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The Roof as an Environmental Assessment Tool: the computational Equation of Thermal Comfort based on Fanger Scale

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Keywords
Fanger scale; thermal comfort; roof environmental tool; environmental roofs variables; roof environmental impact.

Abstract
Purpose –Is to identify the effect of the design variables of roofs on the thermal comfort field according to the Fanger Scale, in order to determine the computational mathematical design relationship of the effect of the design variable on the amount of change in the ratio of the thermal comfort field according to the Fanger Scale. Design/methodology/approach – A simulation study model was designed according to administrative buildings standards. Then, five roof variables were determined with its values to be simulated using design builder software as an approach to study its effect on thermal comfort according to Fanger scale and its computational mathematical relationship. Findings - The results indicated that the design of pergolas is one of the biggest elements that have a positive impact on thermal comfort, followed by the roof cantilever, the plantings, the inclination of the roof, and then the thermal insulation. Based on the simulation model and the mathematical base of Simple Linear Regression, computational equations were found out for the most influential design variables based on the computational form "Change in PPD Fanger = Constant 1 + Constant 2 x Roof Variable", as the research determined the values of the constants in typical equation for the top three design variables. Originality/value – It is a base for calculating the roof variables and its impact on thermal comfort as a roof base design standard.

1. **Introduction**

The world faces many challenges to achieve sustainable development, and perhaps the most important of these challenges is achieving mechanisms to improve energy efficiency, especially in buildings, as one of the resources supporting development and economy. At that time, statistics still indicated a significant increase in energy consumption rates resulting from the use of means of cooling, air conditioning and heating [1], as evidenced by Figure (1): indicators of that increase and expectations of its continuation as a result of population increase and climate changes [2].

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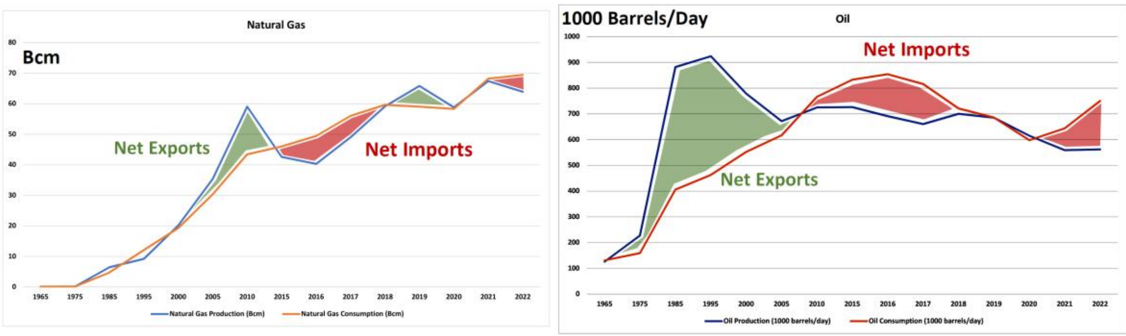


Fig. 1: Gas & Oil Production/Consumption in Egypt.

As buildings suffer from the lack of interest of their designers in the thermal performance of their internal spaces and the negative impact of the increase in energy consumption, there is a need to study the most appropriate design alternatives for the outer shells of buildings, especially with the rapid technological development and the emergence of many modern methods in the field of construction. Roofs are among the most important elements of the outer shell and most closely related to the surrounding external environment, as they are most exposed to various factors such as solar radiation, rain, wind, heat and humidity.

2. Roofs as an environmental stimulus

The limits of thermal comfort for the internal spaces are between the limits of an actual temperature ranging between 20:27 ºC, and a relative humidity of between 20:80%, according to the specifications of the American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE) [3]. A preliminary study was calculated to clarify the effect of the roof on the thermal comfort of the space in the last floor of the building based on the Fanger Scale [4] to clarify its effect on the values of both PMV and PPD [5]. Figure (2) shows the Fanger Scale Model and the relationship between the values of PMV and PPD.

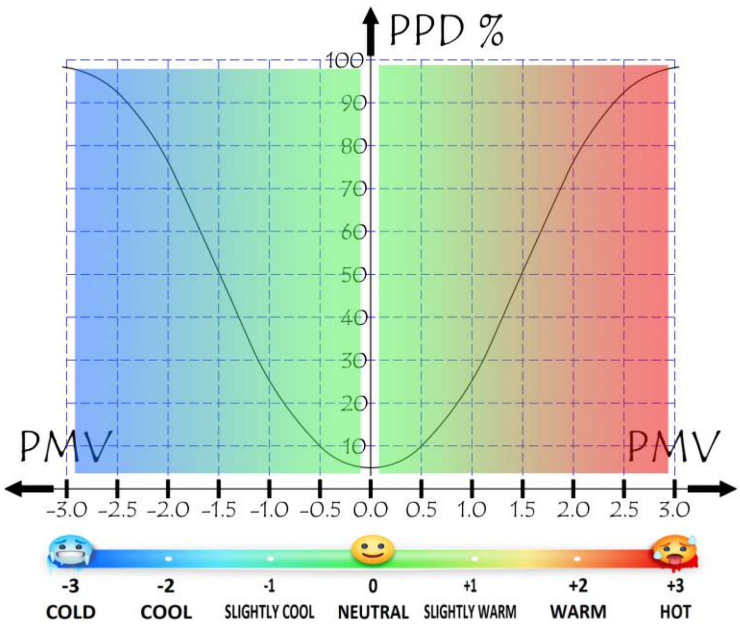


Fig. 2: PMV/PPD Fanger scale.

A simulation model was built for a typical design module for administrative buildings with an open space system, with design dimensions of 8 x 8 x 3.60 m; as shown in Figure (3).Through simulation, using the Design Builder program thermal climate indicators for the climate of Greater Cairo were determined to identify the comfort zone according to the Fanger Scale [3], in the case of the effect of the external walls only on the ground floor, and the effect of the external walls and the roof on the last floor.

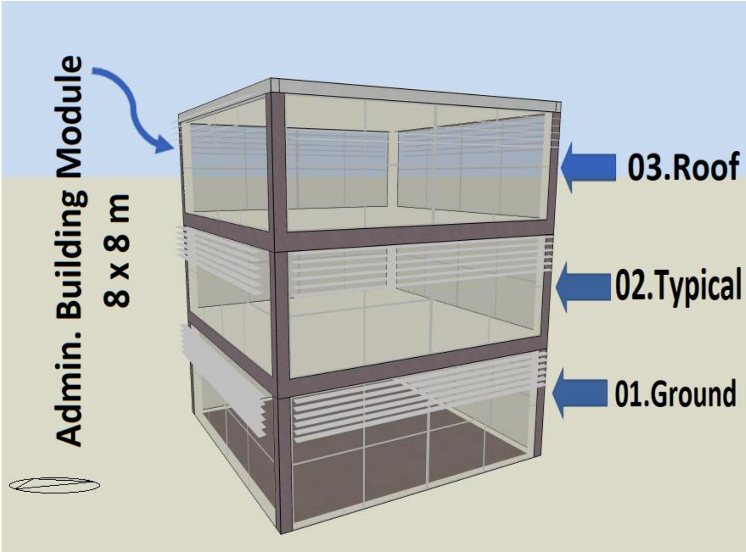


Fig. 3: Administration proposed unit.

As shown in Figure (4), the effect of the roof of the last floor on the thermal comfort of the space is as follows:

- The roof reduces the level of thermal comfort with an average PMV = 0.267, reaching its highest value in August with a PMV = 0.44.
- The value of the percentage of thermal discomfort PPD decreases due to the effect of the roof of the last floor by a percentage with its highest value in summer is PPD = 14.47%.
- The actual operative temperature of the last floor space increases by an average of 0.74:1.00ºC, reaching its highest value in summer by 1.30ºC.

3. The research design model

A typical model for administrative buildings, with an open space system, was chosen as one of the models of public buildings that were largely constructed in recent years as a basic pillar for the investment sector [5]. It was designed, as shown in Figure (5), according to the following typical design criteria:

- The structural design module is 8 x 8 m.
- The dimensions of the building are 32 x 32 m, as one of the typical models for medium-sized administrative buildings.
- The height of the floor is 3.60 m.
- The percentage of service spaces and vertical movement elements is 25% of the floor area.
- The location in Cairo, according to the main climatic trends.
- Layers of the outer shell covering of the walls and the roof according to the typical design common to administrative buildings in Cairo, as shown in Figure (6).

- Relying on achieving natural ventilation of the building without the presence of mechanical means to achieve thermal comfort, as a neutral element through which it is possible to study the extent of the effect of design alternatives to the roof without the intervention of mechanical means to compensate for the difference in the thermal comfort zone.
- Clothing properties and metabolic heat production as per standards [6].

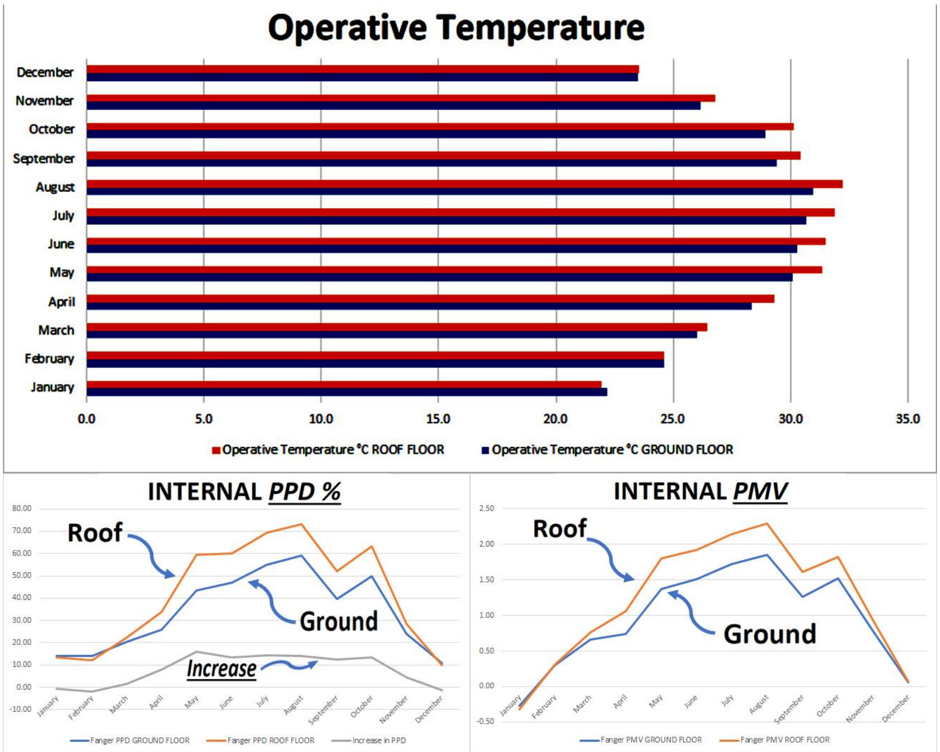


Fig. 4: Thermal comfort roof slab impact according to Fanger scale.

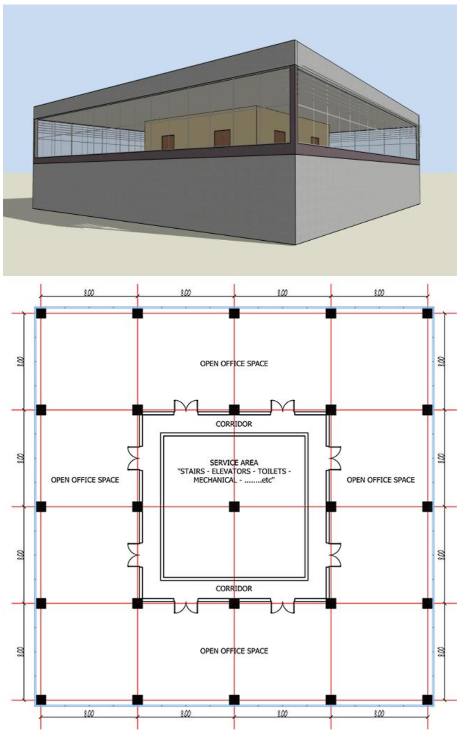


Fig. 5: Proposed simulation administration building.

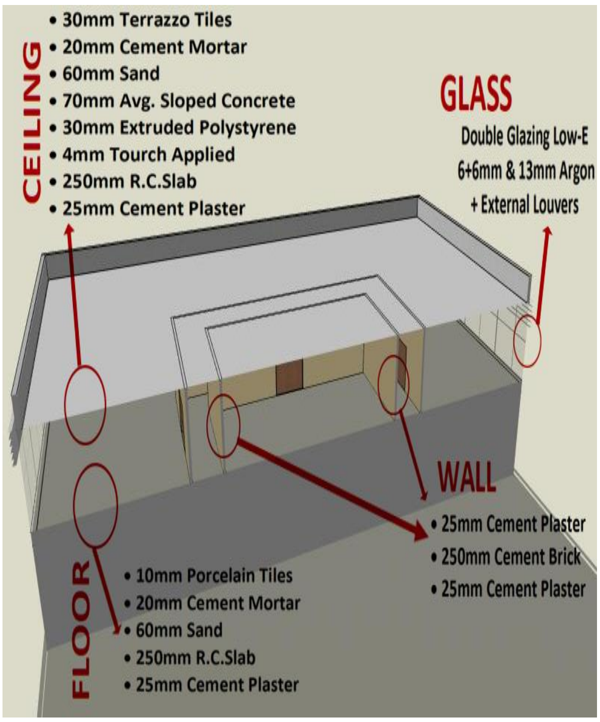


Fig. 6: External envelope layers.

4. Determining the design variables and alternatives for the roof

A number of roof design variables were determined to study their effect on the Fanger Thermal Comfort Scale and its direct effect on the PMV/PPD values. These variables fall under three main design elements illustrated in Figure (7), including the following:

- General formation of the horizontal roof.
- Layers and components of the roof.
- Elements are added to the roof.

Table (1) shows the default values of these variables for the analytical study of the simulation model using the Design Builder Program.

The computational results of building simulation indicated the Base Case Design, as shown in Figure (8), as only the natural ventilation was relied upon without mechanical means.

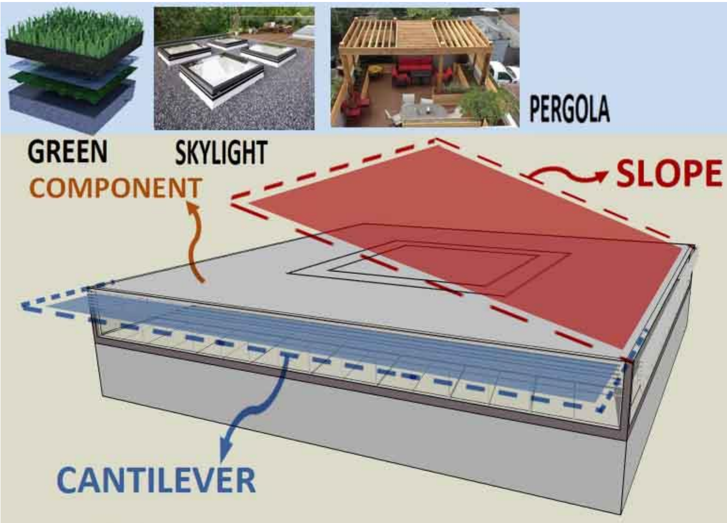


Fig. 7: External envelope alternatives.

Table 1: Roof design variables and alternatives.

Design Variable	Alternative	Base Case	1	2	3	4	5
Roof Form	Slope (%)	0	1	2	3	4	5
	Cantilever toward south (cm)	0	50	100	150	200	250
Roof Layers	Thermal insulation thickness (mm)	30	0	1	2	4	5
	Skylight ratio (%)	0	10	20	30	40	50
Added component	Pergola ratio (%)	0	10	20	30	40	50
	Green ratio (%)	0	10	20	40	60	80

5. Applied results of design variables and alternatives of the roof

The next part deals with the applied results of simulating the design alternatives of the roof, according to what was mentioned in the previous paragraph, as the simulated model was prepared for each alternative using the Design Builder Program, with the calculation of the percentage of the thermal comfort zone per month of the building through the extent to which the PPD values were achieved. The main results of the simulation were as follows:

5.1. The effect of roof inclination

It is noted through the simulation results that the inclination of the roof towards the north, Figure (9), has a noticeable effect on the thermal comfort scale, especially in the summer seasons compared to the winter seasons. The most important results concluded as following:

- The effect of the roof inclination in general leads to an improvement in the thermal comfort zone in summer, with its limited effect in winter.
- With the gradual increase in the roof inclination from 1% to 5%, very limited differences appear in improving the thermal comfort zone, as shown in Figure (10), where the average value of the change in summer is 1.93%, and in winter is 2.59%.
- Figure (11) shows the change in the value of the PPD as a result of adjusting the position of the roof from the horizontal direction to the inclined one by 5% to the north, as it leads to an improvement in the value of the thermal zone in summer by 13%, and in winter by 2.25%.
- The highest effect value of roof inclination is in June, with a rate of 19.16%.

Comfort - Base Case, Main Building

EnergyPlus Output 1 Jan - 31 Dec, Monthly Licensed

Month												
Air Temperature (°C)	21.28	22.94	24.96	28.26	31.02	31.54	32.28	33.02	30.66	31.12	26.21	22.68
Radiant Temperature (°C)	22.02	23.89	26.11	29.81	32.45	33.80	34.48	35.02	32.72	32.46	27.45	23.49
Operative Temperature (°C)	21.65	23.42	25.54	29.04	31.73	32.67	33.38	34.02	31.69	31.79	26.83	23.08
Outside Dry-Bulb Temperature (°C)	13.75	15.89	18.20	21.15	24.76	27.41	28.75	29.24	26.96	24.67	19.56	16.27
Relative Humidity (%)	43.38	37.35	35.48	32.22	32.72	39.97	47.87	48.68	48.74	40.33	42.18	41.64
Fanger PPD (%)	9.26	7.92	17.51	33.55	66.81	80.51	90.56	93.55	73.06	81.05	31.88	7.51
Fanger PMV ()	-0.24	0.14	0.64	1.06	1.98	2.35	2.69	2.94	2.10	2.25	1.02	0.09

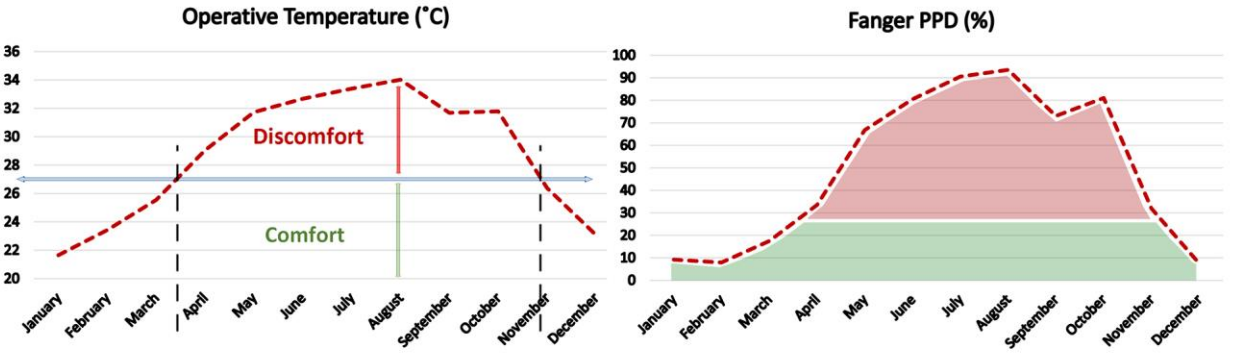


Fig. 8: Building base case simulation results.

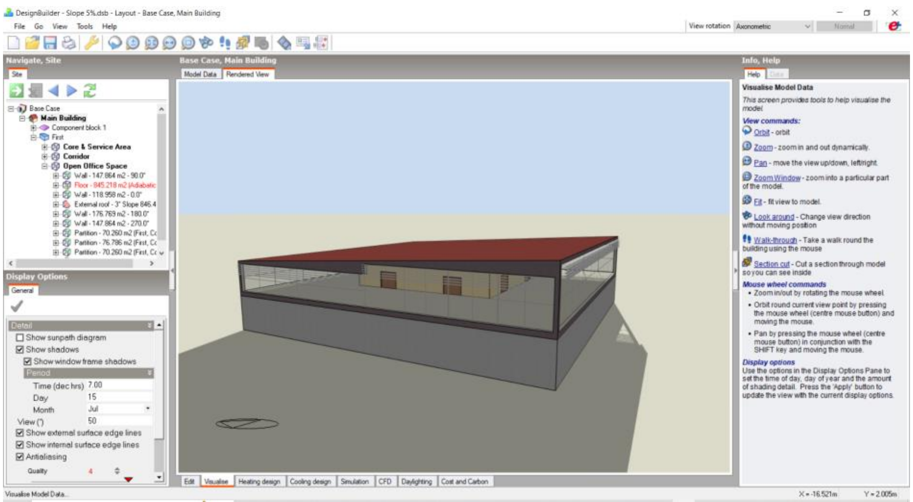


Fig. 9: Sloped roof simulation screen shot.

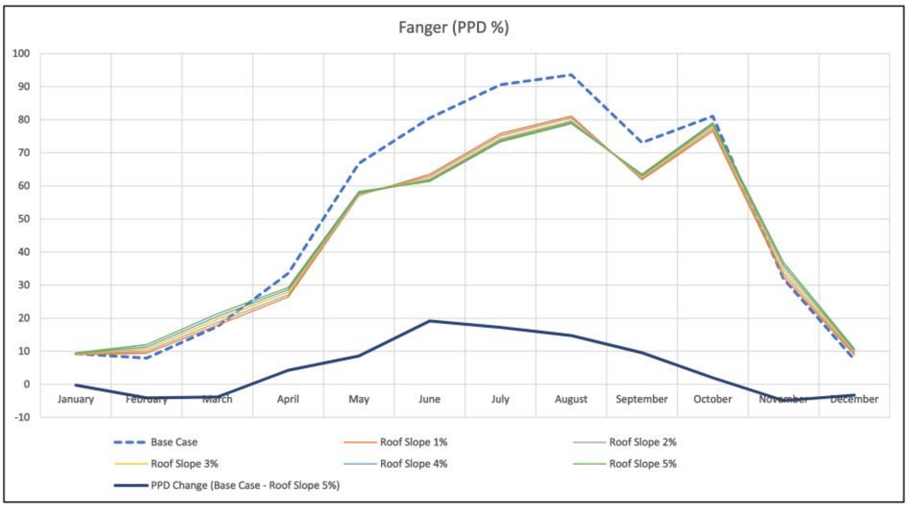


Fig. 10: Sloped roof (PPD) impact.

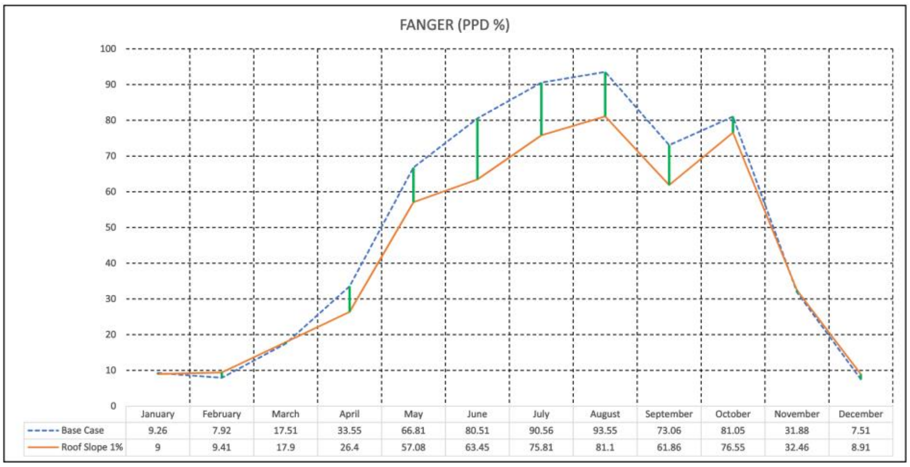


Fig. 11: Horizontal/sloped roof (PPD) change.

5.2. The effect of roof cantilever

According to what was done in the simulation model as in Figure (12) by adding and modifying the roof cantilever to the south, it is clear that it has an average effect compared to other design variables, as this is evident from the following:

- The comfort zone increases with the increase in the roof cantilever in general most months of the year, as the average increase ranges from the cantilever of 50 cm to the cantilever of 250 cm, with an average increase in the comfort zone up to 7.94% in summer and 6.95% in winter. Figure (13) shows the rates of that increase.
- By studying the maximum cantilever that was designed and compared with the typical model, as shown in Figure (14), it is possible to achieve an improvement in the proportion of the comfort zone by an average of 10.31% in summer, 9.03% in winter.
- The highest value of the effect of the roof cantilever under study on the comfort zone is in close proportions for several months, including April 16.85%, May 17.32% and October 16.96%.

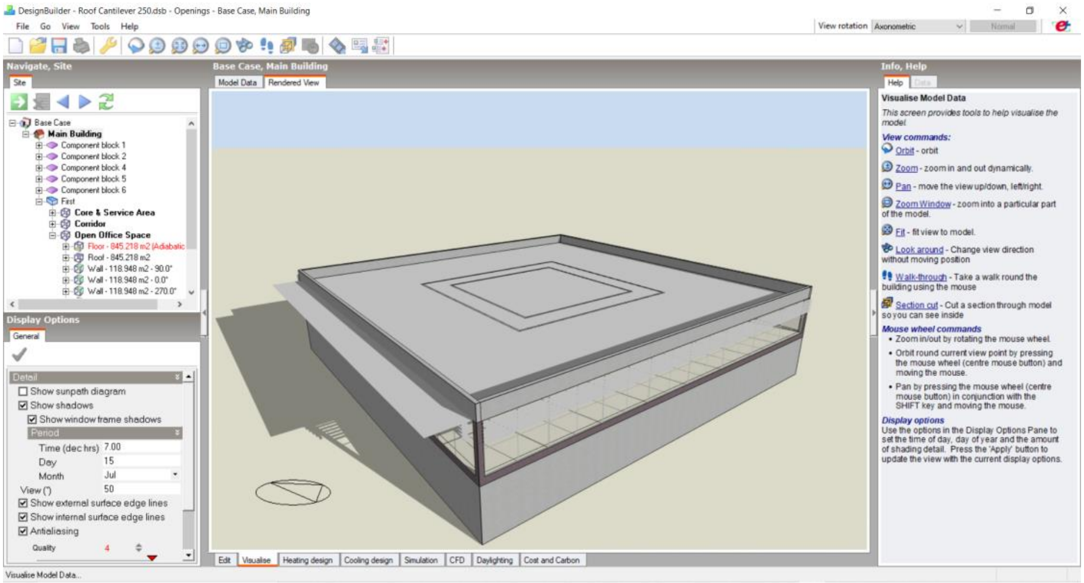


Fig. 12: Roof cantilever simulation screen shot.

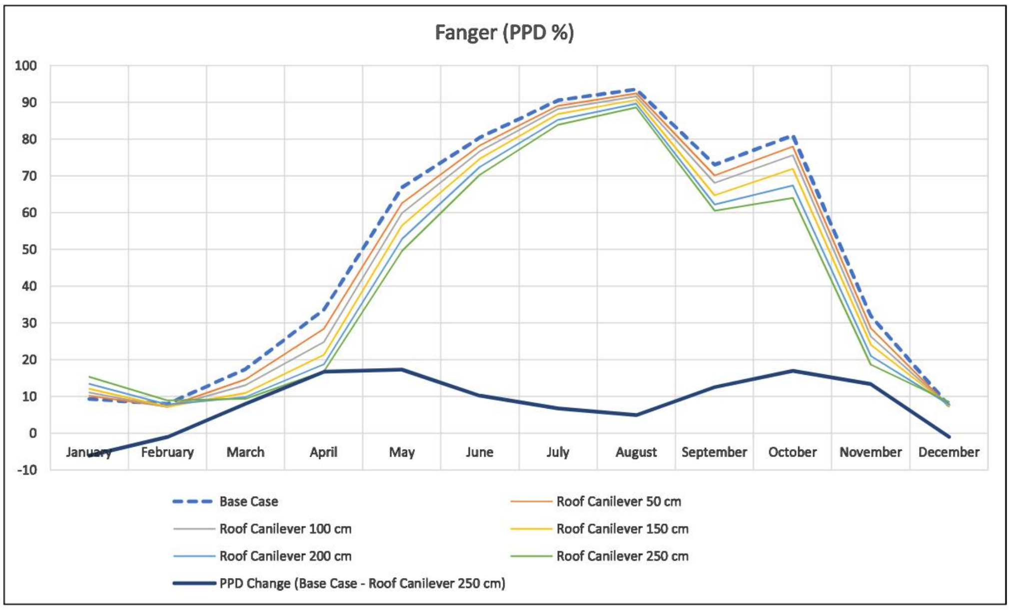
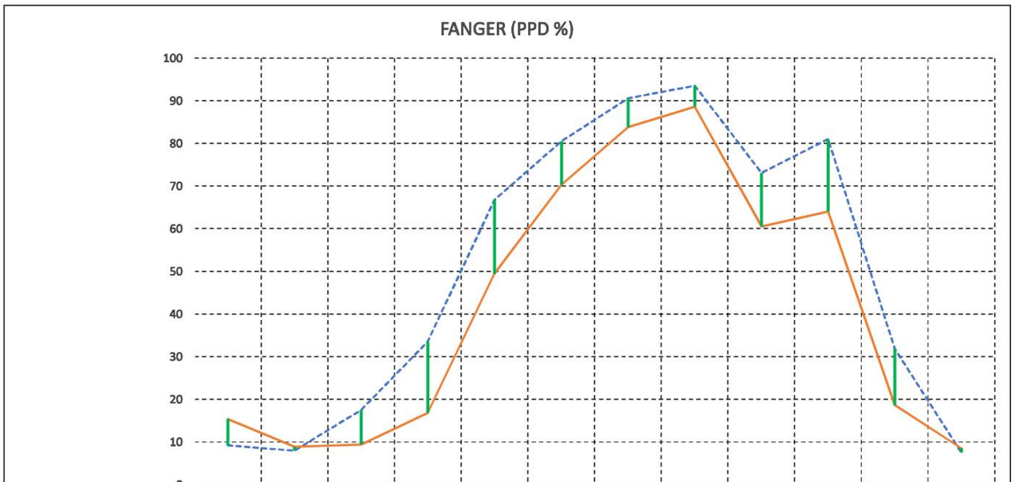


Fig. 13: Roof cantilever (PPD) impact.



	January	February	March	April	May	June	July	August	September	October	November	December
Base Case	9.26	7.92	17.51	33.55	66.81	80.51	90.56	93.55	73.06	81.05	31.88	7.51
Roof Canilever 250 cm	15.36	8.93	9.4	16.85	49.49	70.36	83.89	88.65	60.55	64.09	18.59	8.55

Fig. 14: Roof cantilever/no cantilever (PPD) change.

5.3. The effect of the thickness of the thermal insulation layer

According to the typical layers of the roofs, the typical simulation model includes an Extruded Polystyrene heat insulating layer of 3 cm thickness.

- It is clear from Figure (15) that the limited effect of reducing or increasing the thickness of the thermal insulation layer from the typical thickness specified by 3 cm, as it ranges between 2.33: 5.67%, while it ranges between 2.64: 4.01% in winter.
- The thermal insulation layer in the typical model with a thickness of 3 cm leads to an increase in the thermal comfort zone compared to its absence within the layers by an average of 4.88% in summer and 4.38% in winter, as shown in Figure (16).
- The highest value of the effect of the thermal insulation layer with a thickness of 3 cm on the thermal comfort zone was 10.83% in June, similar to the same effect in November with an amount of 8.15%.

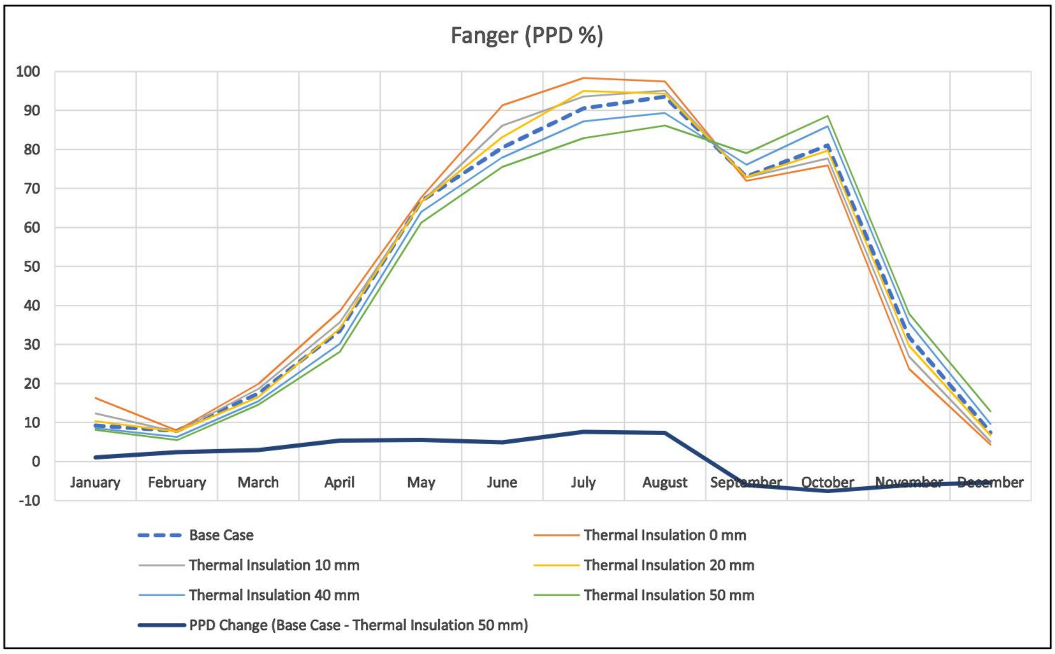
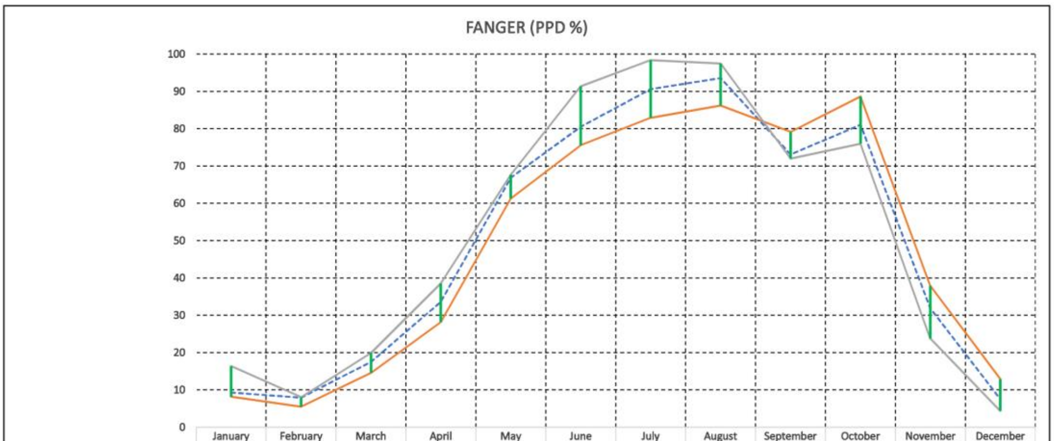


Fig. 15: Roof insulation (PPD) impact.



	January	February	March	April	May	June	July	August	September	October	November	December
Base Case	9.26	7.92	17.51	33.55	66.81	80.51	90.56	93.55	73.06	81.05	31.88	7.51
Thermal Insulation 50 mm	8.17	5.48	14.58	28.15	61.23	75.57	82.93	86.2	79.09	88.62	37.94	12.91
Thermal Insulation 0 mm	16.35	8.01	19.92	38.56	67.67	91.34	98.36	97.43	71.99	75.97	23.73	4.35

Fig. 16: Roof insulation (PPD) change.

5.4. Skylight effect

This design element was chosen as a main aim to study the extent of its impact on the decrease in the thermal comfort zone, contrary to what is expected from other design elements with a positive output, to determine the effect of all design elements. Figure (17) shows one of the design alternatives to the model, and its effect can be mentioned in the following points:

- With the increase in the Sky Light area, the thermal comfort zone decreases, according to what was determined by the change from the area of 10% to 50%, as in Figure (18). The percentage of reducing the thermal comfort zone, as an average, in summer is (-7.24%), and in winter is (-9.8%).
- The highest percentage of Sky Light studied in the model leads to a decrease in the thermal comfort zone compared to the typical model, as shown in Figure (19), with an average value in summer (-12.13%), while it reaches (-13.26%) in winter.
- The highest value for reducing the thermal comfort zone due to the Sky Light is in April, reaching (-36.43%), followed by March, with a rate of (-24.78%).

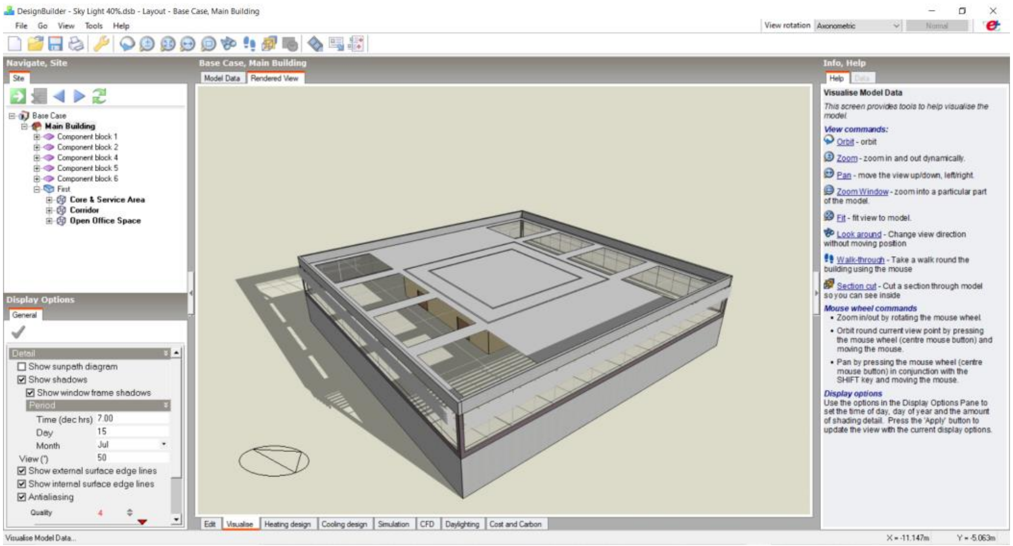


Fig. 17: Roof skylight simulation screen shot.

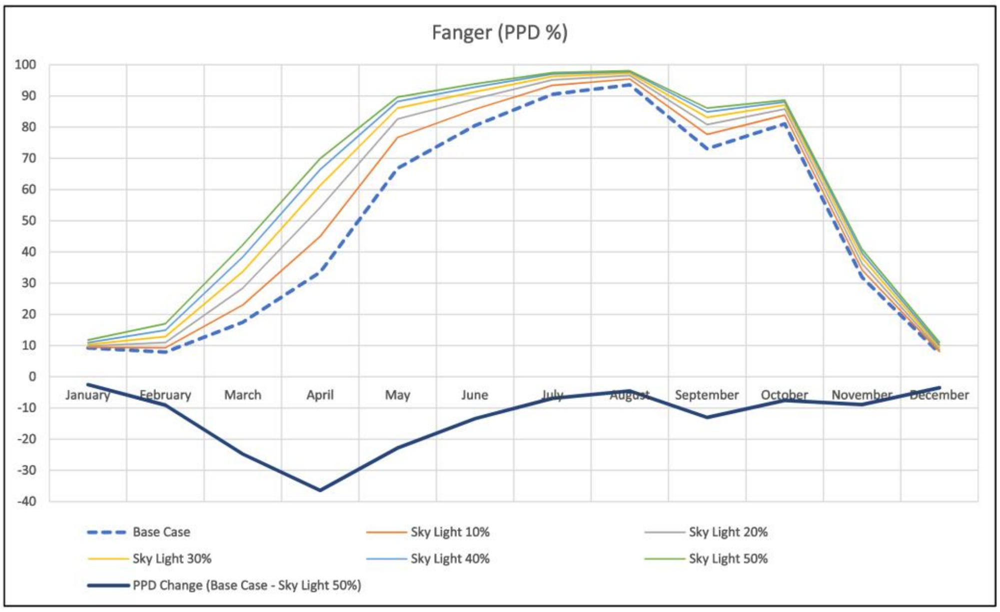


Fig. 18: Roof skylight (PPD) impact.

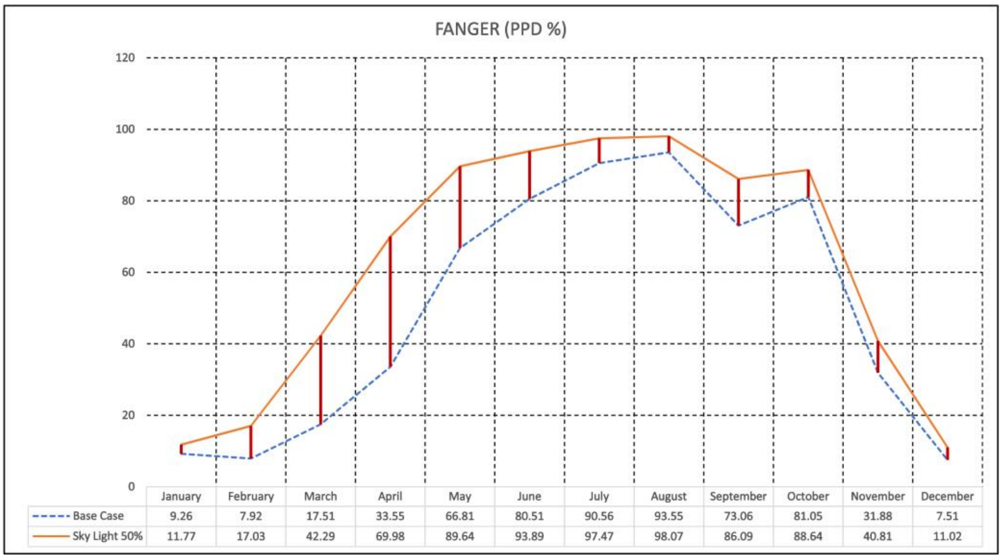


Fig. 19: Roof sky light (PPD) change.

5.5. Pergola effect on the roof

Pergolas are light structures that are common in open areas. They have been used in many administrative buildings above the roof of the last floor as a recreational element of the administrative building. Simulation model studies of alternatives to the pergola area above the roof have shown several points as follows:

- It is clear from Figure (20) that the simulation results for the alternative design of the pergola, where the value of the thermal comfort zone increases with the increase in the percentage of the pergola area from 10% to 50%, with an average value of 20.64% in summer and 6.77% in winter.
- According to the highest percentage of pergola area studied in the model, the results showed an increase in the thermal comfort zone compared to the typical design, as shown in Figure (21), with an average value of 29.79% in summer and 10.49% in winter.
- The highest value studied in the model for the effect of the pergola area in increasing the thermal comfort zone compared to the typical design, for several months, in close proportions as follows: May 27.68%, June 29.99%, July 32.03%, August 32.42%, September 26.85%, and October 28.96%.

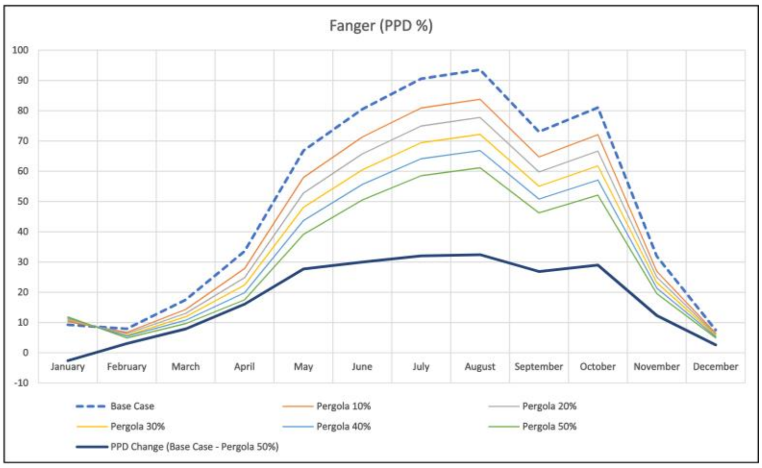


Fig. 20: Roof pergola (PPD) impact.

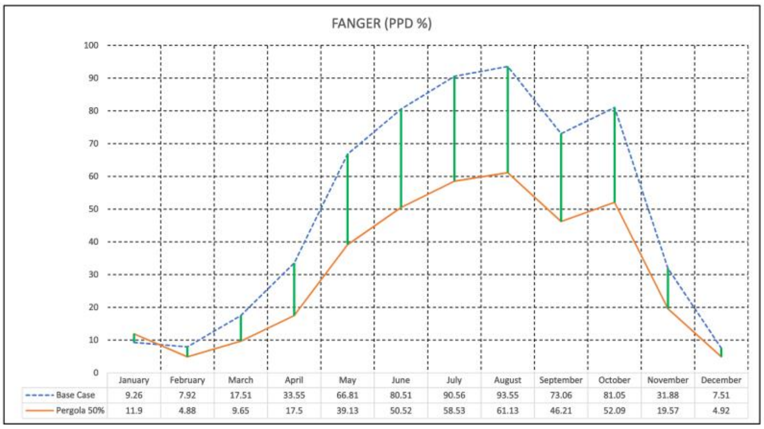


Fig. 21: Roof pergola (PPD) change.

5.6. The effect of the percentage of plantings on the surface

It is noted from the simulation results that by increasing the percentage of plantings on the roof, the thermal comfort zone of the internal spaces increases significantly with the increase in the percentage of the cultivated area of the roof. The most important results concluded the following:

- The thermal comfort zones increase with the gradual increase in the proportion of plantings from 10% to 80%, with an average of 13.88% in summer and 3.5% in winter. Figure (22) shows the results of the simulation effect of design alternative for plantings.
- Figure (22) shows the results of the effect of plantings with a flat rate of 80% of the roof and its comparison with the original design of the Base Case, as it leads to an increase in the thermal comfort zone by an average of 15.95% in summer, while the average in winter is 4.04%.
- The highest value of the effect of summer plantings is in June, with a rate of 17.48%.

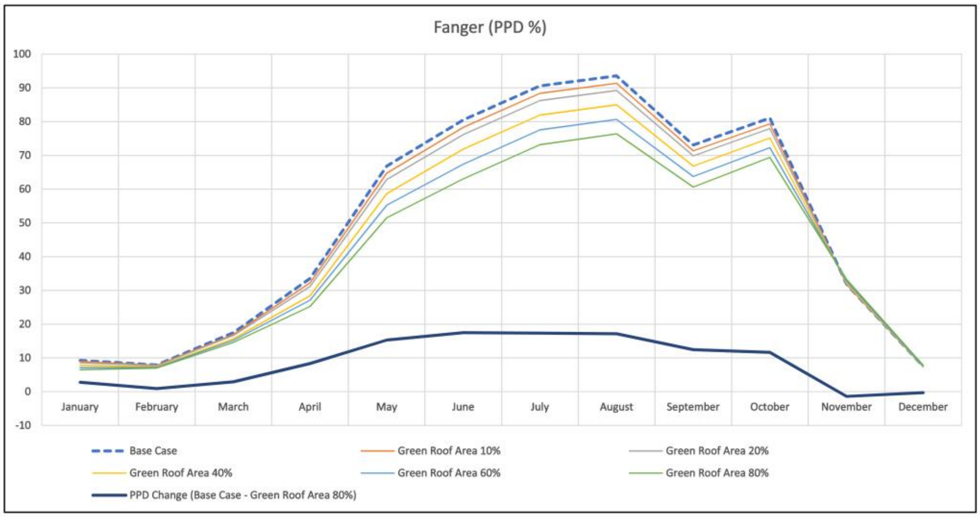


Fig. 22: Green roof (PPD) impact.

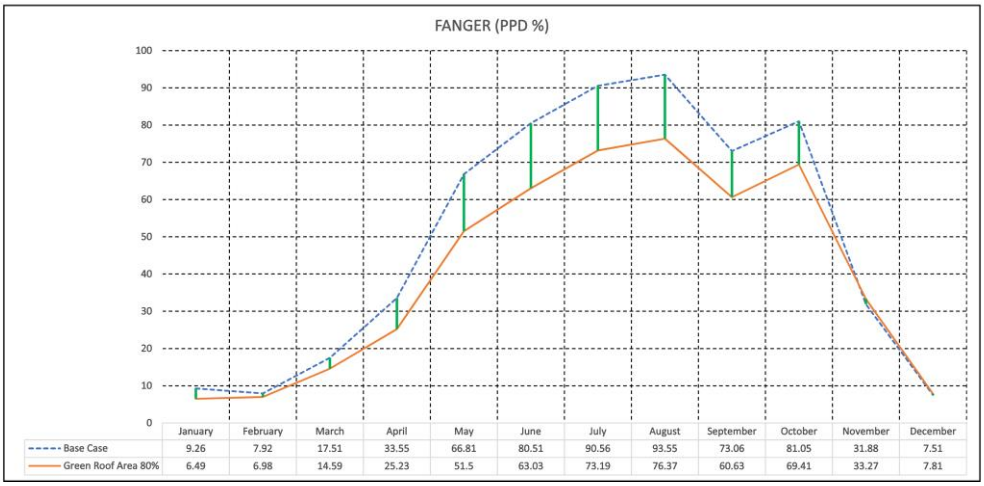


Fig. 23: Green roof (PPD) change.

6. Effective evaluation of the design alternatives for the roof

The following part deals with the combined study of the effect of all the previous design alternatives for the roof, as evidenced by Figures: (24) and (25), which shows the effect of each of the following alternatives:

- The original design of the Base Case.
- The effect of roof inclination by 5%, and a study of its effect difference from the original design ***(PPD.S)***.
- The effect of plantings on the roof with a flat surface of 80% and studying the difference in its effect from the original design ***(PPD.G)***.
- The effect of the thermal insulation layer with a thickness of 5 cm, and a study of its effect difference from the original design ***(PPD.I)***.
- The effect of having a pergola on the roof with a flatness of 50% and studying its effect difference from the original design ***(PPD.P)***.
- The effect of the sky light with a flatness of 50% and studying the difference in its effect from the original design ***(PPD.SK)***.
- The effect of the roof cantilever to the south by 250 cm and studying the difference in its effect from the original design ***(PPD.C)***.

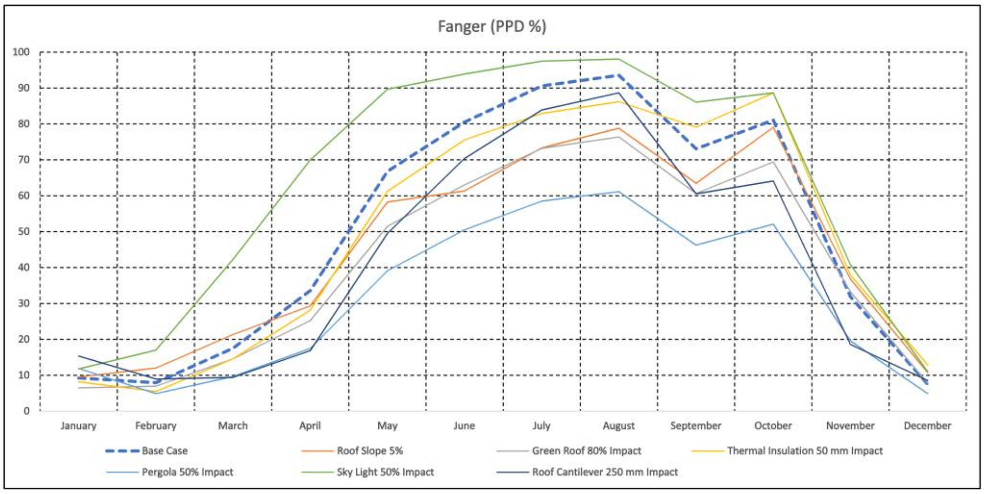
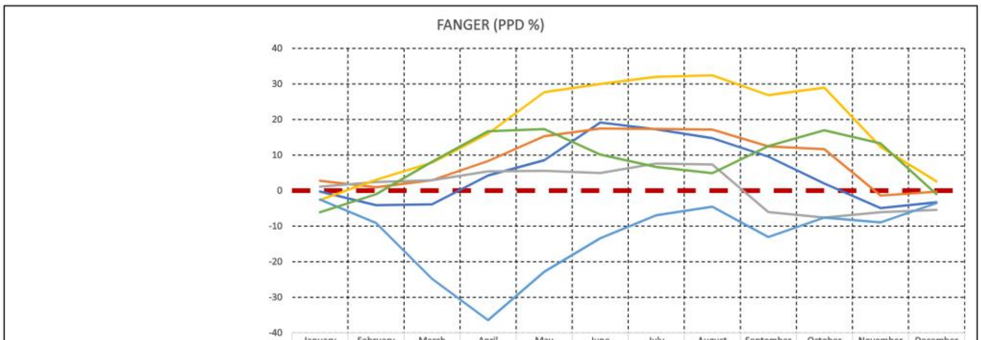


Fig. 24: Maximum design alternatives (PPD) impact.



	January	February	March	April	May	June	July	August	September	October	November	December
PPD.Slope% (PPD.S) = Base Case - Roof Slope	-0.24	-4.08	-3.84	4.25	8.56	19.16	17.22	14.74	9.57	1.98	-4.91	-3.26
PPD.Green% (PPD.G) = Base Case - Green Roof	2.77	0.94	2.92	8.32	15.31	17.48	17.37	17.18	12.43	11.64	-1.39	-0.3
PPD.Insulation% (PPD.I) = Base Case - Thermal Insulation	1.09	2.44	2.93	5.4	5.58	4.94	7.63	7.35	-6.03	-7.57	-6.06	-5.4
PPD.Pergola% (PPD.P) = Base Case - Pergola	-2.64	3.04	7.86	16.05	27.68	29.99	32.03	32.42	26.85	28.96	12.31	2.59
PPD.Skylight% (PPD.SK) = Base Case - Sky Light	-2.51	-9.11	-24.78	-36.43	-22.83	-13.38	-6.91	-4.52	-13.03	-7.59	-8.93	-3.51
PPD.Cantilever% (PPD.C) = Base Case - Roof Cantilever	-6.1	-1.01	8.11	16.7	17.32	10.15	6.67	4.9	12.51	16.96	13.29	-1.04

Fig. 25: Maximum design alternatives (PPD) changes.

By calculating the total effect for all months of the year for each design variable, by aggregating it separately from the total effect for each element for a period of 12 months, with a total percentage of 1200%, as it is clear from Table (2) and Figure (26) as the following:

- The maximum positive effect of the design element was the pergola above the roof, with a total of PPD.P = 222.42.
- The least positive effect of the design element was thermal insulation with a total PPD.I = 62.42. It is worth noting that this does not diminish the importance of thermal insulation but confirms the lack of effect when changing its thickness from 3 cm in the standard design to 5 cm as was done in the simulation model.
- The Sky Light has the only negative effect among other design elements with a total PPD.SK = -153.53, and this confirms the need for good environmental treatment of the element when used in roofs in terms of glass quality, appropriate shading, and other climatic requirements.
- Preference for the design elements in terms of positive overall impact on the arrangement is as follows: Pergolas, cantilever, plantings, inclination, and thermal insulation.

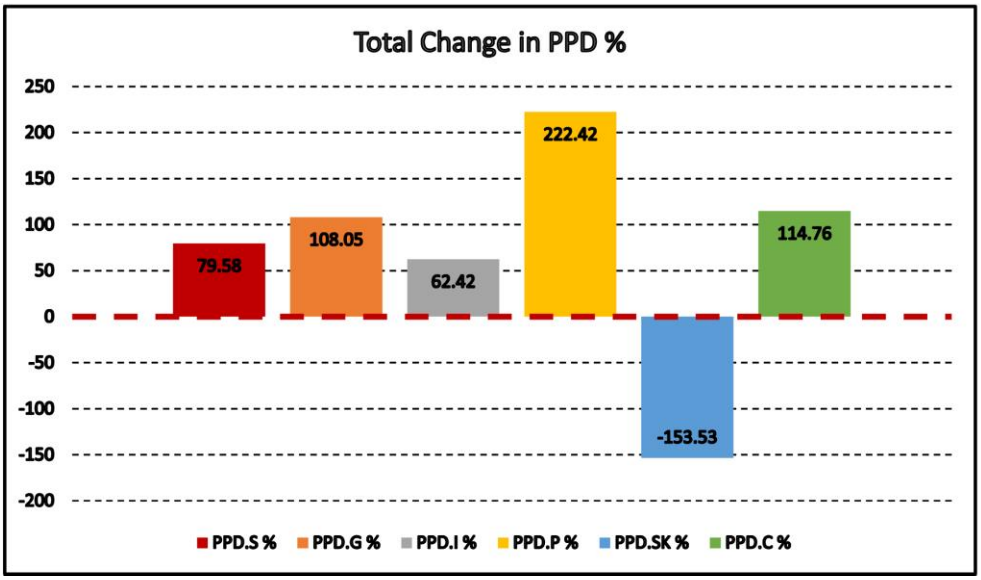


Fig. 26: The total impact of the design elements over the year.

Table 2: The total effect of alternatives to the design elements of the roof.

		Design element					
		Slope 5% (PPD.S)	**Cantilever 250 cm (PPD.C)**	**Insulation 50 mm (PPD.I)**	**Skylight 50 % (PPD.SK)**	**Pergola 50 % (PPD.P)**	**Green 80 % (PPD.G)**
PPD impact per month (%)	Jan.	0.24	6.1	1.09	-2.51	2.64	2.77
	Feb.	1.49	1.01	2.44	-9.11	3.04	0.94
	Mar.	0.39	8.11	2.93	-24.78	7.86	2.92
	Apr.	4.25	16.7	5.4	-36.43	16.04	8.32
	May	8.56	17.32	5.58	-22.83	27.68	15.31
	June	19.16	10.15	4.94	-13.38	29.99	17.48
	July	17.22	6.67	7.63	-6.91	32.03	17.37
	Aug.	14.74	4.9	7.35	-4.52	32.42	17.18
	Sep.	9.57	12.51	6.03	-13.03	26.85	12.43
	Oct.	1.98	16.96	7.57	-7.59	28.96	11.64
	Nov.	0.58	13.29	6.06	-8.93	12.31	1.39
	Dec.	1.4	1.04	5.4	-3.51	2.59	0.30
Total impact over the year		**79.58**	**114.76**	**62.42**	**-153.53**	**222.42**	**108.05**
Element priority		**4**	**2**	**5**	**--**	**1**	**3**

7. Computatıonal Mathematıcal Relatıonshıp for Roof Desıgn

Based on the previous simulation models and their results, the next part deals with deducing the approximate mathematical relationship through which the expected value of the change in the amount of thermal comfort zone can be predicted depending on the amount of change in the value of the design alternatives for the roof.

The top three design alternatives for the roof were selected from the previously studied alternatives to derive their mathematical equation as models from which the rest of the mathematical equations for all other design variables can be deduced. The mathematical and computational foundations were relied upon in deducing the relationship on each of the following:

- Simple Linear Regression equations.
- Pearson Linear Correlation Coefficient.

6.1. SIMPLE LINEAR REGRESSION EQUATIONS:

It is a method through which the value of a variable called the dependent variable can be predicted through the information of another variable, which is the independent one [7]. This simplified equation can be formulated through the following relationship:

Y = a + b X

Where:

Y = dependent value

X = independent value

a = Y-intercept “value of Y when X is zero”

b = slope “rate of predicted ↑/↓ for Y values for each unit increase in X”

Figure(27) shows the graphic representation of the mathematical relationship of the simple linear regression, as it is noted that this relationship does not depend mainly on an explicit linear relationship, but is deduced through a set of points resulting from the relationship between two variables that are not linked by an explicit straight line that collects all the points, which in this case, it is called the scattered plots, and therefore the relationship is deduced by defining the most suitable straight line as close as possible to all the scattered plots, where this line is called the best fit line [7,8], and the value of the dependent variable in this case is an estimated value symbolized by the symbol "Ŷ" with a value as close as possible to the actual value "Y", the relationship becomes:

$$\hat{Y} = a + b\,X,$$

where "Ŷ = predicted values of Y"

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$a = \frac{\sum Y - b\sum X}{n}$$

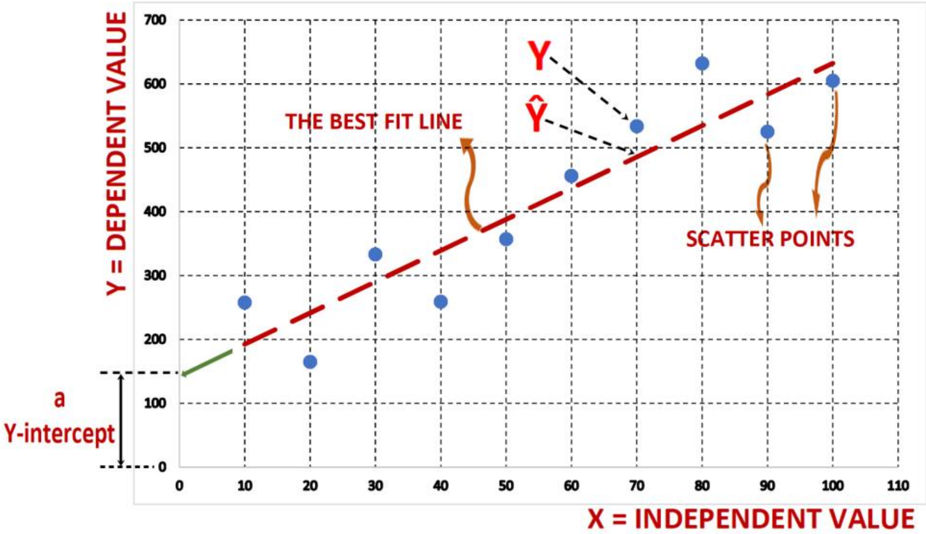


Fig. 27: The simple linear regression.

6.2. Pearson Lınear Correlatıon Coeffıcıent

It is a numeric measure used to determine the strength of the linear correlation between two variables, and it is calculated from the following equation [9]:

$$r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{(n\sum X^2 - (\sum X)^2)\,.\,(n\sum Y^2 - (\sum Y)^2)}}$$

Its value ranges between (+1) and (-1), where the correlation strength can be determined according to the value of r [9,10], as shown in Figure (28).

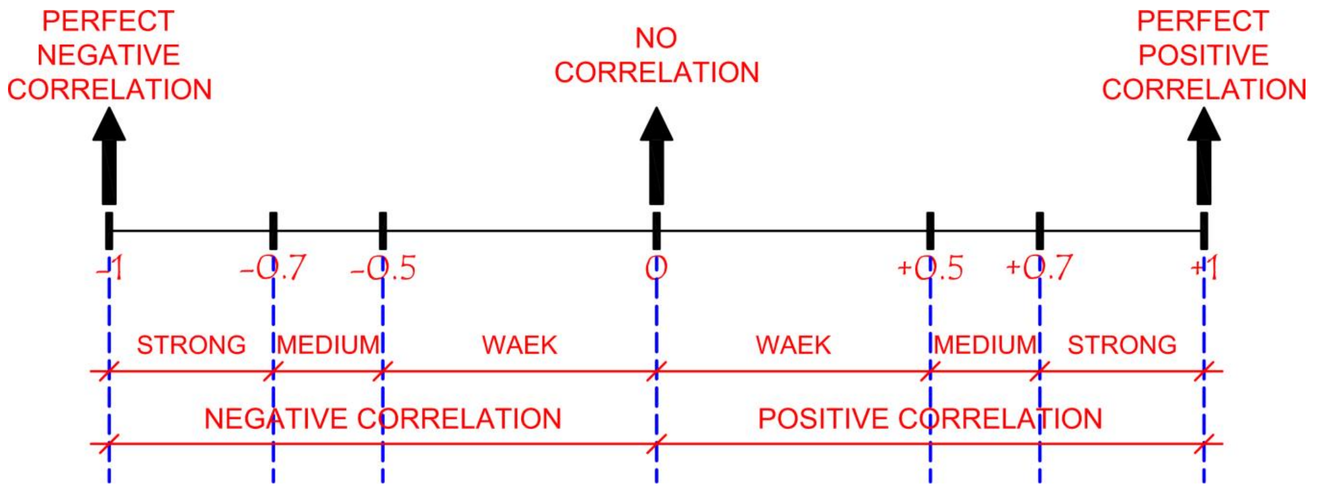


Fig. 28: Pearson Linear Correlation Coefficient.

6.3. Computatıonal Mathematıcal Relatıonshıps for Desıgnıng the Roof and Thermal Comfort Varıables

Based on the previous mathematical steps, it is possible to deduce the computational mathematical relationships for the impact of the three highest design alternatives for the roof on the thermal comfort rates of the last floor, which include: pergolas, cantilever, and plantings, where the roof variable was determined by the independent variable (X), and the percentage change in the thermal comfort zone by the dependent variable (Y). Tables: (3), (4), and (5) show the basic calculations to derive these mathematical equations.

Table 3: The mathematical calculations for the roof pergola equation, Source: The author.

	X	Y	XY	X^2	Y^2
	10	523.45	5234.50	100	273999.90
	20	483.84	9676.80	400	234101.15
	30	447.04	13411.20	900	199844.76
	40	412.40	16496.00	1600	170073.76
	50	376.03	18801.50	2500	141398.56
∑	**150**	**2242.76**	**63620**	**5500**	**1019418.13**
a	**558.436**	**Pergola equation**			
b	**-3.6628**	**PPD.P = 558.436 – (3.6628 x Pergola Area %)**			
r	**-1.00**	**PERFECT NEGATIVE CORRELATION**			

Table 4: The mathematical calculations for the roof cantilever equation.

	X	Y	XY	X^2	Y^2
	50	566.95	28347.5	2500	321432.30
	100	549.93	54993	10000	302423.00
	150	528.43	79264.5	22500	279238.26
	200	508.57	101714	40000	258643.44
	250	494.71	123677.5	62500	244737.98
∑	**750**	**2648.59**	**387996.5**	**137500**	**1406475.00**
a	**585.47**	**Cantilever equation**			
b	**-0.37168**	**PPD.C = 585.47 – (0.37168 x Roof Cantilever cm)**			
r	**-0.998**	**STRONG NEGATIVE CORRELATION**			

Table 5: The mathematical calculations for the green roof equation.

	X	Y	XY	X^2	Y^2
	10	578.72	5787.2	100	334916.84
	20	565.58	11311.6	400	319880.74
	40	538.32	21532.8	1600	289788.42
	60	513.66	30819.6	3600	263846.60
	80	488.50	39080	6400	238632.25
∑	**210**	**2684.78**	**108531.2**	**12100**	**1447064.84**

a	**591.115**	**Green equation** **PPD.G = 591.115 – (1.2895 x Green Area %)**
b	**-1.2895**	
r	**1.00**	**PERFECT NEGATIVE CORRELATION**

The previous equations have been applied and approximate re-calculations of the amount of change in the thermal comfort zone ($\hat{Y}$) and compared with the actual value of the change in the thermal comfort zone (Y) ranging between (+0.44%) and (-0.50%), meaning that the accuracy of the equation reaches approximately 99.50% of the actual calculations.

Table 6: The error estimating of the mathematical equations for the roof cantilever, pergola and green roof.

X	Y	$\hat{Y}$	Y - $\hat{Y}$	Change %
Pergola equation accuracy				
10	523.45	521.808	1.642	**0.31**
20	483.84	485.18	-1.34	**-0.28**
30	447.04	448.552	-1.512	**-0.34**
40	412.40	411.924	0.476	**0.12**
50	376.03	375.296	0.734	**0.20**
Cantilever equation accuracy				
50	566.95	566.886	0.064	**0.01**
100	549.93	548.302	1.628	**0.30**
150	528.43	529.718	-1.288	**-0.24**
200	508.57	511.134	-2.564	**-0.50**
250	494.71	492.55	2.16	**0.44**
Green equation accuracy				
10	578.72	578.22	0.50	**0.09**
20	565.58	565.325	0.255	**0.05**
40	538.32	539.535	-1.215	**-0.23**
60	513.66	513.745	-0.085	**-0.02**
80	488.50	487.955	0.545	**0.11**

8. Research results:

The research dealt with the effect of the direct relationship of the roof design on the thermal comfort zone for the spaces of the last floor of the administrative buildings in Egypt. The study found out several practical results that can be mentioned in the following points:

- The typical roof of the administrative buildings causes a decrease in the thermal comfort zone by up to 14.47% in summer.
- The roof causes an increase in the actual operative temperature by 1.3˚ C in the spaces of the last floor compared to the repeated ones.
- The design variables of the roof have a positive effect on the thermal comfort zone, according to the order of the effect rate from the highest to the least effective, as follows: pergolas, cantilever, plantings, thermal insulation and inclinations.
- The design variable of the roof has a negative effect on the thermal comfort zone, including the Sky Light.
- The average values for the percentage of comfort zone improvement through the roof design elements are as follows:
  - The roof inclination is 13% in summer - 2.25% in winter.
  - The roof cantilever is 10.31% in summer - 9.03% in winter.
  - The heat insulating layer is 4.88% in summer - 4.38% in winter.
  - Pergolas are 29.79% in summer - 10.49% in winter.
  - Plantings are 13.88% in summer - 3.5% in winter.
- The Sky Light reduces the thermal comfort zone by an average value of (-12.13%) in summer and (-13.26%) in winter.
- The maximum positive effect on the thermal comfort zone was through the pergolas above the roof with a total percentage effect of 222.42% throughout the year, totaling 1200%.
- Mathematical computational equations have been deduced for the effect of the highest design variables of the roof on the thermal comfort zone as follows:
  - **Pergola equation:** PPD.P = 558.436 – (3.6628 x Pergola Area %)
  - **Cantilever equation:** PPD.C = 585.47 – (0.37168 x Roof Cantilever cm)
  - **Green equation:** PPD.G = 591.115 – (1.2895 x Green Area %)
- The percentage of error in the previous equations does not exceed 0.50%.
- The accuracy of the previous computational equations reaches 99.50% compared to the simulation model.
- The strength of the correlation between the previous variables and the increase in the thermal comfort zone can be considered as Perfect Positive Correlation.

9. Conclusıons

The roof has a direct effect with a complete positive correlation force to raise the thermal efficiency and the amount of thermal zone suitable for the internal spaces, with its positive effect on the users of those spaces and the operational cost of the building. With the multiplicity of design alternatives governing the roof, they open the appropriate environmental design space for the spaces of the last floor to reduce the environmental negative effects of the outer shell of the building. Furthermore, the research sheds light on the importance of specific future studies of the economic return of the roof design alternatives as one of the most important elements governing the choice among alternatives and determining the most appropriate design alternative.

Abbreviations

PMV	Predicted Mean Vote
PPD	Predicted Percentage Dissatisfied, units of %
PPD.S	Predicted Percentage Dissatisfied due to roof Slope
PPD.G	Predicted Percentage Dissatisfied due to green area
PPD.I	Predicted Percentage Dissatisfied due to roof Insulation thickness
PPD.P	Predicted Percentage Dissatisfied due to roof Pergola
PPD.SK	Predicted Percentage Dissatisfied due to Skylight
PPD.C	Predicted Percentage Dissatisfied due to roof Cantilever
r	Pearson linear correlation coefficient

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السقف كمحدد تقييمي بيئي
العلاقة الحسابية لتأثير المتغيرات التصميمية على معدل الراحة الحرارية بمقياس Fanger

الملخص

تعتبر الأسقف من أحد أهم عناصر الغلاف الخارجي للمبنى، وله دور مباشر في تشكيل البيئة الحرارية لفراغات الدور الأخير ومدى تأثيره على الراحة الحرارية لمستخدمي تلك الفراغات، وبالرغم من ذلك فقد تم إهمال التصميم البيئي للأسقف إلا من بعض المعالجات النمطية التي تتدرج أغلبها في العزل الحراري والزرعات أعلى الأسقف في الوقت الذي تتوفر فيه العديد من البدائل التصميمية الأخرى للسقف ذات الأثر الإيجابي البيئي.

وتهدف الدراسة البحثية إلى تحديد تأثير المتغيرات التصميمية للأسقف على مجال الراحة الحرارية طبقا لمقياس *Fanger*، وذلك لتحديد العلاقة التصميمية الرياضية الحسابية لتأثير المتغير التصميمي على مقدار التغير في نسبة مجال الراحة الحرارية التابع لمقياس *Fanger*.

هذا وتشير نتائج الدراسة البحثية إلى أن تصميم البرجولات على السقف من أكبر العناصر ذات التأثير الإيجابي على لاراحة الحرارية يليها بروز السقف، الزراعات، ميول السقف ثم العزل الحراري. وبناء على نموذج المحاكاة والأسس الرياضية الخاصة بـ *Simple Linear Regression* تم التوصل إلى المعادلات الرياضية الحسابية لأعلى المتغيرات التصميمية تأثيرا إعتمادا على الشكل الرياضي الحسابي *"Change in PPD Fanger = Constant 1 + Constant 2 x Roof Variable"*، حيث توصل البحث إلى تحديد قيم الثوابت في المعادلة النمطية لأعلى ثلاث متغيرات تصميمية للأسقف.