out how to an expense driver a lot more straightforward in building cost assessments [10]. The unit approach is the most frequently utilized gauge technique. The most well-known unit gauge for the building is the expense per square foot. Region very affects costs. Broadly, the unit technique is utilized like middle-of-the-road material costs, worker hours, and work costs. The shallow strategy is a solitary cost rate technique relying upon the expense per square meter of the structure. This methodology ought to be utilized distinctly in the beginning stages of the planning cycle and it is the most useful technique to around appraise the expense. The technique is speedy and simple to apply, on account of the unit strategy, there are likewise many advantages where the unit of estimation is significate for the client and the planning group. Despite the region for this technique is somewhat easy to work out, it needs some ability in deciding the cost information [9].

The area is calculated using the following rules:

- All dimensions are obtained from the internal face of external walls. Interior walls, elevator shafts, stairwells, stairwells, etc. are not deducted from the gross internal floor area.
- When the components of the building differ in function, the areas are estimated independently.
- External works and non-standard elements, such as piling, are estimated individually and then affix to the estimate. Subcontractors and particulars contractors may be able to provide estimates for specialized work [11].

Linear programming is a mathematical technique for optimum use of the available resources. Linear programming has been used successfully in life's different aspects, for example, in the military, industry, agriculture, transportation, economics, health systems, and even the behavioural and social sciences [12]. But we must be aware that there are many life problems that are difficult to formulate and solve using linear programming. When one or more of the assumptions of linear programming are significantly breached, another mathematical programming paradigm, such as integer programming or nonlinear programming, may be used instead [13].

Nonlinear programming is a mathematical technique in which the objective and constraint functions are both nonlinear. Nonlinear and linear programming have comparable

advantages and applications. To solve complicated issues, linear and nonlinear programming software is often utilised in personal computers nowadays. A nonlinear programming life's application of multi-criteria optimization in the design of a building plan was studied by [14]. Objective functions have a major role in minimizing the cost of construction and maximizing usable area and aspect ratio of building area, but constraint functions are formulated according to many conditions, for example, dimensions and room sizes required. The unit expenses of construction costs per room, as well as the construction costs per exterior and interior walls, were used to measure the disparities across rooms. The results showed that room dimensions were optimized based on trade-offs between construction cost, useable area, and building aspect ratio.

3. Research Methodology

The mathematical technique was used in this paper, which consisted of three parts:

3.1. Issue formulation

Issues in building plan configuration were researched, and it was found that the choice in making aspects and room sizes rely upon a variable development cost. The expense of development has two areas. Right off the bat, the development cost of the floor by taking into thought various unit expenses of floor development for elements of the relative multitude of rooms of the plan. Besides, development expenses of dividers by thinking about different unit expenses of divider development for different kinds of outer and interior dividers. All components of the structure affect the absolute development cost of the venture.

3.2. Data Gathering

Data from the examined issues in the issue formulation were utilized in the analysis of the issues that the researchers used to create the mathematical models that contained nonlinear programming.

- From the preliminary plan of the building, set up decision variables in all dimensions in the building plan on the vertical and horizontal axis of the plan.
- Collection of data that achieves the owner's needs in dimensions and room sizes of the building plan.
- Collection of data according to building laws and regulations.
- Obtaining information on the unit expenses of floor construction for each usage of the seven rooms.
- Collection of data according to unit costs of wall construction for external and internal walls.

Implementing data collection by requesting the building owner to measure dimensions and room sizes. Furthermore, in building designs, seek local rules and regulations.

Rooms	Design Restrictions
Room of the	Requirements of owner: The two dimensions of the living room are not
Living	less than 5 m.
Kitchen	Owner needs: The two dimensions of the kitchen are not less than 3m and
	its area is not less than 12 m^2 .
Bathroom	Owner requirements: The two dimensions of a bathroom are not less than
	2 m, and its area is not less than 6 m^2 .
Hall	According to the size of the doors, design constraints from door openings
	to bedroom1,2 are not less than 0.9 m.
Bed Room1,2,3	Owner requirement:
	• Case (1); Bedrooms are the same size.
	• Case (2): Bedrooms are not the same size.
	Design constraints from building laws and regulations: The two dimensions of every bedroom must be not less than 2.5 m and the area of every bedroom must be not less than 8 m^2 .

Table 1. Constraints in the residential design restrictions in the building plan design.

Then, using historical cost information of similarly constructed buildings, look for construction costs. The construction expense was divided into two sections. Firstly, the building cost of floors is calculated by dividing the unit cost of construction by the useable area (L.E/m²) of each room, as shown in Table 2. Secondly, as indicated in Table 3, the construction cost of walls takes into account unit costs of construction per wall area (L.E/m²) by rating kinds of exterior and interior walls, where C_1 , C_2 , C_3 , C_4 , C_5 , C_6 , C_7 , C_8 and C_9 the construction cost of each room.

Table 2. Unit costs	for the construct	tion of floors in	the residential building
			U

Rooms	Bedroom1,2,3	living room	Bathroom	Kitchen	Hall
Cost of	$C_{k L.E}/m^2$, k=1,2, and 3	$C_{4 L.E}/m^2$	$C_{5 L.E}/m^2$	$C_{6L.E}\!/m^2$	C_7
Construction					$L.E/m^2$

Table 3. Unit costs for the construction of walls in the residential building

Walls	Exterior wall	Interior wall
Cost of Construction	$C_{8 L.E}/m^2$	$C_{9 L.E}/m^2$

3.3. Model formulation

In this study, nonlinear programming was used to create mathematical models, which consisted of three major components:

3.3.1. Decision variables

Setting up decision factors in all aspects of the structure plan in the vertical and even tomahawks of the arrangement from the primer arrangement of the structure. Choice factors were aspects of the rooms in the structure plan. Horizontal axis: x_4 , x_5 , x_6 , x_7 , and x_8 , and *vertical* axis: x_1 , x_2 , and x_3 as shown in Fig. 4



Fig.4. Condominium construction preliminary plan with dimensions of choice variables and cost of each unit

3.3.2. Objective function

In this study, the objective function was to minimize total construction cost. The following is the formula for the objective function, which is a nonlinear programming equation, based on the data obtained:

Minimum construction cost = (Bedroom 1 area × Unit cost of bedroom 1) +(Bedroom 2 area × Unit cost of bedroom 2) + (Bedroom 3 area × Unit cost of bedroom 3) + (Living room area × Unit cost of the living room) + (Bathroom area × Unit cost of the bathroom) + (kitchen area × Unit cost of the kitchen) + (Hall area × Unit cost of the hall) + (Total length of external walls × Height of external wall × Unit cost of the external wall) + (Total length of internal walls × Height of internal wall × Unit cost of the internal wall).

Minimum construction cost = $x_8(x_1 + x_2) C_1 + x_3 (x_7 + x_8) C_2 + x_3(x_5 + x_6) C_3 + x_4 (x_1 + x_2 + x_3) C_4 + x_1 x_5 C_5 + x_1(x_6 + x_7) C_6 + x_2 (x_5 + x_6 + x_7) C_7 + 6 (x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8) C_8 + 3 (3x_1 + x_2 + 2x_3 + 2x_5 + 2x_6 + 2x_7 + x_8) C_9$ (1)

3.3.3. Constraint functions

3.3.3.1. Case 1: Bedrooms are the same size.

$x_1 \ge 3$	(2)
$x_2 \ge 0.9$	(3)
$x_3 \ge 2.5$	(4)
$x_4 \ge 5$	(5)

$x_5 \ge 2$	(6)
$x_7 \ge 0.9$	(7)
$x_8 \ge 2.5$	(8)
$x_1 + x_2 \ge 2.5$	(9)
$x_6 + x_7 \ge 3$	(10)
$x_7 + x_8 \ge 2.5$	(11)
$x_5 + x_6 \ge 2.5$	(12)
$x_2 - x_7 = 0$	(13)
$x_3 - x_8 = 0$	(14)
$x_1 + x_2 + x_3 \ge 5$	(15)
$x_8(x_1 + x_2) \ge 8$	(16)
$x_1(x_6 + x_7) \ge 12$	(17)
$x_3(x_7 + x_8) \ge 8$	(18)
$x_3(x_5 + x_6) \ge 8$	(19)
$x_1x_5 \ge 6$	(20)
$x_1 + x_2 = x_5 + x_6$	(21)
$x_5 + x_6 = x_7 + x_8$	(22)

3.3.3.2. Case 2: Bedrooms are not the same size.

$x_1 \ge 3$	(2*)
$x_2 \ge 0.9$	(3*)
$x_3 \ge 2.5$	(4*)
$x_4 \ge 5$	(5*)
$x_5 \ge 2$	(6*)
$x_7 \ge 0.9$	(7*)
$x_8 \ge 2.5$	(8*)
$x_1 + x_2 \ge 2.5$	(9*)
$x_6 + x_7 \ge 3$	(10*)
$x_7 + x_8 \ge 2.5$	(11*)
$x_5 + x_6 \ge 2.5$	(12*)
$x_2 - x_7 = 0$	(13*)
$x_1 + x_2 + x_3 \ge 5$	(14*)
$x_8(x_1 + x_2) \ge 8$	(15*)
$x_1(x_6 + x_7) \ge 12$	(16*)
$x_3(x_7 + x_8) \ge 8$	(17*)
$x_3(x_5 + x_6) \ge 8$	(18*)
$x_1x_5 \ge 6$	(19*)

4. Results and Discussion

The system of the nonlinear objective function (1), nonlinear constraints of the case (1) (2-22), and nonlinear constraints of the case (2) (2^*-19^*) are solved by using Mathematica and GAMS software. Tables and graphical representations of the results are very useful to demonstrate the efficiency and accuracy of (Mathematica) and (GAMS) software for the problem stated in this work. As illustrated in Figures (5-6), the condominium building plan was examined and designed for the two different cases. Results in tables (4-5) compatible and verified nonlinear constrains (2-22) and (2*-19*). The dimensions shown in tables (4-5), achieve the minimum construction cost in each case, and through which we can form a general equation (23) for case (1) and a general equation (20*) for case (2) to calculate the minimum construction cost for the two cases (1) and (2) according to the latest construction cost prices.

Table (4): Results for case (1).

<i>x</i> ₁	<i>x</i> ₂	<i>x</i> ₃	<i>x</i> ₄	<i>x</i> ₅	<i>x</i> ₆	x_7	<i>x</i> ₈	Cost of construction	Area usable (m ²)
3	1.5	3	5	2	2.5	1.5	3	Eq. (23)	105

Minimum construction cost

 $= 13.5 C_1 + 13.5 C_2 + 13.5 C_3 + 37.5 C_4 + 6 C_5 + 12 C_6 + 9 C_7 + 129 C_8 + 103.5 C_9$ (23)



Fig. 5. The proposal for a residential building was studied and designed for case (1)

Table (5): Results for case (2).									
<i>x</i> ₁	<i>x</i> ₂	<i>x</i> ₃	x_4	<i>x</i> ₅	<i>x</i> ₆	<i>x</i> ₇	x ₈	Cost of construction	Area usable (m ²)
3	0.9	2.5	5	2	3.1	0.9	2.5	Eq. (20*)	86.4

5 0.9 2.5 5 2

Minimum construction cost

 $= 9.75 C_1 + 8.5 C_2 + 12.75 C_3 + 32 C_4 + 6 C_5 + 12 C_6 + 5.4 C_7 + 119.4 C_8 + 95.7 C_9 \qquad (20^*)$



Fig. 6. The proposal for a residential building was studied and designed for case (2)

5. Conclusion

This article describes a strategy for making decisions in the design of a building plan. To obtain the dimensions and areas of the different design rooms that comply with all the constrains surrounding the design through applied a nonlinear programming mathematical model in the present case study (one of the projects of Egypt's New Urban Communities Authority), which was studied and estimated. The case study's mathematical models were formulated for two different cases as given in equations (1) and constrains (2-22) for the first case and constrains (2^{*} -19^{*}) for the second case. The results are shown in tables (4-5) and figures (5-6). The outcomes of applying the model to the current case study were as follows:

- Mathematical model controls the construction cost of architectural design considering customer requirements and building laws regulations.
- Mathematical model (nonlinear programming) achieves the minimum construction cost, usable area, and suitable dimensions according to the design constraints assigned to each project.
- ➢ Form a general equation (23) for case (1) and a general equation (20*) for case (2) to calculate the minimum construction cost for the two cases (1) and (2) according to the latest construction cost prices.
- Always be construction cost of case (1) more than the construction cost of case (2).
- Reducing the total construction cost, offset by the abandonment of the requirement (bedrooms of the same size).
- > The mathematical model's applications might be utilized for other more complex structure plans, for example:
 - ✓ Piece of circle building plans,
 - ✓ Triangle building plans,
 - ✓ Other mathematical plans where the structure plan regions might be developed and assessed,

- ✓ Besides, different models, for example, required room sizes for furniture sizes, or different limitations can be added to the imperative capacities.
- > The applications of the mathematical model can also be applied to other objective functions such as:
 - ✓ Maximum usable area in the building within construction cost budget,
 - ✓ maximized usable area and aspect ratio of building area,
 - \checkmark The most building plan close to land as much as possible.

Future work

Since the various sources of energy are on the way out one day, so we have to search from now on for alternatives or access to ways to extend the life of the available energy and this is what all European Union countries are striving for after the Russian invasion of Ukraine. Therefore, my research friends and I are looking forward to publishing a research paper soon explaining the possibility of linking the architectural designs of different buildings and reducing the energy used.

Abbreviations

- GAMS General Algebraic Modeling System
- Ck Cost of Construction
- **L.E** Egyptian Pound

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دراسة نموذج تنبؤي للاستغلال الأمثل للتصميم المعماري لتقليل تكلفة البناء مع الالتزام بقيود التصميم

الملخص العربى:

في العديد من مشاريع التطوير المختلفة، نحتاج إلى تقليل تكاليف البناء وزيادة المساحة الصالحة للاستخدام وفقًا لقيود التصميم المحددة لكل مشروع. الهدف من البحث هو إيجاد نموذج رياضي لحساب أبعاد ومساحات الغرف في مخطط سكني لتحقيق أقل تكلفة إنشاء وإدارة مثلي لموارد البناء. تقدم هذه الورقة نموذجًا رياضيًا (البرمجة غير الخطية) لتحقيق الأهداف المذكورة سابقا وفقًا لقيود وشروط التصميم المختلفة، اعتمادًا على طبيعة المشروع والهدف من إنشائه. للتحقق من كفاءة ودقة النتائج، تم حل النموذج الرياضي غير الخطي والقيود غير الخطية باستخدام برامج Mathematica ورسومات مغيرة محل الدراسة. تم تمثيل النتائج التي حصانا علي هيئه جدول ورسومات مفيدة جدا للمشكلة محل الدراسة. الدراسة تم تطبيقها على أحد مشروعات هيئة المجتمعات العمرانية الجديدة في مصر.